STANDARD GEAR BOOK

Working Formulas and Tables
In Gear Design

BY
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PREFACE

The purpose of this book is to supply the facts behind the important advances in commercial gear-production practices during the past quarter century, to reiterate the basic principles upon which the operation of the modern gear-generating machines and the art of gear designing are founded, and to show how and why transmission efficiencies of 99 per cent plus have been attained. The book aims to meet the needs of the day, to be of assistance to the machine builder as well as to the gear designer.

Twenty-five years ago Charles H. Logue wrote the first "American Machinist Gear Book" with the expressed idea of filling a then pressing need, making the book, as he expressed it, one for the "man behind the machine" and a book for the student of gearing. The volume proved all its author hoped, becoming a recognized standard reference on gearing. It was revised in 1915 and again in 1922, each revision recording all noteworthy progress and keeping the volume well abreast of developments of demonstrated merit. It remains a valuable reference book, a record of achievements and upbuilding in one of the most important branches of the mechanic arts.

A fourth edition of the work could well have been published, now that the new economic order has made it imperative for industry to cut all reducible costs, among the more outstanding of which are those concerned with the transmission and utilization of mechanical power, but a careful survey of the industrial field showed that the tempo of the times has changed; that the present demand is for a work of somewhat different character; a practical treatise on gearing that is presented simply, clearly, and in a manner that is helpful to the builder of gear-production equipment, the gear designer, and the commercial user of gears. The "Standard Gear Book" has been written to meet this need.

Much of the historical background of the earlier book by Logue and of the revisions by Logue and Trautschold is lacking

in this newer work, sacrificed to heed the numerous suggestions and helpful advice tendered by present leaders in the gear-manufacturing industry. The keen interest shown by these many cooperators has been an inspiration in the preparation of this record of modern practice in gear design and production.

REGINALD TRAUTSCHOLD.

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STANDARD GEAR BOOK

SECTION I

TOOTH FORMS

The trend toward higher efficiency and greater simplicity that marks all real progress in the mechanic arts is well exemplified in the refinements and developments characterizing the advance that has been made in the production of modern commercial toothed gearing. Mechanical efficiencies of such high order as to leave little in the way of attainable improvement have been secured with standard gear assemblages, and marked simplicity in commercial gear production has been attained through the general adoption of the generating method of gear-tooth formation and reproduction.

The almost universal application of the principles of generation to the machining of gear teeth, the development chiefly responsible for the betterments in efficiencies developed by modern gear assemblages, has brought about such decided changes in commercial gear production, in fact, that the importance of correct gear-tooth design is apt to be overlooked. The skill of the gear designer seems to have been usurped by the gear-generating machine, with the result that the design of the gear tooth, instead of serving as a model of the form desired has come to be looked upon as simply a chart of the tooth form produced. In a way this is quite natural, the skill of the gear designer being evidenced in the refinements incorporated in the gear-tooth generating machine, but this really adds to the importance of a thorough understanding of those basic laws that govern the principles of gear transmission. Familiarity with all standard forms of gear teeth is essential, moreover, for a proper appraisal of the various accepted modifications in gear-tooth profile curvature found to be expedient in the manufacture of commercial gearing.

Many forms of gear teeth and classes of gear-tooth contours could be used. Many of these would lend themselves readily to reproduction by generating methods, but, as the essential purpose of gear teeth is to transmit motion from one shaft to another, usually at as uniform a speed as possible (*conjugate gear-tooth action*), and because interchangeability of gears in a drive assemblage is a desirable if not an essential requirement, the number of accepted forms is quite limited in commercial practice. Definite demands are imposed, furthermore, and these have to be appreciated in order to understand the reasons for, and the respective advantages of, the various recognized systems of standard gearing.

The profile curvature of the teeth of mating gears could be pretty freely selected were it not for the important fact that the line of gear action, or path of tooth contact, should advisably be symmetrical in respect to the pitch point of the gears: this, in order that the gears in a drive assemblage may be interchangeable. The choice of tooth contour, consequently, is customarily limited to one of the more common of the regular geometric curves.

Among the first of these curves to be selected for gear-tooth profiles was that of the cycloidal form; gears with teeth of such contour having inherently decided merits, or advantages, that would seem to make them highly desirable for a satisfactory system of commercial gearing. The engagement of meshing cycloidal teeth, for example, is affected by pure rolling action; and cycloidal gears, when generated by describing circles of the same size, are interchangeable. Practical difficulties in accurately producing these gears, however, have led to their being generally displaced in the field of commercial gearing by gears with teeth of involute profile; *i.e.*, the *involute system of gearing*.

This transition, incidentally, has not been so radical as might be supposed, for the involutes are, in reality, limiting forms of cycloidal curves, being generated by a describing circle of infinite radius. For a clear conception of the laws of tooth gearing, consequently, familiarity with these two classes of regular geometric curves, the cycloidals and the involutes, is necessary. Other forms of regular curves are used occasionally for gear-tooth profiles in specific systems of gearing, but these can be taken up to better advantage when presenting expositions of their approved application.

CYCLOIDAL CURVES

So far as the cycloidal curves are concerned, the profile of the tooth contour is traced by a point on the circumference of a describing, or generating, circle rolled above and below the pitch circle of the gear or the pitch line of a rack. The exterior cycloid that traced as the describing circle rolls on the outside of the pitch circle is an epicycloid, and the interior cycloid is termed a hypocycloid. The full cycloids are not usually developed in commercial gearing but simply that section serving as the active profile of the gear teeth, *i.e.*, the section between the base and outer circles of the gear. Ordinarily, the diameter of the describing circle is made equal to the pitch radius of a 15-tooth pinion of the gear generated.

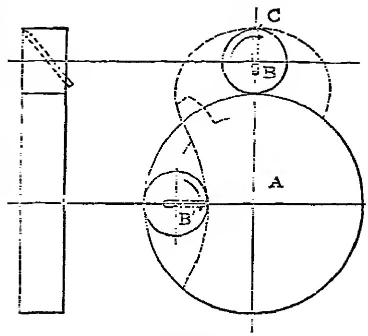


FIG. 1.—Generating the cycloidal tooth.

In this cycloidal system of gearing, the angularity at which the teeth engage is not constant but varies from zero at the point of cycloidal origin on the pitch line to about 22 deg. in the case of a standard gear at the end of its engagement with a standard cycloidal rack tooth. Where the full cycloids are represented in the active profiles of the gears, as in cycloidal rotors for blowers, the pressure angle becomes so great that the mating rotors have to depend upon supplementary gears for their rotation. The chief disadvantage of the cycloidal system of gearing is, however, not the limitation imposed by excessive angularity of tooth engagement but lies rather in the practical difficulty of producing the teeth.

In Fig. 1, the circumference of the wheel *A* represents the pitch circle of a cycloidal gear, the generating circle tracing the epicycloid at *B* and the hypocycloid at *B'*, the tracing point *C* being on the pitch circle at the point of origin of the respective curves. The rotation of the describing circle is in the same direction when generating the epicycloid and hypocycloid profiles; in one case rolling on the outside of the pitch circle, and in the other on the inside of the same circle.

THE INVOLUTE

In the involute system of gearing, the path of action along which the pressure on the engaging teeth of meshing gears is transmitted is a straight line passing through the pitch point *P* (Fig. 2) and tangent to the so-called "base circle," upon which the involute tooth profiles originate. The inclination of this line, in respect to the plane tangent to the pitch circle at the pitch point, measures the pressure angle (angle of obliquity) of the gears, and this inclination is constant throughout the engagement of the meshing teeth.

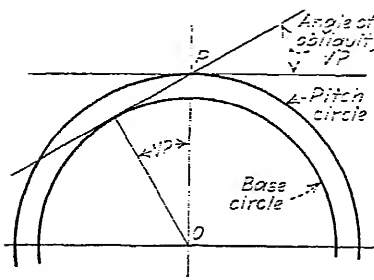


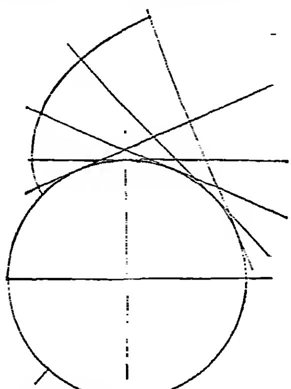
FIG. 2.—Angle of tooth obliquity (pressure angle), involute system.

The involute contour of the gear teeth is described by the end of the generating line rolling on the base circle and may be likened to the curve traced by the end of a string unwinding from the circumference of the same circle. The functioning, or active, curvature of the tooth profile does not extend below the base circle, on which the involute contour of the gear teeth originates and on which the generating straight line rolls. In the case of the involute rack, the active tooth profile becomes a straight line normal to the line of action, or pressure: an involute curve of infinite radius.

The behavior of a pair of engaging involute gears closely resembles that which would occur were two pulleys of corresponding diameters connected by an endless crossbelt, the point of belt-crossing depicting the contact, or pitch, point on the pitch circles of the gears, and the straight sections of the crossbelt the lines of action for either direction of rotation. This comparison of the action of a pair of engaging involute gears is most apt, incidentally, for it also serves to illustrate a distinctive characteristic of the involute system of gearing, in respect to flexibility in the center spacing (separation of axes) of engaging gears.

It is quite obvious that, if the distance separating two pulleys connected by an endless crossbelt should be increased or decreased, the obliquity of the straight runs of belt, corre-

sponding to the lines of action of comparable engaging involute gears, would change, and the location of the point at which the belt sections cross, corresponding to the contact point of the gears on their pitch circles, *i.e.*, the pitch point, would shift. The speed ratio of the transmission, on the other hand, would remain the same. Similarly, if the center distance of engaging involute gears should be altered slightly, the obliquity of the lines of action determining the pressure angle of the gears and the diameters of the respective pitch circles are modified propor-



Base Circle

FIG. 3.—Involute curve generated by straight line rolling on base circle.

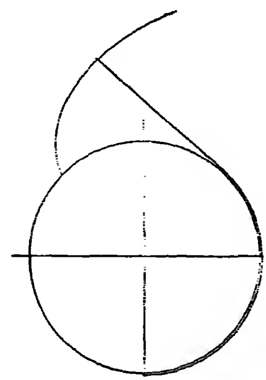
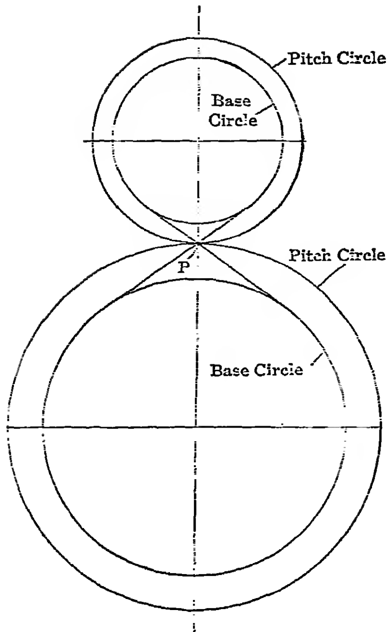


FIG. 4.—Involute curve traced by unwinding string.

tionally, and there is no change in the speed ratio of the gear assemblage.

This peculiarity of the involute system of gearing permits some little latitude in the center adjustment of mating gears, also of angular adjustment, without disturbing the smooth-rolling action of the engaging teeth, for, while there is always a definite relationship between the pitch diameter and the pressure angle of involute gears, the exact pitch diameters of engaging gears are established only when the gears are placed in mesh. Expressed somewhat differently, the pitch diameters of involute gears in engagement are directly proportional to the diameters of their respective base circles, and, no matter what may be the distance separating the centers of mating gears of true involute form or the obliquity of their pressure angle, engagement of teeth occurs only along the tangent path of action and the relative rates of gear rotation remain constant.



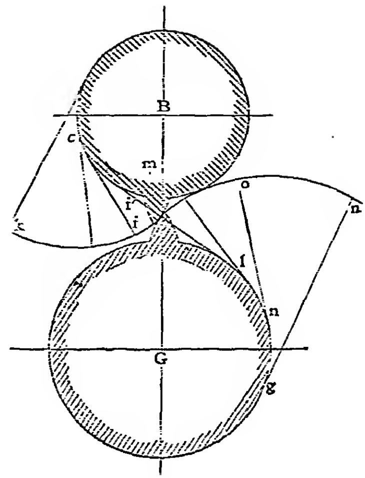


FIG. 5.—Action of involute teeth illustrated by crossed belt connecting base circles.

FIG. 6.—Action of the involute tooth.

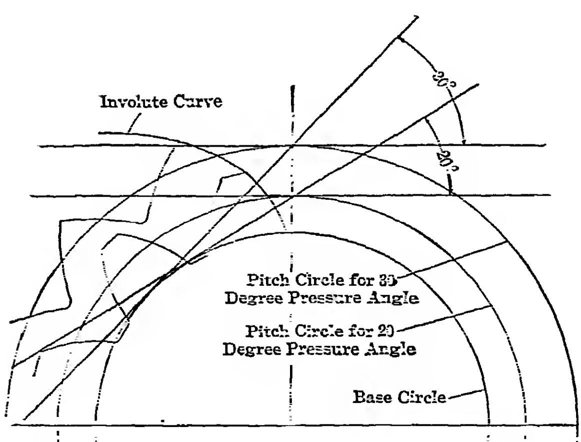


FIG. 7.—Proper action maintained as gear axes are separated.

The illustration showing a method of laying out an involute tooth profile also portrays the generation of the contour curve by the end of a line rolling on the base circle of the gear. This base circle is divided into any number of equal spaces, such as a–1′, 1′–2′, 2′–3′, 3′–4′, 4′–5′, and 5′–6′ (Fig. 8). Radial lines are drawn from points a, 1′, 2′, 3′, 4′, 5′, and 6′, as shown, and a corresponding series of tangents to the base circle at the respective points.

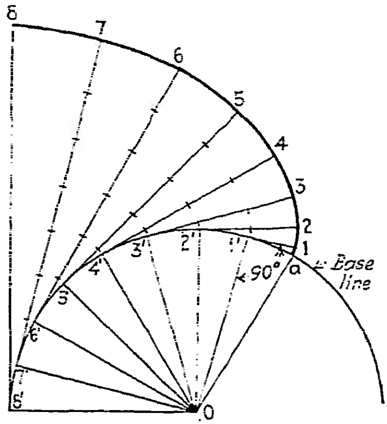


FIG. 8.—Laying out the involute curve.

With point a taken as the origin of the involute, the points 1, 2, 3, 4, 5, and 6 on the profile curve are located by making the tangent $1' - 1$ equal in length to the arc $1' - a$; tangent $2' - 2$ equal to arc $2' - a$; $3' - 3$ equal to $3' - a$; etc. This procedure establishes definite points on the involute curve by locating successive positions of the generating line. The involute curve does not extend within the base circle on which it originates, and a small clearance is provided for the purpose of avoiding interference with the teeth of a mating gear.

INTERFERENCE IN INVOLUTE GEARING

The origin of the involute curve being on the base circle, and conjugate gear-tooth action possible only along the path of action on the line of pressure, the active involute tooth profile is a small section of the involute curve in proximity to its origin. The permissible working (active) profile of the involute gear tooth is limited, in fact, not only by the tooth proportions of its gear but also by the proportions of its mating gear or pinion.

This is graphically depicted in the diagrammatic illustration showing a 5-pitch, 15-tooth pinion mating with a 5-pitch, 48-tooth gear and demonstrating conditions as they pertain with 14½- and with 20-deg. paths of action. It will be noted that points F and F' on the 14½- and 20-deg. pressure lines respectively fall within and on the addendum (outer) circle of the gear. Under the latter condition, the gear and pinion will

mate without interference, but with the proportions of the gear combination or the obliquity of the line of pressure such as to cause this important point to fall inside the addendum circle of the gear, interference will occur, necessitating the modification of the tooth forms by undercutting the pinion teeth or rounding off the corners of the gear teeth in order to provide essential clearance. This interference may occur by reason of

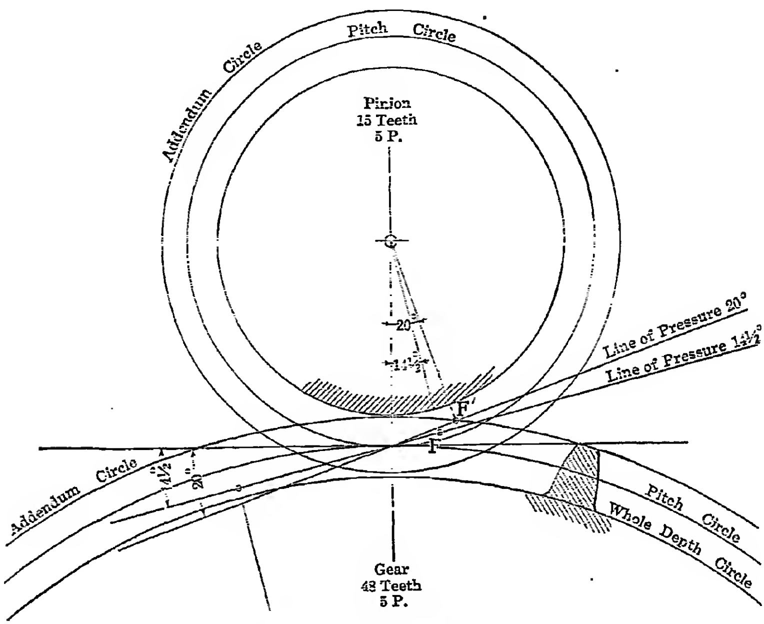


FIG. 9.—Interference of involute spur gears.

an excessive obliquity to the line of pressure, depth of teeth, or speed ratio of the gear assemblage.

The importance of these dominant considerations is well exemplified on the chart depicting the amount of interference (distance from the point where interference commences to the addendum circle of the gear) occurring between a 1-pitch, 12-tooth involute pinion and mating gears of from 10 to 135 teeth. Curve *A* depicts conditions when 14½-deg. standard involute teeth are employed; curve *B*, when 14½-deg. stubbed teeth are used; curve *C*, for 20-deg. standard involute teeth; and curve *D*,

interference when employing 20-deg. stubbed involute teeth. With $14\frac{1}{2}$-deg. standard teeth, interference is not only evident for all speed ratios in excess of unity, but it is decidedly marked, while with 20-deg. stubbed teeth interference commences only at a considerably higher speed ratio and is much less serious.

The amounts of interference developed at these particular speed ratios with gear combinations of other than one diametral pitch are obtained by dividing the values charted on the diagram by the diametral pitch in question, but, as the number of teeth on the pinion member is an influencing consideration, an equation

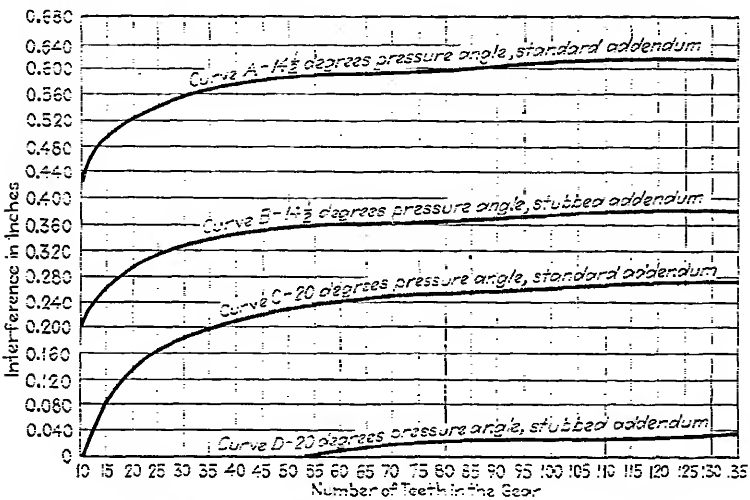


FIG. 10.—Involute interference between 12-tooth, 1 diametral pitch, pinion and mating gears with from 10 to 135 teeth.

of universal application, giving the limiting maximum outside (addendum) radii of mating gear and pinion that will avoid interference, is found much more convenient [see formulas (1) and (1')].

In Fig. 11, let

OR = radius of maximum outer circle of gear without interference.

or = radius of maximum outer circle of pinion without interference.

CD = center distance.

PR = pitch radius of gear.

pr = pitch radius of pinion.

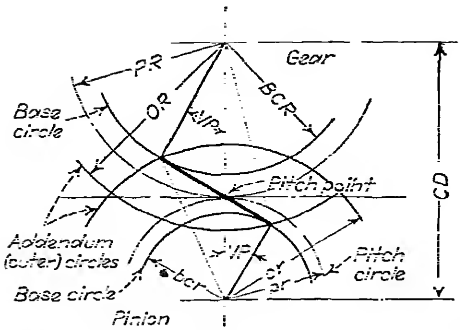


FIG. 11.—Limiting outside gear diameter to avoid interference.

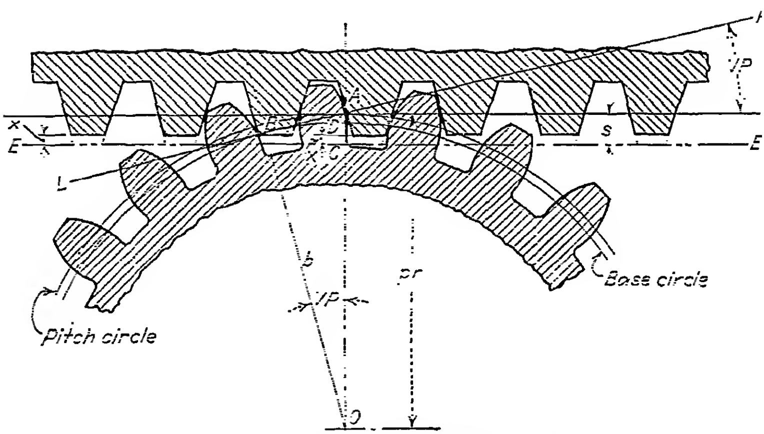


FIG. 12.—Interference of gear and rack.

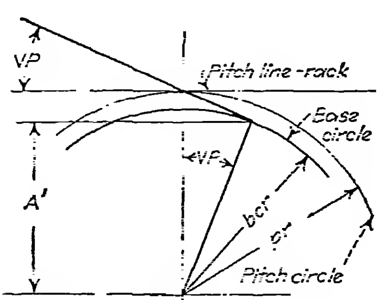


FIG. 13.—Limiting proximity of pinion to involute rack.

BCR = base-circle radius of gear.
bcr = base-circle radius of pinion.
VP = pressure angle.

Then

$$OR = \sqrt{(BCR)^2 + (CD \times \sin\ VP)^2} \tag{1}$$

and

$$or = \sqrt{(bcr)^2 + (CD \times \sin\ VP)^2} \tag{1'}$$

The limiting situation occurs for externally meshing spur gears, obviously, when there is interference between an involute pinion and a mating rack, *i.e.*, when the commencement of the path of action on the line of pressure (point B, Fig. 12) falls inside the rack addendum line. Should this occur, as it will under certain conditions of pinion engagement and obliquity of line of pressure, the customary remedy lies in shortening the rack teeth by the amount of the interference DC.

In the diagrammatic depiction showing the limiting proximity of the pinion to the involute rack that will avoid interference without trimming the rack teeth (Fig. 13), A' is the minimum permissible distance between the face of the rack and the center of the mating pinion, the equation for which is

$$A' = bcr \times \cos VP = pr \times \cos^2 VP \tag{2}$$

where A' = minimum distance between rack face and pinion center.
pr = radius of pitch circle of pinion.
bcr = base-circle radius of pinion.
VP = pressure angle.

PRESSURE COMPONENTS

Since the pressure (T) on involute gear teeth is always normal to the tooth profile and exerted along the path of action, or line of pressure, it is constant as regards both direction and effectiveness on successive teeth. The tangential driving force T_1 and the pressure components T_2 tending to separate the axes of mating gears are definite proportions of the total tooth pressure and both are influenced by the obliquity of the line of pressure.

T = total tooth pressure.
T_1 = tangential force at pitch circle of involute gear.
T_2 = force tending to separate axes of mating gears.
VP = pressure angle (obliquity of path of action).

$$T_1 = T \times \cos VP \tag{3}$$
$$T_2 = T_1 \times \tan VP \tag{3'}$$

For a $14\frac{1}{2}$-deg. pressure angle, the tangential driving force is equal to the total tooth pressure multiplied by 0.9681 and for a 20-deg. pressure angle to the total tooth pressure multiplied by 0.9397. These are the two obliquities of path of action most frequently employed in commercial gearing, and it will be noted

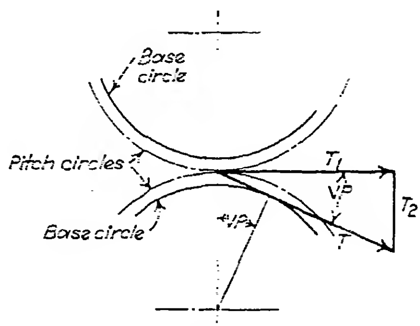


FIG. 14.—Pressure components.

that, while the intensity of the tangential driving force decreases slightly with the increase in obliquity of pressure application, the component force tending to separate the axes of the gears increases correspondingly. However, the total load on the bearings does not change but remains constant and equal to the total tooth pressure plus the load imposed by the weight of the gearing. This is an important consideration in the design of gear assemblages, for the mistake is frequently made of taking the load on the bearings as entailing, in addition to the weight of the gears, simply that component of the total tooth pressure tending to force the bearings apart.

SPUR-GEAR TOOTH PARTS

The terminology of spur gearing is employed quite universally for all forms and systems of gearing and, while certain specific terms are required for the specifications of bevel, helical, worm, spiral, and other special varieties of gear transmissions, familiarity with the technical names of spur-gear tooth parts and important gear dimensions is essential for a comprehensive grasp of even the elements of gearing and entailed gear problems.

Addendum.—The addendum of a gear tooth is the height of that part of the tooth extending outside the pitch circle.

Dedendum.—The dedendum is the depth of the tooth inside the pitch circle.

Active Profile.—The active profile of a gear tooth (involute) is that part of its profile which comes in contact with the profile of the mating tooth on the line, or path, of action; *i.e.*, the section of tooth profile that actually functions in transmitting motion.

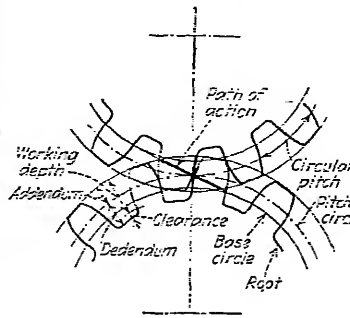


Fig. 15.—Gear-tooth nomenclature.

Angle of Action.—The angle of action is the angle through which the point of contact travels on the line, or path of action during the engagement of mating gear teeth.

Base Circle.—The base circle of an involute gear is the circle on which the involute tooth profiles originate.

Center Distance.—The center distance of mating gears is the distance between their respective centers.

Circular Pitch.—The circular pitch of a gear is the length of the arc on its pitch circle corresponding to one complete tooth interval.

Clearance.—The clearance is the space between the top of the tooth of one gear and the bottom of the tooth space of its mating gear.

Diametral Pitch.—The diametral pitch of a gear is the ratio of the number of its teeth to its pitch diameter. It is the number of teeth per inch of pitch diameter.

Interference.—Interference is the condition that permits contact of teeth elsewhere than on the line, or path, of action.

Line, or Path, of Action.—The path of action is the line along which the effective contact between mating gear teeth takes place, the result of which is the transmission of uniform motion from one gear to the other.

Module.—The module of a gear is the ratio of its pitch diameter with its number of teeth. It is the reciprocal of the diametral pitch.

Normal Pitch.—The normal pitch of an involute gear is the distance between two successive tooth profiles of similar curva-

ture. It is equal to the circumference of the base circle divided by the number of teeth on the gear.

Pitch Circle.—The pitch circle of a gear is the circle corresponding to a smooth disk that would transmit the same relative motion by friction. It is tangent to the pitch circle of a mating gear on the line of action; *i.e.*, the line of action passes through the tangent point of the pitch circles of mating gears.

Pressure Angle.—The pressure angle of involute gears is the angle between the line of action and a normal to the common center line of mating gears.

Whole-tooth Depth.—The whole-tooth depth is the radial distance between the circle of the gear enveloping the tops of the teeth and that which bounds the bottoms of the tooth spaces. It is equal to the sum of the addendum and dedendum dimensions of the gear teeth.

Working Depth.—The working depth of gear teeth is the depth that the teeth of one gear extend into the tooth spaces of its mating gears. It is equal to the sum of the addenda dimensions of the mating gears, *i.e.*, to the whole tooth depth minus the clearance.

PRODUCTION OF GEARS

The evolution through which the production of commercial gearing has passed has played an important part, quite naturally, in gear design, and various modifications in gear-tooth profiles have resulted. At first, practically all gears were cast, the cut tooth gear being in reality a relatively modern achievement. These early gears were made from forms carefully shaped by the patternmakers to secure as close an approximation of conjugate gear-tooth action as possible, at a time when both the cycloidal and involute forms of gear teeth were employed commercially.

The development of the so-termed *standard* 14½*-deg. involute system* marked the first real advancement in gear design, and this is in fact a composite system in which the profiles of the gear teeth are only of true involute contour on the section of the profile in proximity to the pitch circle, the upper and lower sections of the teeth being of cycloidal curvature.

This system, adopted by the American Gear Manufacturers' Association and certain other standardization organizations, is a fully interchangeable one and is employed generally in form-milling gear production. The obliquity of 14½ deg. was origi-

nally selected as the pressure angle of the system, for the reason that the natural sine of a 14½-deg. angle is so close to 0.25 (0.25038) that for all practical purposes it may be taken as the convenient common fraction, ¼, by the gear designer, pattern-maker, and machinist.

The system is based upon a pinion of 12 teeth, and the limitations this imposes in cut-gear production is the underlying reason for the cycloidal curvature of the upper and lower portions of the gear-tooth profiles. A pinion of only 12 teeth of full involute form, with a pressure angle of 14½ deg., has teeth so seriously undercut that it was deemed expedient to modify the

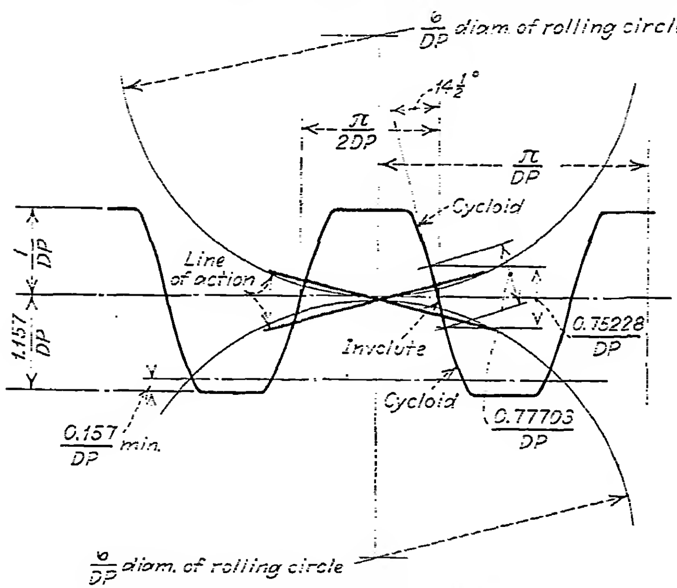


FIG. 16.—Basic rack for 14½-deg. composite system.

profile of the lower portions of the teeth by making them of cycloidal, instead of involute, curvature, the basic-pinion teeth of such form being bound below the base circle by radial lines. In order to maintain an interchangeable system, the profiles of the upper sections of the gear teeth are, naturally, modified in like manner.

FORM MILLING OF GEAR TEETH

The profiles of the teeth of the basic rack of this composite system are also of mixed curvature, flat in proximity to the

pitch line, the true form of the full involute rack tooth, and of cycloidal curvature on upper and lower sections. These curved portions of the rack-tooth profiles are difficult to reproduce accurately, but a close approximation is secured by making the curved sections arcs of a circle having a radius equal to $3\frac{3}{4}$ times the module of the rack, the reciprocal of the diametral pitch employed. This is the basic rack of the composite, or standard 14½-deg. system of gearing used in practice, conforming to which the sets of standard involute milling cutters are proportioned.

These so-called involute cutters were formerly made up in sets of eight for the form milling of gears with from 12 teeth to rack form, but in order to secure somewhat greater accuracy, the modern set now consists of 15 cutters, as follows.

Table 1.—Range of Standard (14½-deg.) Involute Milling Cutters

Number	Range
1	135 teeth to a rack
1½	80–134
2	55– 79
2½	42– 54
3	35– 41
3½	30– 34
4	26– 29
4½	23– 25
5	21– 22
5½	19– 20
6	17– 18
6½	15– 16
7	14
7½	13
8	12

Each cutter is proportioned for the accurate production only of gears having the minimum number of teeth in its specified range, and for the production of gears with any other number of teeth the skilled machinist depends upon a certain amount of "shake."

A delicate adjustment of milling cutter and gear blank is entailed if form milling of gear teeth is to be performed with reasonable accuracy, as well as the use of precision templets

for checking two of the critical tooth dimensions. The first of these measurements is the *chordal thickness* of the teeth on the pitch circle, and the other is the location of the pitch line on the *sides of the teeth*. Customarily, these dimensions are referred to as thickness and addendum (corrected) measurements, values for which are given in Table 2 for diametral pitches of from 1 to 24; and in Table 3 for ⅝- to 2-in. circular pitches.

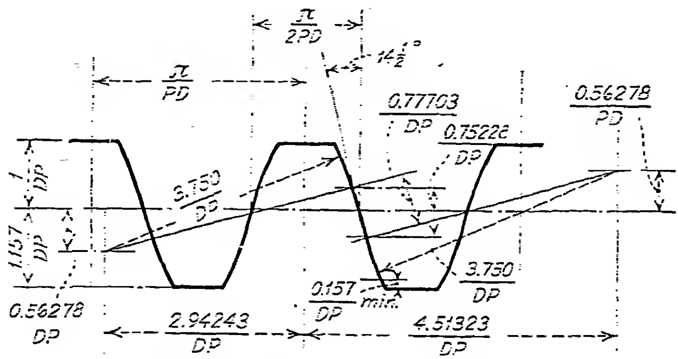


FIG. 17.—Approximation of basic rack for 14½-deg. composite system.

The pitch diameters of gears of any other circular pitch are obtained by multiplying the value given in Table 6 for a gear of the same number of teeth by the circular pitch in question. The pitch diameters of gears of diametral pitch are obtained by dividing the number of teeth by the diametral pitch.

GENERATED GEAR-TOOTH SYSTEMS

While the form milling of gear teeth is still quite extensively employed, the demand for cheaper and more rapid methods of gear production, brought about by the unprecedented needs of industries making use of pairs and trains of gears in large quantities, has been responsible for the development of highly efficient gear-generating machines and the general introduction of systems of gearing with tooth profiles and proportions that depart considerably from those of the standard 14½-deg. composite system. The main difference in most of these newer systems of generated tooth gearing lies in the fact that the profiles of the gear teeth are of more extended involute curvature than are the contours of gear teeth in the standard 14½-deg. composite system.

Number of teeth	1DP Thickness, in.	1DP Addendum, in.	1½DP Thickness, in.	1½DP Addendum, in.	2DP Thickness, in.	2DP Addendum, in.	2½DP Thickness, in.	2½DP Addendum, in.
8	1.5607	1.0769	1.0405	0.7179	0.7804	0.5385	0.6243	0.4308
9	1.5628	1.0684	1.0419	.7123	.7814	.5342	.6251	.4273
10	1.5643	1.0616	1.0429	.7077	.7821	.5308	.6257	.4246
11	1.5654	1.0559	1.0436	.7039	.7827	.5279	.6261	.4224
12	1.5663	1.0514	1.0442	.7009	.7831	.5257	.6265	.4206
14	1.5675	1.0440	1.0450	.6960	.7837	.5220	.6270	.4176
17	1.5686	1.0362	1.0457	.6908	.7843	.5181	.6274	.4145
21	1.5694	1.0294	1.0463	.6863	.7847	.5147	.6277	.4118
26	1.5698	1.0237	1.0465	.6825	.7849	.5118	.6279	.4095
35	1.5702	1.0176	1.0468	.6784	.7851	.5088	.6281	.4070
55	1.5706	1.0112	1.0471	.6741	.7853	.5056	.6282	.4045
135	1.5707	1.0047	1.0471	.6698	.7853	.5023	.6283	.4019

Number of teeth	3DP Thickness, in.	3DP Addendum, in.	3½DP Thickness, in.	3½DP Addendum, in.	4DP Thickness, in.	4DP Addendum, in.	5DP Thickness, in.	5DP Addendum, in.
8	0.5202	0.3589	0.4459	0.3077	0.3902	0.2692	0.3121	0.2154
9	.5209	.3561	.4465	.3052	.3907	.2671	.3126	.2137
10	.5214	.3538	.4469	.3033	.3911	.2654	.3129	.2123
11	.5218	.3519	.4473	.3017	.3913	.2640	.3131	.2112
12	.5221	.3505	.4475	.3004	.3916	.2628	.3133	.2103
14	.5225	.3480	.4479	.2983	.3919	.2610	.3135	.2088
17	.5228	.3454	.4482	.2961	.3921	.2590	.3137	.2072
21	.5231	.3431	.4485	.2941	.3923	.2573	.3139	.2059
26	.5233	.3412	.4485	.2925	.3925	.2559	.3140	.2047
35	.5234	.3392	.4486	.2907	.3926	.2544	.3140	.2035
55	.5235	.3371	.4487	.2889	.3927	.2528	.3141	.2022
135	.5236	.3349	.4488	.2871	.3927	.2512	.3141	.2009

Number of teeth	6DP Thickness, in.	6DP Addendum, in.	7DP Thickness, in.	7DP Addendum, in.	8DP Thickness, in.	8DP Addendum, in.	9DP Thickness, in.	9DP Addendum, in.
8	0.2601	0.1795	0.2230	0.1538	0.1951	0.1346	0.1734	0.1197
9	.2605	.1781	.2233	.1526	.1954	.1336	.1736	.1187
10	.2607	.1769	.2235	.1517	.1955	.1327	.1738	.1180
11	.2609	.1760	.2236	.1508	.1957	.1320	.1739	.1173
12	.2610	.1752	.2238	.1502	.1958	.1314	.1740	.1168
14	.2612	.1740	.2239	.1491	.1959	.1305	.1742	.1160
17	.2614	.1727	.2241	.1480	.1961	.1295	.1743	.1151
21	.2616	.1716	.2242	.1471	.1962	.1287	.1744	.1144
26	.2616	.1706	.2243	.1462	.1962	.1280	.1744	.1137
35	.2617	.1696	.2243	.1454	.1963	.1272	.1745	.1131
55	.2618	.1685	.2244	.1445	.1963	.1264	.1745	.1124
135	.2618	.1675	.2244	.1435	.1963	.1256	.1745	.1116

TABLE 2.—CHORDAL THICKNESSES AND (CORRECTED) ADDENDA OF GEAR-TEETH DIAMETRAL PITCH

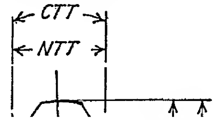


TABLE 2.—CHORDAL THICKNESSES AND (CORRECTED) ADDENDA OF GEAR-TEETH DIAMETRAL PITCH.—(*Concluded*)

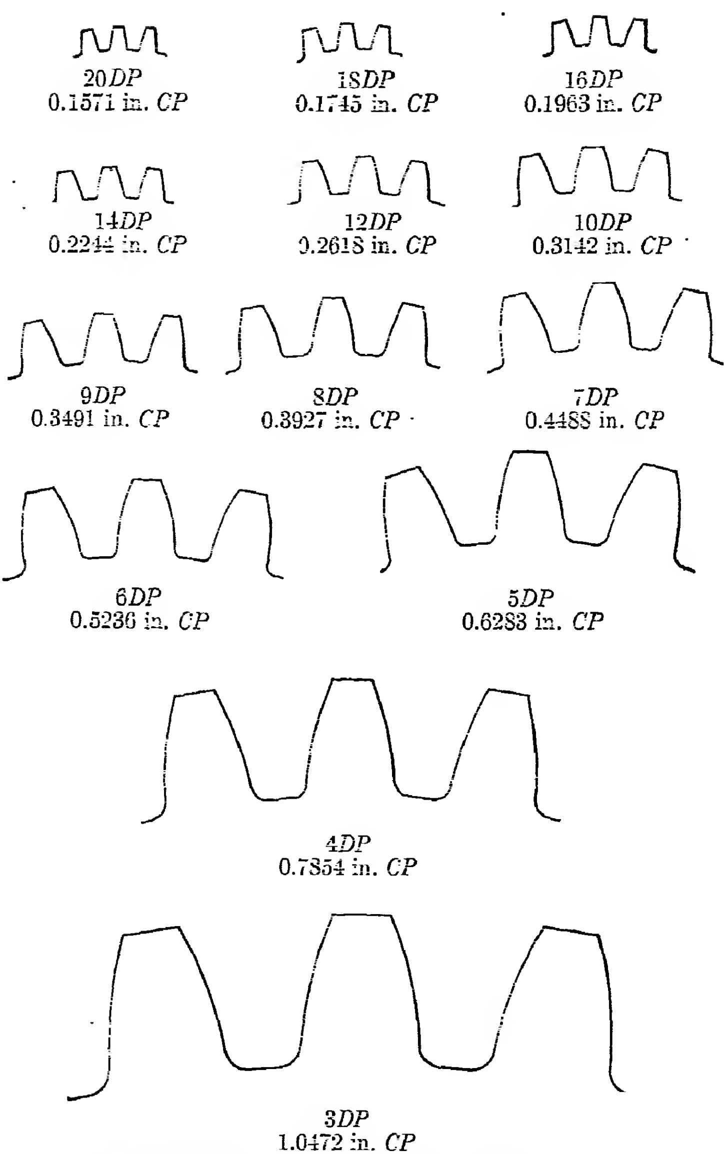
Number of teeth	10*DP*		11*DP*		12*DP*		13*DP*	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.

TABLE 3.—CHORDAL THICKNESSES AND CORRECTED ADDENDA OF GEAR-TEETH CIRCULAR PITCH

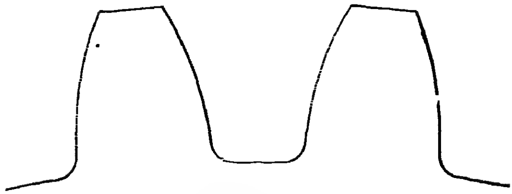
Number of teeth	5/8-in. *CP*		3/4-in. *CP*		7/8-in. *CP*		1-in. *CP*	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.
8	0.3105	0.2142	0.3725	0.2570	0.4347	0.2997	0.4968	0.3426
9	.3109	.2125	.3730	.2550	.4353	.2976	.4974	.3400
10	.3112	.2112	.3734	.2534	.4357	.2957	.4978	.3379
11	.3114	.2100	.3737	.2520	.4360	.2941	.4982	.3360
12	.3116	.2091	.3739	.2510	.4363	.2938	.4986	.3343
14	.3118	.2077	.3741	.2492	.4366	.2908	.4988	.3322
17	.3120	.2061	.3744	.2473	.4369	.2886	.4992	.3298
21	.3122	.2048	.3746	.2457	.4371	.2868	.4994	.3276
26	.3123	.2036	.3748	.2443	.4372	.2851	.4997	.3258
35	.3124	.2024	.3748	.2429	.4373	.2833	.4999	.3238
55	.3124	.2011	.3748	.2414	.4374	.2816	.4999	.3218
135	.3124	.1999	.3748	.2398	.4374	.2798	.4999	.3198

Number of teeth	1 1/4-in. *CP*		1 1/2-in. *CP*		1 3/4-in. *CP*		2-in. *CP*	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.
8	0.6210	0.4284	0.7450	0.5140	0.8694	0.5994	0.9936	0.6852
9	.6218	.4250	.7460	.5100	.8706	.5952	.9948	.6800
10	.6224	.4224	.7468	.5068	.8714	.5914	.9956	.6756
11	.6228	.4200	.7474	.5040	.8720	.5882	.9964	.6720
12	.6232	.4182	.7478	.5020	.8726	.5876	.9972	.6692
14	.6236	.4154	.7482	.4984	.8732	.5816	.9976	.6644
17	.6240	.4122	.7488	.4946	.8738	.5772	.9984	.6596
21	.6244	.4096	.7492	.4914	.8742	.5736	.9988	.6552
26	.6246	.4072	.7496	.4886	.8744	.5702	.9994	.6516
35	.6248	.4048	.7498	.4858	.8746	.5666	.9998	.6476
55	.6250	.4022	.7499	.4828	.8748	.5632	.9999	.6436
135	.6250	.3998	.7499	.4796	.8748	.5596	.9999	.6396

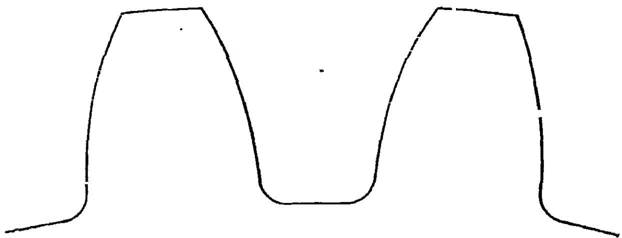
CP = circular pitch.



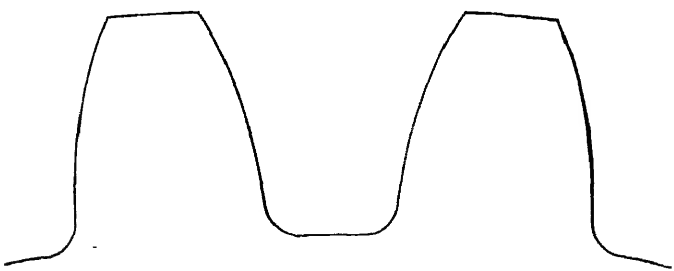
Comparative sizes of gear teeth standard 14½-deg. involute forms.



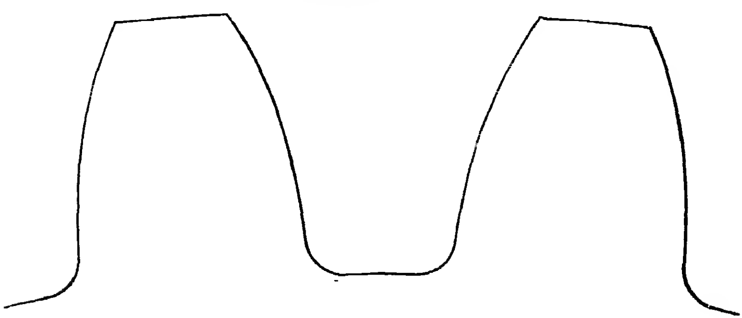
$2\frac{1}{2}DP$
1.2566 in. CP



$2DP$
1.5708 in. CP

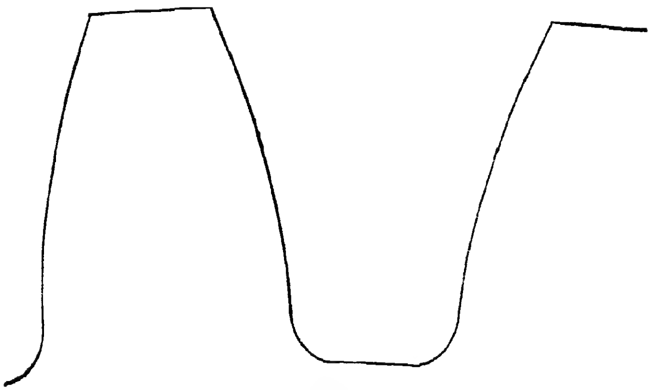


$1\frac{3}{4}DP$
1.7952 in. CP

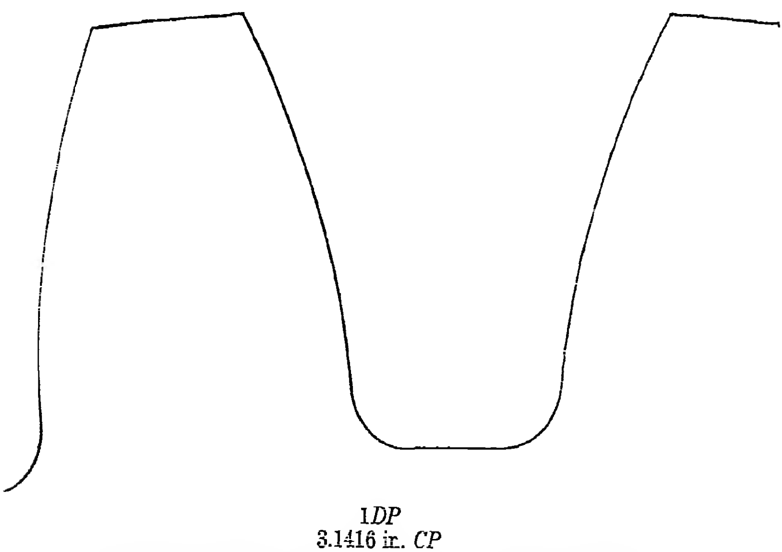


$1\frac{1}{2}DP$
2.0944 in. CP

Comparative sizes of gear teeth standard 14½-deg. involute forms.



$1\frac{1}{4}DP$
2.5133 in. CP



$1DP$
3.1416 in. CP

Comparative sizes of gear teeth standard $14\frac{1}{2}$-deg. involute forms.

Table 4.—Diametral Pitch

Relation between diametral and circular pitches, with corresponding tooth dimensions for standard 14½-deg. teeth (composite system)

Diametral pitch	Circular pitch, inches	Thickness of tooth of pitch line, inches	Whole depth, inches	Dedendum, inches	Addendum, inches
½	6.2832	3.1416	4.3142	2.3142	2.0000
¾	4.1888	2.0944	2.8761	1.5428	1.3333
1	3.1416	1.5708	2.1571	1.1571	1.0000
1¼	2.5133	1.2566	1.7257	.9257	.8000
1½	2.0944	1.0472	1.4381	.7714	.6666
1¾	1.7952	.8976	1.2326	.6612	.5714
2	1.5708	.7854	1.0785	.5785	.5000
2¼	1.3963	.6981	.9587	.5143	.4444
2½	1.2566	.6283	.8628	.4628	.4000
2¾	1.1424	.5712	.7844	.4208	.3636
3	1.0472	.5236	.7190	.3857	.3333
3½	.8976	.4488	.6163	.3306	.2857
4	.7854	.3927	.5393	.2893	.2500
5	.6283	.3142	.4314	.2314	.2000
6	.5236	.2618	.3595	.1928	.1666
7	.4488	.2244	.3081	.1653	.1429
8	.3927	.1963	.2696	.1446	.1250
9	.3491	.1745	.2397	.1286	.1111
10	.3142	.1571	.2157	.1157	.1000
11	.2856	.1428	.1961	.1052	.0909
12	.2618	.1309	.1798	.0964	.0833
13	.2417	.1208	.1659	.0890	.0769
14	.2244	.1122	.1541	.0826	.0714
15	.2094	.1047	.1438	.0771	.0666
16	.1963	.0982	.1348	.0723	.0625
17	.1848	.0924	.1269	.0681	.0588
18	.1745	.0873	.1198	.0643	.0555
19	.1653	.0827	.1135	.0609	.0526
20	.1571	.0785	.1079	.0579	.0500
22	.1428	.0714	.0980	.0526	.0455
24	.1309	.0654	.0898	.0482	.0417
26	.1208	.0604	.0829	.0445	.0385
28	.1122	.0561	.0770	.0413	.0357
30	.1047	.0524	.0719	.0386	.0333
32	.0982	.0491	.0674	.0362	.0312
34	.0924	.0462	.0634	.0340	.0294
36	.0873	.0436	.0599	.0321	.0278
38	.0827	.0413	.0568	.0304	.0263
40	.0785	.0393	.0539	.0289	.0250
42	.0748	.0374	.0514	.0275	.0238
44	.0714	.0357	.0490	.0263	.0227
46	.0683	.0341	.0469	.0252	.0217
48	.0654	.0327	.0449	.0241	.0208
50	.0628	.0314	.0431	.0231	.0200
56	.0561	.0280	.0385	.0207	.0178
60	.0524	.0262	.0360	.0193	.0166

Table 5.—Circular Pitch

Relation between circular and diametral pitches, with corresponding tooth dimensions for standard 14½-deg. teeth (composite system)

Circular pitch, inches	Diametral pitch	Thickness of tooth of pitch line, inches	Whole depth, inches	Dedendum, inches	Addendum, inches
6	0.5236	3.0000	4.1196	2.2098	1.9098
5	.6283	2.5000	3.4330	1.8415	1.5915
4	.7854	2.0000	2.7464	1.4732	1.2732
3½	.8976	1.7500	2.4031	1.2890	1.1140
3	1.0472	1.5000	2.0598	1.1049	.9550
2¾	1.1424	1.3750	1.8882	1.0128	.8754
2½	1.2566	1.2500	1.7165	.9207	.7958
2¼	1.3963	1.1250	1.5449	.8287	.7162
2	1.5708	1.0000	1.3732	.7366	.6366
1⅞	1.6755	.9375	1.2874	.6906	.5968
1¾	1.7952	.8750	1.2016	.6445	.5570
1⅝	1.9333	.8125	1.1158	.5985	.5173
1½	2.0944	.7500	1.0299	.5525	.4775
1 7/16	2.1855	.7187	.9870	.5294	.4576
1⅜	2.2848	.6875	.9441	.5064	.4377
1 5/16	2.3936	.6562	.9012	.4834	.4178
1¼	2.5133	.6250	.8583	.4604	.3979
1 3/16	2.6456	.5937	.8156	.4374	.3780
1⅛	2.7925	.5625	.7724	.4143	.3581
1 1/16	2.9568	.5312	.7295	.3913	.3382
1	3.1416	.5000	.6866	.3683	.3183
15/16	3.3510	.4687	.6437	.3453	.2984
⅞	3.5904	.4375	.6007	.3223	.2785
13/16	3.8666	.4062	.5579	.2993	.2586
¾	4.1888	.3750	.5150	.2762	.2387
11/16	4.5696	.3437	.4720	.2532	.2189
⅝	5.0265	.3125	.4291	.2301	.1989
9/16	5.5851	.2812	.3862	.2071	.1790
½	6.2832	.2500	.3433	.1842	.1592
7/16	7.1808	.2187	.3003	.1611	.1393
⅖	7.8540	.2000	.2746	.1473	.1273
⅜	8.3776	.1875	.2575	.1381	.1194
⅓	9.4248	.1666	.2287	.1228	.1061
5/16	10.0531	.1562	.2146	.1151	.0995
2/7	10.9956	.1429	.1962	.1052	.0909
¼	12.5664	.1250	.1716	.0921	.0796
2/9	14.1372	.1111	.1526	.0818	.0707
⅕	15.7080	.1000	.1373	.0737	.0637
3/16	16.7552	.0937	.1287	.0690	.0592
⅙	18.8496	.0833	.1144	.0614	.0531
1/7	21.9911	.0714	.0981	.0526	.0455
⅛	25.1327	.0625	.0858	.0460	.0398
1/9	28.2743	.0555	.0763	.0409	.0354
1/10	31.4159	.0500	.0687	.0368	.0318
1/16	50.2655	.0312	.0429	.0230	.0199

TABLE 6.—PITCH DIAMETERS FOR 1-IN. CIRCULAR PITCH

Number of teeth	Pitch diameter	Number of teeth	Pitch diameter	Number of teeth	Pitch diameter	Number of teeth	Pitch diameter
8	2.550	43	13.687	78	24.828	113	35.968
9	2.870	44	14.006	79	25.146	114	36.286
10	3.183	45	14.324	80	25.465	115	36.605
11	3.501	46	14.642	81	25.783	116	36.923
12	3.820	47	14.961	82	26.101	117	37.241
13	4.138	48	15.279	83	26.420	118	37.560
14	4.456	49	15.597	84	26.738	119	37.878
15	4.775	50	15.915	85	27.056	120	38.196
16	5.093	51	16.234	86	27.375	121	38.514
17	5.411	52	16.552	87	27.693	122	38.833
18	5.730	53	16.870	88	28.011	123	39.151
19	6.048	54	17.189	89	28.330	124	39.469
20	6.366	55	17.507	90	28.648	125	39.788
21	6.684	56	17.825	91	28.966	126	40.106
22	7.003	57	18.144	92	29.284	127	40.424
23	7.321	58	18.462	93	29.603	128	40.743
24	7.639	59	18.780	94	29.921	129	41.061
25	7.958	60	19.099	95	30.239	130	41.379
26	8.276	61	19.417	96	30.558	131	41.697
27	8.594	62	19.735	97	30.876	132	42.016
28	8.913	63	20.053	98	31.194	133	42.334
29	9.231	64	20.372	99	31.513	134	42.652
30	9.549	65	20.690	100	31.831	135	42.971
31	9.868	66	21.008	101	32.148	136	43.289
32	10.186	67	21.327	102	32.468	137	43.607
33	10.504	68	21.645	103	32.785	138	43.926
34	10.822	69	21.963	104	33.103	139	44.243
35	11.141	70	22.282	105	33.421	140	44.562
36	11.459	71	22.600	106	33.740	141	44.881
37	11.777	72	22.918	107	34.058	142	45.199
38	12.096	73	23.237	108	34.376	143	45.517
39	12.414	74	23.555	109	34.695	144	45.835
40	12.732	75	23.873	110	35.013	145	46.154
41	13.051	76	24.192	111	35.331	146	46.472
42	13.369	77	24.510	112	35.650	147	46.790

The basic rack of most approved systems of generated tooth gearing is that with teeth of the flat involute profile, the form that can be most easily and accurately reproduced. These systems are eminently satisfactory if the generated gears are supplied with an adequate number of teeth, but, if the gears have only a few teeth, excessive undercutting results, destroying much of the involute tooth profile inside the pitch circle of the gears. This drawback is compensated for in practice, with more or less success, by variations in the obliquity of the teeth (pressure angle) and by modifications in the depth, or height, of gear teeth.

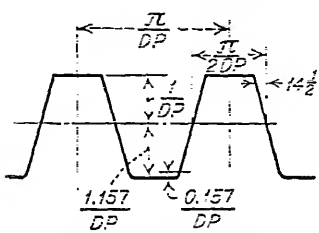


Fig. 18.—Basic rack of 14½-deg. generated gear-tooth system.

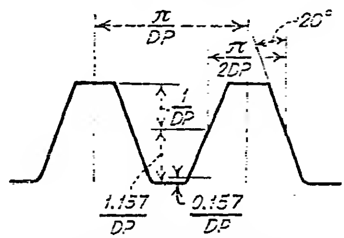


Fig. 19.—Basic rack of 20-deg., full depth, gear-tooth system.

For gears with a relatively large number of teeth, say 40 or more, the 14½-deg. generated gear-tooth system is quite customarily employed, but, for gears with fewer teeth, the 20-deg. generated gear-tooth system is now given the preference, both systems, however, having teeth of full depth, *i.e.*, with addendum and dedendum dimensions similar to those of the standard 14½-deg. composite system. Undercutting, with its resulting shortening of the line of tooth action is not entirely avoided by the 20-deg. pressure angle, but it is much reduced.

For reasonably quiet gear operation, it has been found that one of the requisites is a tooth contact of at least 1.4 tooth intervals and this is secured with 20-deg. full-depth generated-gear teeth when the pinion of a pair of mating gears has a minimum of 14 teeth. With 14½-deg. teeth, such extent of tooth contact is secured only when the pinion has at least 20, 21, or 22 teeth, the exact number depending upon the number of teeth on a mating gear. Pinions with fewer teeth are sometimes employed, it is true, but, when this is necessary, extreme accuracy in gear-tooth proportions is required for smooth and quiet gear operation.

Where the tooth numbers are relatively small in both pinion and gear, as in automobile-transmission assemblies, and for heavy mill gears where great strains are placed on the gear teeth, the required gear-tooth strength is secured ordinarily by a modification of the tooth proportions, as well as the use of the heavier 20-deg. pressure angle. The so-termed 20-deg. stub-tooth gear system is the one commonly employed, in which the obliquity of the teeth is made 20 deg. and the tooth height is reduced.

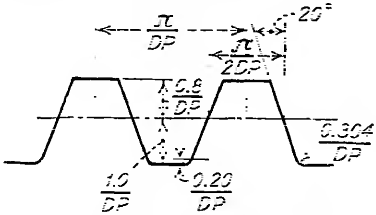


Fig. 20.—Basic rack of 20-deg. stub-tooth gear system.

As adopted by the American Gear Manufacturers' Association, the tooth heights for the four modern systems of commercial gearing are, in terms of the module, or reciprocal of the diametral pitch, as follows:

Table 7.—Tooth Heights in Standard 14½-deg. Composite System (*A*); 14½-deg. Generated Gear-tooth System (*B*); 20-deg., Full-depth, Gear-tooth System (*C*); and 20-deg. Stub-tooth Gear System (*D*)

Dimension	*A*	*B*	*C*	*D*
Addendum.....	1.0*M*	1.0*M*	1.0*M*	0.8*M*
Dedendum.....	1.157*M*	1.157*M*	1.157*M*	1.0*M*
Working depth.	2.0*M*	2.0*M*	2.0*M*	1.6*M*
Whole depth...	2.157*M*	2.157*M*	2.157*M*	1.8*M*
Clearance......	0.157*M*	0.157*M*	0.157*M*	0.2*M*

M = module.

While the dimensions given in Table 7 are recognized standards for the respective systems of modern commercial gearing, certain of the leading gear manufacturers have adopted slightly different proportions for 20-deg. stub-tooth gears, based upon the use of two diametral pitches: one for establishing the thickness and number of teeth and the other for determining the tooth depth. The composite diametral pitch is expressed in the form of a fraction, or ratio, the numerator of which designates the diametral pitch used for determining the number of teeth, etc. and the denominator the diametral pitch for establishing the depth, or heights, of the teeth.

TABLE 8.—DIMENSIONS OF 20-DEG. STUB-GEAR TEETH
(Fellows Gear Shaper Company's System)

Diametral pitch	Thickness of tooth inches	Addendum, inches	Working depth, inches	Depth of space below pitch line, inches	Clearance, inches	Whole depth of tooth, inches
4/5	0.3927	0.2000	0.4000	0.2500	0.0500	0.4500
5/7	.3142	.1429	.2858	.1786	.0357	.3214
6/8	.2618	.1250	.2500	.1562	.0312	.2812
7/9	.2244	.1111	.2222	.1389	.0278	.2500
8/10	.1963	.1000	.2000	.1250	.0250	.2250
9/11	.1745	.0909	.1818	.1136	.0227	.2045
10/12	.1571	.0833	.1667	.1041	.0208	.1875
12/14	.1309	.0714	.1429	.0893	.0179	.1607

In the stub-tooth systems, the reduction in the height, or depth, of the gear teeth is effected in the larger member by a decrease in the addenda of the gear teeth and in the smaller member by a corresponding decrease in the dedenda of the pinion teeth. This long- and short-tooth addenda system of gear teeth, while it tends to reduce the duration, or extent, of tooth contact, avoids much of the undercutting in gears with small numbers of teeth. For instance, in the A.G.M.A., 20-deg., stub-tooth gear system only 12- and 13-tooth pinions of one diametral pitch are at all undercut, and the amount of undercut is not sufficient to exert any appreciable influence upon the action of the gears. The greater strength of the shorter teeth is also a decided advantage.

Combinations of straight, 20-deg., stub-tooth gears (A.G.M.A. standard), with small numbers of teeth, ordinarily do not give a contact of 1.40 tooth intervals, as a rule necessitating greater care in gear production to secure reasonably quiet gear operation. Nevertheless, the 20-deg. stub-tooth form of gear is used extensively for automobile-transmission gears, etc. In general, however, the 14½-deg. generated gear system is preferable for gears with 40 or more teeth, and the 20-deg. full-depth tooth system for gears of a small number of teeth.

SECTION II

SPEEDS AND POWERS

As the essential purpose of employing toothed gears is to transmit motion from one shaft to another, even the simplest gear assemblage consists of two externally (spur) gear units, one attached to each shaft. More complicated gearing may

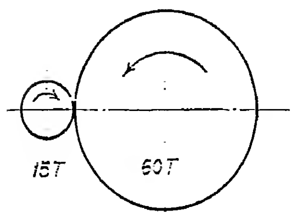


FIG. 21.—Mating gears rotate in opposite directions.

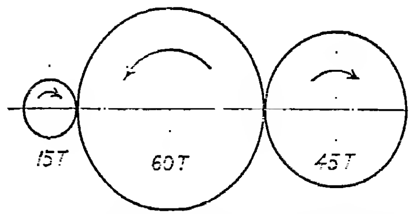


FIG. 22.—Driver and driven members rotate in the same direction in simple gear trains with an odd number of intermediate gears.

make use of a considerably larger number of gears, simple or compound units, but, so far as the act of transmitting power is concerned, the operating gear members function in *pairs* and the mating gears in all cases rotate in opposite directions.

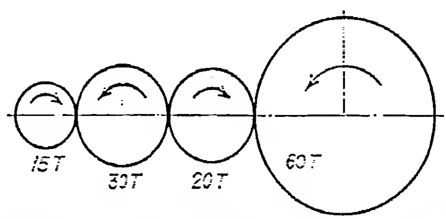


FIG. 23.—Driver and driven members rotate in opposite directions when the number of intermediate gears in a simple train is even.

The introduction of intermediate gears to form simple gear trains does not modify the respective speeds of the first and last gears—the driver and driven units of the assemblage—as the peripheral speed of all the engaging gears is the same, but it may alter their respective directions of rotation. If one or any odd number of intermediate gears are employed, for instance, the

direction of rotation of the driver and driven members is the same, but, if the number of intermediate gears is even, the driver and driven members of the simple gear train rotate in opposite directions, as in the case of a pair of mating spur gears.

Where compounded gears, gears of different sizes attached to a common shaft, are employed to secure greater changes in speed between the driver and driven members of a gear train, the compounding of the intermediate gears has no effect upon the directions of rotation of the driver and driven gear members. That is, if an odd number of compounded intermediate gears are employed, the direction of rotation of the first and last gears of the train will be the same, while, if an even number of compounded gears are incorporated in the assemblage, the driver and driven members rotate in opposite directions.

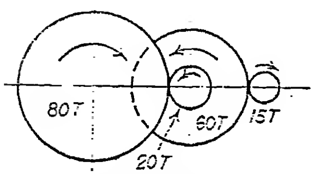


Fig. 24.—An odd number of compounded gears causes the driver and driven members to rotate in the same direction.

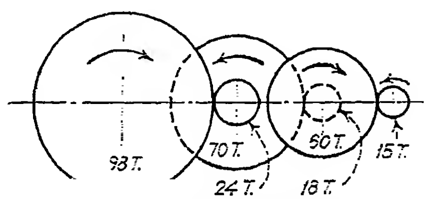


Fig. 25.—An even number of compounded gears causes the driver and driven members to rotate in opposite directions.

SPEED RATIO

In determining the respective speeds and speed ratios of gear trains, simple or compound, the individual gear members may be designated, or represented, by the number of their teeth or by any factor of their diameters. The rotating speed of the engaging gears is inversely proportional to their size, number of teeth or diameter, so, if the sizes of the respective mating gear units are known and the relative speed of one of the members, the speed of the mating gear is readily ascertained by direct proportion:

$$n:N::R:r$$

where n = number of teeth in pinion (size of unit).
N = number of teeth in gear (size of unit).
R = r.p.m. of gear.
r = r.p.m. of pinion.

For example, if a pair of mating gears have 60 and 15 teeth, respectively, and the gear member makes 300 r.p.m., the speed of the pinion, 1,200 r.p.m., is

$$15:60::300:r$$
$$r = \frac{60 \times 300}{15} = 1{,}200 \text{ r.p.m.}$$

and the speed ratio of the combination is 15 to 60, or 1 to 4.

In the case of a multiple-speed-change assemblage, or compound reduction, employing compounded gear units, the respective speeds of mating gears may be obtained in like manner, by direct proportion methods for each individual pair of engaging gears, or the continued product of the pinion-member sizes and the continued product of the gear-member sizes may be substituted in the ratio. To illustrate: if the gear members of a triple-reduction assemblage have 98, 70, and 60 teeth (N, N', and N''), respectively, and the pinion members 15, 18, and 24 teeth (n, n', and n''), respectively, the rotating speed r of the pinion n'', knowing the speed of rotation (R = 10 r.p.m.) of the first gear N, is

$$NN'N'', \text{ or } N \times N' \times N'' = 98 \times 70 \times 60 = 411{,}600$$
$$nn'n'', \text{ or } n \times n' \times n'' = 15 \times 18 \times 24 = 6{,}480$$
$$6{,}480:411{,}600::10:r$$
$$r = 635 \text{ r.p.m.}$$

and the speed ratio of the assemblage is 10 to 635, or approximately 1 to 64.

In practice, to secure a speed ratio of 1 to 64 with a triple-reduction compound-gear train, better results would be obtained by dividing the total speed ratio into three equal steps of 4 to 1; *i.e.*, by making each of the three steps, or stages, equal to the cube root of the total speed ratio. Three pinions of 15 teeth each and three gear members of 60 teeth each could be employed in such compound-gear train. Then:

$$NN'N'' = 60 \times 60 \times 60 = 216{,}000$$
$$nn'n'' = 15 \times 15 \times 15 = 3{,}375$$
$$3{,}375:216{,}000::10:r$$
$$r = 640 \text{ r.p.m.}$$

making the speed ratio exactly 1 to 64.

Should a similar speed ratio be desired employing a double-, instead of a triple-, reduction compound-gear train, the most

satisfactory results are secured when the speed ratios of the two stages of reduction are the same and made equal to the square root of the total speed ratio of the train. The stage speed ratios would then be 1 to the square root of 64, *i.e.*, 1 to 8, and, if two 15-tooth pinions were employed, the two mating gears should be 120-tooth units.

$$NN' = 120 \times 120 = 14{,}400$$
$$nn' = 15 \times 15 = 225$$
$$225:14{,}400::10:r$$
$$r = 640 \text{ r.p.m.}$$

the speed ratio being, consequently, the desired 1 to 64.

POWER RATIO

The relative powers of a train of gears are inversely proportional to their peripheral, or circumferential, velocities and, as the peripheral velocities of mating gears are always the same, so also are the loads caried by their respective teeth. In computing the power ratio of a simple gear drive, such as an ordinary single-stage drum-hoist mechanism (Fig. 26), consequently, the problem resolves itself into determining the required amount of power applied on the gear teeth to operate simply the drum mechanism and elevate the imposed load. Ignoring the frictional load entailed in operating the drum hoist, a simple direct proportion expresses the relationship between the four governing factors (load, size of functioning gear, applied power, and the size of the drum):

$$W:D::F:d$$

where W = load.

D = diameter (size) of functioning gear.

F = force, or applied power.

d = diameter of drum.

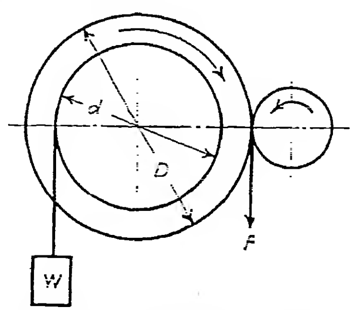


FIG. 26.—Simple-geared drum-hoist mechanism.

The applied power required to lift 300 lb. (W) with a 27-in. drum-hoist actuated by a 36-in. gear, for example, would be (disregarding the frictional load and mechanical losses):

$$300:36::F:27$$
$$F = 225 \text{ lb.}$$

A simple gear drive being involved and the mechanism such as to introduce little resistance, the frictional load and other mechanical inefficiencies may be taken at 3 per cent. This would add the equivalent of 9 lb. to the load to be raised, so the force required to operate the hoist and elevate the load would be

$$309:36::F:27$$
$$F = 231.8 \text{ lb.}$$

For a multiple-reduction drive, one entailing the use of compound gears, *the continued product of the driving and driven gear sizes* should be substituted for the simple gear-drum values, much as when computing the speed ratio of a compound-gear train. For instance, in the case of a triple-reduction 18-in.-geared drum hoist actuated by a gear train composed of 36-, 30-, and 24-in. gear members (D, D', and D'') and compounded 9- and 10-in. pinions (d' and d''), the computations entailed in the determination of the necessary applied force to raise 2,500 lb. and overcome a frictional load of 7½ per cent are as follows:

$$DD'D'' = 36 \times 30 \times 24 = 25{,}920$$
$$dd'd'' = 18 \times 9 \times 10 = 1{,}620$$
$$W = 2{,}500 \times 1.075 = 26{,}875 \text{ lb.}$$
$$26{,}875:25{,}920::F:1{,}620$$
$$F = 1{,}675.8 \text{ lb.}$$

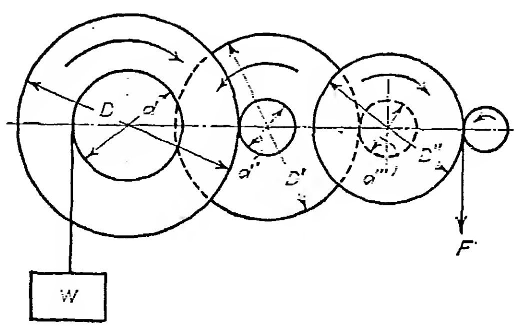


FIG. 27.—Triple-reduction drum-hoist mechanism.

In these two examples, the size of the driving pinion in the gearing assemblage has not been a factor, the time element not being a consideration, but, when the speed of the operation is of importance, the size of the driving pinion is of outstanding concern. A simple example in hoist gearing will serve to make this point clear.

A typical case would be the computations entailed in the selection of the gearing for a hoist to raise 2,400 lb. at a uniform speed, when making use of a 10-hp. motor running at a speed of 1,120 r.p.m. and driving through a rawhide pinion of 4-in. pitch diameter.

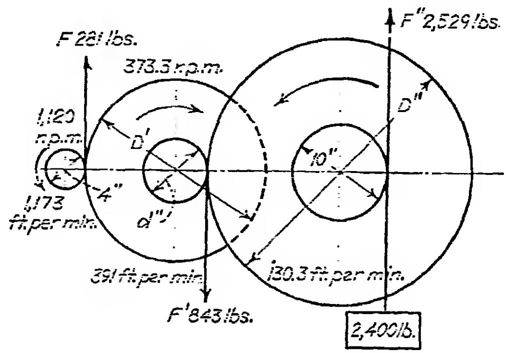


FIG. 28.—Ten-horsepower drum-hoist gear drive.

The velocity of the driving pinion in feet per minute (V) and the safe load for the pinion (F) are respectively:

$$V = d'0.2618 \quad \text{r.p.m.} = 4 \times 0.2618 \times 1{,}120 = 1{,}173 \quad \text{ft. per minute}$$

$$F = \frac{Hp. \times 33{,}000}{V} = \frac{10 \times 33{,}000}{1{,}173} = 281 \text{ lb.}$$

Assuming that a 5 per cent power loss is occasioned by frictional resistance and mechanical inefficiencies, the total load to be raised by the force of 281 lb. at the pitch circumference of the driving pinion becomes

$$2{,}400 \times 1.05 = 2{,}520 \text{ lb.}$$

The necessary total velocity ratio of the gears in a reduction train to secure this ratio of power is consequently:

$$2{,}520 : 281 :: 9 : 1$$

This reduction should be made in two stages, as indicated in Fig. 28, and, since it is always desirable to make the reduction in even steps, the most suitable reduction in each stage is 3 to 1, the square root of the total required velocity ratio of 9 to 1.

Then, if the diameter of the hoisting drum is made 10 in., the pitch diameter of the driven gear D'' to effect a velocity ratio of

3 to 1 will be 30 in. Similarly, if the pitch diameter of the intermediate pinion d'' is 7 in., the pitch diameter of the gear D' will have to be 21 in. in order to effect the first 3-to-1 reduction.

The circumferential force (F), or power, exerted by the intermediate gear D' at normal full-load speed is, naturally, the same as is exerted by the rawhide driving-motor pinion, or 281 lb.; that exerted by the pinion d'' three times as much, or 843 lb. (F'); and, finally, that exerted by the hoisting drum, three times as much as is exerted by the intermediate pinion, or 2,529 lb. (F''). *The latter is substantially that required to elevate the* 2,400-lb. load at a uniform rate of speed and overcome the frictional resistance of the drive, etc. The respective rotary speeds and circumferential velocities at which these critical forces are exerted are 1,120, 373.3, and 124.4 r.p.m.; 1,173, 391, and 130.3 ft. per minute; the latter being the uniform speed at which the load is raised.

While these computations establish suitable sizes for the required gear members, they do not in themselves determine the gear-tooth proportions. A suitable pitch has to be selected for the respective gears and the faces of the gears proportioned for adequate strength.

For the first gear reduction, that between the rawhide driving pinion and the intermediate gear D', a four diametral pitch would be suitable, if the gearing was of standard 14½-deg., composite or generated, systems, the safe working stress for rawhide being taken as 5,000 lb. per square inch. As the rawhide pinion is of 4-in. pitch diameter, a 16-tooth pinion of 0.7852 circular pitch is indicated, the safe load for which per inch of face (14½-deg. teeth) at a speed of 1,173 ft. per minute is 107.5 lb. A 2¾-in. face rawhide driving-motor pinion of 16 four diametral-pitch, teeth (14½ deg.), would consequently be satisfactory.

The intermediate gear D', operating at the same peripheral velocity and, of necessity, of the same pitch as the rawhide driving pinion, may be constructed of cast iron and be safely made with a slightly narrower face, since the metal has a considerably higher tensile strength than the rawhide material. The 7-in. intermediate pinion d'', however, operating at only one-third the peripheral speed of the intermediate gear and exerting a force three times as great, has to be of coarser pitch, if also constructed of cast iron. If proportioned for three diametral pitch, it would have 21 (14½-deg.) teeth of 1.0472-in.

circular pitch, the safe load for which at an operating speed of 391 ft. per minute is 466.3 lb. per inch of face.

The same tooth proportions apply, of course, to the driven gear D'', the pinion and gear mating in the drive, and, while both the pinion and gear would have adequate strength if made of cast iron with faces only slightly greater than $1\frac{3}{4}$ in. (1.8 + in.), it would be preferable to make the face of such a pinion, the weaker of the two members, $2\frac{1}{4}$ in. and the face of the mating gear D'' 2 in.; or the face of the pinion could be made the same as that of the gear D' with which it is compounded, $2\frac{1}{2}$ in., and the face of the driven gear D'', 2 or $2\frac{1}{4}$ in. Any of these combinations would prove satisfactory and the choice in gear-face dimensions would be governed doubtless by the sizes of the available stock gears.

STRENGTH OF GEAR TEETH

The question of safe gear load, strength of gear teeth, and the amount of power that can be safely transmitted by them, is naturally of outstanding importance and is one in which many variable and uncertain conditions play a part. Numerous empirical formulas have been developed at one time or another for determining the safe strength and durability of gear teeth, but it is probable that none of these equations takes, or can take, into account any exact appraisal of the many conditions involved. The uncertainty necessarily existing in respect to the strength characteristics and qualities of the various materials of which the gear blanks are made alone makes it highly problematic whether any rule of universal and easy application can be developed. These variations that cannot be sensitively controlled consequently make necessary, and account for, the general use of "factors of safety" in determinations of gear-tooth strengths.

These factors, ranging in value from 3 to 6 or more, provide ample margin to care for the unforeseen or neglected conditions, and the general use of certain of these empirical formulas is approved, the results secured being considered dependable. Those accorded general recognition are for the most part developments and modifications of the Lewis formula, propounded by Wilfred Lewis in 1892. This notable contribution takes the form of

$$W = SWS \times CP \times FW \times Y \tag{4}$$

where W = transmitted tooth load, pounds.

SWS = safe working stress of material, pounds per square inch.

CP = circular pitch of gear, inches.

FW = width of face of gear, inches.

Y = factor for number and form of teeth.

The factor SWS, the safe working stress for the gear material, is the ultimate tensile strength of the material in question in pounds per square inch divided by the proper *factor of safety.* Wilfred Lewis presented a table incorporating safe working stresses, values of SWS, for cast-iron and steel gears at pitch-line velocities ranging from 100 to 2,400 ft. per minute. This table was subsequently supplanted by an equation evolved by Carl G. Barth in which the variables of safe working stress and pitch-line velocity, or speed, are incorporated:

$$W = \frac{600 \times SWS \times CP \times FW \times Y}{600 + PLV} \tag{4a}$$

where PLV = pitch-line velocity of gear in feet per minute.

This modification of the Lewis formula is the one still commonly accepted for ordinary commercial metal gearing, but, where the gear teeth are carefully proportioned and finished, as is essential in the case of high-speed gearing, the basic formula is usually still further modified by substituting 1,200 for the constant 600 in formula (4a).

$$W = \frac{1{,}200 \times SWS \times CP \times FW \times Y}{1{,}200 - PLV} \tag{4b}$$

For high-speed metal gears running at pitch-line velocities of 4,000 ft. per minute and over, the American Gear Manufacturers' Association, however, has adopted a somewhat different formula:

$$W = \frac{\qquad}{78 - \sqrt{PLV}} \tag{4c}$$

while a modification of the basic Lewis equation developed for nonmetallic (rawhide, laminated phenolic materials, etc.) gears is

$$W = \left(\frac{150}{200 + PLV} + 0.25\right) SWS \times CP \times FW \times Y \tag{4d}$$

TABLE 9.—SAFE WORKING STRESS *SWS*

Material	Tensile strength, lb. per sq. in.	Safe stress,[1] lb. per sq. in., 0 speed
Metallic:		
Cast iron	24,000	8,000
Mild steel	36,000	12,000
Bronze	36,000	12,000
Cast steel (S.A.E. 1235)	45,000	15,000
Forged steel (S.A.E. 1030)	60,000	20,000
(S.A.E. 1045)	90,000	30,000
(S.A.E. 3245)	120,000	40,000
Nonmetallic:		
Rawhide, etc.	....	6,000

American Gear Manufacturers' Association Recommendations Nonmetallic Gears

PLV	*SWS*	*PLV*	*SWS*	*PLV*	*SWS*
100	4,500	700	2,500	1,700	1,974
150	4,071	800	2,400	1,800	1,950
200	3,650	900	2,318	1,900	1,929
250	3,500	1,000	2,250	2,000	1,909
300	3,300	1,100	2,192	2,200	1,875
350	3,136	1,200	2,143	2,300	1,860
400	3,000	1,300	2,100	2,400	1,846
450	2,885	1,400	2,063	2,600	1,821
500	2,786	1,500	2,029	2,800	1,800
600	2,625	1,600	2,000	3,000	1,781

[1] For steady loads on single pairs of gears,
suddenly applied loads on single gears, discount .... 25 per cent
steady loads on gear trains beyond first mesh, discount .... 40 per cent
suddenly applied loads on gear trains beyond first mesh, discount .. 50 per cent

The safe working stresses given in Table 9 are those that are customarily employed, but when the physical properties of the materials are definitely known, the specific tensile strength of the material in question should be employed. The safe working stress for steady loads on single pairs of gears is obtained by dividing the tensile strength of the material by a factor of safety of 3.

In the case of herringbone-type gears, the gear teeth being accurately finished, and the double obliquity of the teeth balancing the axial thrust, the use of formulas (4*b*) and (4*c*) is

entailed in gear-tooth-strength computations, but for helical- and spiral-type gears the unbalanced axial thrust introduced by the oblique arrangement of the gear teeth necessitates a further modification of the developed Lewis formulas. The cosine of the

TABLE 10.—VALUES OF FORM FACTOR Y IN LEWIS FORMULAS

Number of teeth	Gear-tooth system: 14½-deg. composite and generated	Gear-tooth system: 20-deg. full depth	Gear-tooth system: 20-deg. stub tooth
10	0.056	0.064	0.083
11	.061	.072	.092
12	.067	.078	.099
13	.071	.083	.103
14	.075	.088	.108
15	.078	.092	.111
16	.081	.094	.115
17	.084	.096	.117
18	.086	.098	.120
19	.088	.100	.123
20	.090	.102	.125
21	.092	.104	.127
22	.093	.105	.128
23	.094	.106	.130
24	.096	.107	.131
25	.097	.108	.133
26	.098	.110	.135
28	.100	.113	.138
30	.101	.114	.139
35	.105	.120	.143
40	.107	.124	.146
50	.110	.130	.151
60	.113	.134	.154
75	.115	.138	.158
100	.117	.142	.161
150	.119	.146	.165
Rack	.124	.154	.175

TABLE 11.—VELOCITY FACTORS

Ft. per min.	$\frac{600}{600 + PLV}$	Ft. per min.	$\frac{1,200}{1,200 + PLV}$	Ft. per min.	$\frac{78}{78 + \sqrt{PLV}}$
100	0.857	1,200	0.500	4,000	0.553
200	.750	1,400	.461	4,200	.545
300	.667	1,600	.429	4,400	.540
400	.600	1,800	.400	4,600	.535
500	.545	2,000	.375	4,800	.530
600	.500	2,200	.353	5,000	.525
700	.461	2,400	.333	5,200	.520
800	.429	2,600	.316	5,400	.515
900	.400	2,800	.300	5,600	.510
1,000	.375	3,000	.286	5,800	.506
1,100	.353	3,200	.273	6,000	.502
1,200	.333	3,400	.261	6,200	.498
1,300	.316	3,600	.250	6,400	.494
1,400	.300	3,800	.240	6,600	.490
1,500	.286	4,000	.231	6,800	.486
1,600	.273	.....	.....	7,000	.482
1,700	.261	.....	.....	7,200	.479
1,800	.250	.....	.....	7,400	.475
1,900	.240	.....	.....	7,600	.472
2,000	.231	.....	.....	7,800	.468
				8,000	.465
				8,200	.462
				8,400	.459
				8,600	.456
				8,800	.454
				9,000	.451
				9,200	.448
				9,400	.446
				9,600	.443
				9,800	.441
				10,000	.438

angle of tooth obliquity, or helix angle, becomes a factor requiring consideration:

$$W = \frac{1,200 \times SWS \times CP \times FW \times Y \times \cos VH}{1,200 + PLV} \qquad (4e)$$

$$W = \frac{78 \times SWS \times CP \times FW \times Y \times \cos VH}{78 + \sqrt{PLV}} \tag{4f}$$

where VH = angle of tooth obliquity (helix angle).

HORSEPOWER FORMULAS

The total transmitted load W in pounds multiplied by the pitch-line velocity in feet per minute and the result divided by 33,000 gives the horsepower-transmitting capacity of a gear, the Lewis formula for horsepower taking the form:

$$HP = \frac{W \times PLV}{33{,}000} \tag{5}$$

The practice recommended by the American Gear Manufacturers' Association, however, is to break down the transmitted load into component factors, any one of which may be influenced by special conditions and require suitable modification. In terms of the circular pitch (CP) and of diametral pitch (DP), respectively, the A.G.M.A. formulas for horsepower are:

$$HP = \frac{SWS \times CP \times FW \times Y \times PLV}{33{,}000} \tag{5a}$$

$$HP = \frac{0.000095 \times SWS \times FW \times Y \times PLV}{DP} \tag{5b}$$

where HP = horsepower.
CP = circular pitch, inches.
DP = diametral pitch.

WEAR OF GEAR TEETH

The various formulas presented for computing the strength of gear teeth and horsepower of gears fail, however, to make any suitable allowance for the unavoidable weakening of the gear teeth in service through destructive wear, a consideration that may assume great importance, particularly in high-speed gearing. The load transmitted, sliding action of the teeth, vibration, and lubrication are some of the considerations that influence gear-tooth wear, as well as the wear-resisting qualities of the gear-tooth material and the precision in the profile finish of the teeth.

A limitation in load is naturally a logical precaution, and a number of formulas for determining what this should be under certain conditions have been developed and have come into quite general use. Two of these are:

$$\frac{W}{fw} = LC \times pd \tag{6a}$$

and

$$\frac{W}{fw} = LC' \times \sqrt{pd} \tag{6b}$$

where W/fW = load per inch face, pounds.

LC [formula (6a)] = factor depending on load conditions = 62.5 to 100.0 for single-reduction, heat-treated steel gears in constant service.

LC' [formula (6b)] = factor depending on load conditions = 175 to 250 for single-reduction, heat-treated steel gears in constant service.

pd = pitch diameter of pinion, inches.

These formulas in effect simply refine still further the accepted modifications of the basic Lewis formula, and similar limitations in transmitted load may be made by employing a somewhat larger factor of safety in computing the value of the allowable safe-working-stress factor in the various equations for strength of gear teeth. This latter method of making provision for wear compensation is, in fact, often employed.

A third formula for determining a proper load limitation for suitable gear-tooth durability is one that takes into account the number of teeth in the pinion and the speed ratio of the mating gears as well as the pitch-line velocity in service:

$$\frac{W}{fw} = \frac{88{,}800 \times LC'' \times CP}{PLV + 32.8} \tag{6c}$$

where LC'' = factor depending upon number of teeth in pinion and speed ratio (see Table 12).

CP = circular pitch, inches.

PLV = pitch-line velocity, feet per minute.

Which one of these various formulas for determining the limiting load consistent with satisfactory gear-tooth wear should be employed remains an open question. Formula (6a) is the one more commonly used in the United States and formula (6b) the one favored by English producers of high-quality, heat-treated steel gears. The former places a somewhat higher limitation on the permissible transmitted load and imposes, consequently, a higher premium upon accuracy in gear-tooth proportions and finish.

Table 12.—Values of LC'' for Use in Formula (6c)
Lubricated gear teeth

Number of pinion teeth	Speed ratio								
	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8	1:10
12	2.80	3.40	3.80	4.20	4.36	4.54	4.68	4.80	5.00
14	3.20	3.80	4.20	4.60	4.88	5.08	5.24	5.40	5.60
16	3.50	4.20	4.64	5.06	5.36	5.58	5.70	5.84	6.10
18	3.80	4.40	5.00	5.40	5.76	5.96	6.10	6.24	6.44
20	4.20	4.90	5.40	5.90	6.20	6.40	6.64	6.88	6.90
22	4.60	5.34	5.86	6.36	6.62	6.86	7.06	7.24	7.36
24	5.00	5.76	6.30	6.80	7.04	7.30	7.46	7.60	7.80
26	5.36	6.08	6.68	7.20	7.46	7.72	7.90	8.06	8.24
28	5.70	6.40	7.04	7.60	7.88	8.14	8.32	8.50	8.64
30	6.06	6.84	7.48	8.00	8.34	8.60	8.74	8.96	
32	6.40	7.28	7.92	8.40	8.80	9.04	9.24	9.40	
34	6.80	7.70	8.34	8.82	9.20	9.46			
36	7.20	8.10	8.76	9.24	9.60	9.88			
38	7.56	8.48	9.16	9.76	10.02				
40	7.90	8.84	9.56	10.28	10.44				

Precision in gear design and production are, in fact, the controlling considerations in combating gear-tooth wear successfully, formulas for limitation of load, etc., being of value simply as guides or recommendations. Tooth wear cannot be entirely avoided, for a certain sliding between engaging gear teeth is inherent in all commercial systems of toothed gearing, because of the varying lengths of equal angular increments on the tooth profiles and the fact that the proportional amount of sliding action, as compared with the amount of the much less destructive rolling action, is different for each speed ratio and variation in gear-tooth proportions. The amount of sliding can be computed, but, as it cannot be controlled, its measure is of little practical value. Accurate finish of tooth profiles, supplemented by suitable gear-tooth lubrication, constitutes the only feasible control over the sliding wear of engaging gear teeth of specified material and hardness. Tooth wear traceable to vibration can in most cases be largely controlled, for it is almost always due to lack of necessary rigidity in gear supports and mountings, or to faulty gear proportions.

As it is desirable, obviously, to have the gear and pinion members in a gear drive wear equally, the pinion, being subjected to more concentrated wearing action on account of its higher speed of rotation, should be constructed of a material possessing greater wear-resisting qualities than that used for the gear. Since the speed ratio of a pair of mating gears is a measure of the relative speeds of gear and pinion rotation, the same measure may also be taken as a relative index of the desired wear-resisting capacities for the teeth of the respective gear and pinion members. In other words, the wear-resisting capacities of the teeth of engaging gears should be proportional to their speed ratio, if the teeth of the gear and pinion members are to wear equally.

The chief wear-resisting quality of a gear-tooth metal, if the load transmitted by the gear assemblage does not impose strains in excess of the elastic limit of the material, is its hardness, and this quality is also customarily taken as an index of the wear-resisting capacity of the gear teeth. The measure of hardness, its value, is gaged usually by the elastic limit of the material, which follows closely the quality of hardness. Consequently, to secure equal gear- and pinion-tooth durability, the hardness of the gear and pinion teeth should be proportional to their respective speeds of rotation and the ratio between the elastic limits of the respective gear-tooth materials made the same as the speed ratio of the gear combination.

In the case of a pair of mating gears having a speed ratio of 4 to 1, for example, the teeth of the pinion member should be four times as hard as the teeth of the gear member. If the latter should be made of cast steel having an elastic limit of about 30,000 lb. per square inch, the desired elastic limit for the pinion-tooth material would be 120,000 lb., necessitating the use of some heat-treated chrome-nickel or other high-grade steel.

It is impractical, of course, to have pinions with teeth of just the right hardness for equalizing the wear of gear and pinion teeth in all assemblages of gearing, but the relative hardness ratio is nevertheless a major consideration where longevity of gearing is of importance. Others of the more important questions having to do with gear wear are service conditions and the liability of modifications in structural formation of the gear metals, *i.e.*, the possibilities of the development of dangerous crystallization. This latter is of especial importance where high-speed gearing is involved, particularly if the gears operate continuously, for

which reason the use of the high-carbon steels, as a rule, is to be avoided for gears and pinions, and the required hardness secured by case-hardening or by the employment of alloyed steels containing such hardening ingredients as manganese, nickel, chromium, or vanadium.

The duration of tooth contact and the number of teeth in engagement at the same time, *i.e.*, the contact ratio, also influence greatly the questions of strength and gear-tooth durability. When the teeth are arranged obliquely across the face of the gears, as in herringbone, helical, spiral, and worm gearing, finer pitches may be employed, as the total tooth load is distributed over the several engaging teeth. To a somewhat less extent, this is also true of ordinary straight-tooth externally meshing spur gearing, for which reason it is desirable to have the duration of contact between mating gears as long as possible and to select the pitch with the view of having as many teeth as feasible in engagement at the same time.

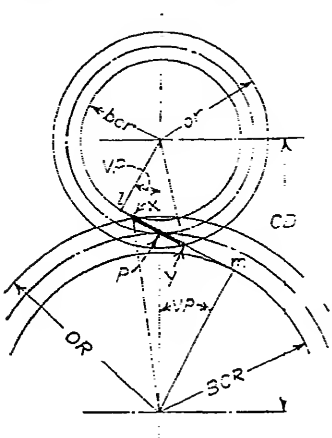


FIG. 29.—Duration of tooth contact.

The duration of tooth contact (DTC) is measured by that portion (xy) of the line of action (lm) encompassed by the outer circles of the engaging gears and is a function of the speed ratio. It is equal to

$$DTC = \sqrt{(OR)^2 - (BCR)^2} + \sqrt{(or)^2 - (bcr)^2} \quad CD \times \sin VP \tag{7}$$

where DTC = duration of tooth contact for a specific speed ratio.
OR = outer-circle radius of gear.
BCR = base-circle radius of gear.
or = outer-circle radius of pinion.
bcr = base-circle radius of pinion.
CD = center distance.
VP = pressure angle.

This distance divided by the normal pitch of the gear teeth gives quite obviously the number of engaging teeth that can be in

contact at the same time. In the involute system of gearing, the normal pitch (NIP) is measured by the length of the arc of the base circle between the origins of two successive involute tooth profiles, or involutes, and is equal to the circumference of the base circle divided by the total number of teeth on the gear:

$$NIP = \frac{3.1416 \times BCD}{N} \tag{8}$$

where NIP = normal (involute) pitch.
BCD = base-circle diameter.
N = number of teeth.

The duration of tooth contact (DTC) divided by the normal involute pitch (NIP) then gives the number of gear teeth in contact at the one time, a highly important consideration since the total tooth load is not concentrated on a single tooth or pair of teeth but is distributed over all the engaging teeth:

$$NTC = \frac{DTC}{NIP} \tag{9}$$

This advantage of having several sets of engaging gear teeth in contact at the one time, distributing the total tooth load over a number of pairs of meshing teeth, places a high premium upon accuracy in tooth form and precision in tooth spacing, for, not only does the number of teeth in contact influence the questions of required tooth strength, pitch, and durability but also the speed at which the gearing can be safely operated. Shock, noise, vibration, and excessive gear-tooth wear result when there are even relatively minor irregularities in tooth proportions and spacings, or slight imperfections in profile finish. The actual strength of the individual gear teeth may even become of secondary importance, the major points, provided the gear teeth are well formed, having to do more with tooth wear, proper proportions of gear diameters, hardness of gear teeth, and the best combination of hardness for wear.

GEAR EFFICIENCY

Under ordinary working conditions, the frictional losses between the teeth of engaging, high-quality, cut gears and pinions should not exceed more than 1 or 2 per cent of the power transmitted. This quite moderate loss is influenced more by the length of the tooth addendum (increasing slippage) than by the

obliquity of the gear teeth, and the degree of efficiency attained is, for all practical purposes, independent of the load transmitted by the gear assemblage. Furthermore, the differences in efficiency of the several standard tooth forms, or systems of gearing, are really so small as to exercise little or no controlling influence on the particular tooth form to be recommended for any specific service.

In general, as the efficiency of gearing depends for the most part upon the uniformity of the angular velocity of the pitch surfaces of the engaging gears, it follows that the finer the pitch of the gears and the more numerous the teeth, the more uniform is apt to be the angular velocity of the engaging pitch surfaces and the higher the operating efficiency of the gearing. For a given pitch, the efficiency of the gearing tends to increase with the number of teeth, so that large gears, if properly mounted, balanced, and supported, are relatively more efficient than smaller gears. This improvement is because the inaccuracies in profile finish, form, and tooth spacings, the chief causes of variations in angular velocity of the pitch surfaces, become relatively less disturbing as the number of teeth increases.

SECTION III

GEAR PROPORTIONS AND DESIGN

For reasons of economy, the element about which industrial gears of standard design are proportioned and about which they function is the hole, or bore, of the gear. On this basis, gear members of the minimum size, providing a sufficient amount of metal under the root of the pinion teeth to avoid dangers of fracture and guard against failures attributable to metal fatigue, are secured.

The controlling factors in determining this bore diameter are those of amount of power transmitted and the speed of gear rotation, the equation generally employed in calculating the nominal bores of pinions and gears being

$$BD = \frac{HP \times 80}{RPM} \tag{10}$$

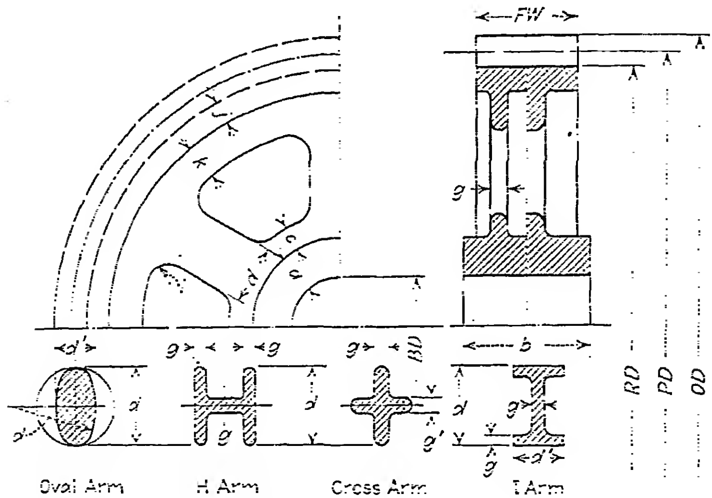
where BD = bore diameter, inches.
HP = horsepower transmitted.
RPM = revolutions per minute.

To provide adequate accommodation for the essential keyway, the diameter of the gear hub should be at least 1.8 times this (nominal) bore diameter (BD) and the minimum amount of metal above the keyway for the smallest permissible pinion, *i.e.*, the distance from the top of the keyway to the root circumference of a hubless pinion, should not be less than the square root of one-fifth the number of pinion teeth divided by the diametral pitch $\left(\frac{\sqrt{0.2 \times N}}{DP}\right)$ These requirements place an empirically determined limitation upon the minimum permissible pitch diameter of a hubless pinion having standard-bore and keyway dimensions, the equation for which is

$$\text{Minimum } PD \text{ (pinion)} = BD + 2\left(KWD + D + \sqrt{\frac{0.2 \times N}{DP}}\right) \tag{11}$$

where PD = pitch diameter.
BD = bore diameter.
KWD = keyway depth.
D = dedendum.
N = number of teeth.
DP = diametral pitch.

In the design of larger gears, the proportions shown in Fig. 30 have been found entirely satisfactory in service, and the following empirical equations for determining the more important gear dimensions give results that conform well with approved gear-design practices:



$a = 0.4BD, \qquad b = 1.25BD$

FIG. 30.—Gear proportions.

A six-arm design is customarily employed for solid-type gears up to approximately 120 in. pitch diameter and an eight-arm construction for larger gears. For split gears of under 40 in. diameter, however, a four-arm design is commonly used, with a narrow face and short hub, in order to avoid the necessity of providing special bolting. In calculating the arm sizes, the arm being considered a cantilever beam with the load distributed equally on each arm, the section modulus of the material is obtained by dividing the product of the load at zero speed and pitch radius by the number of arms multiplied by the stress of the

material. Since the stress is a common factor, the formula for the section modulus of the material takes the form:

$$Z = \frac{CP \times FW \times Y \times PR}{N} \tag{12}$$

where Z = section modulus.
CP = circular pitch.
FW = face width of gear.
Y = form factor (Table 10).
PR = pitch radius.
N = number of teeth.

Having the section modulus, a satisfactory arm width d for any of the various standard forms—oval, H-section, cross, and I-shaped—is

$$d = 2.$$

In the oval-run construction, the thickness of the arm d' is usually made equal to half the arm width, as is also the flange width of I-shaped arms, and the tapering of the arms toward the rim is made $\frac{3}{4}$ in. per foot.

$$d' = 0.5d = \quad 0.3977$$

The thicknesses of the H-section, cross, and I-shaped arms are, respectively,

$$g(\text{H-arm}) = \frac{6Z}{d'}, \qquad g(\text{crossarm}) = \frac{6Z}{d'}, \qquad g(\text{I-shaped}) = \frac{3Z}{d'}$$

while the thickness of the stiffening member in the H-arm construction should be

$$g' = 0.75g$$

The rim thickness j, which is governed by the size of the gear and the number of gear blank arms, is determined by the empirical equation

$$j = \frac{\dfrac{0.5N}{\text{number of arms}}}{DP}$$

The stiffening rib k under the rim should be made 25 per cent greater in depth than the rim thickness, and the radius of the fillets l, joining the arms with the stiffening rib, should vary with the pitch of the gear and the number of arms:

$$k = 1.25j, \qquad l = \frac{PD}{4.25 \times N}$$

For high-efficiency, long-wearing gears, the American Gear Manufacturers' Association recommends making the face width of the gears 10 times the circular pitch, or

$$FW = \frac{31.416}{DP}$$

This makes the face width somewhat greater than has heretofore been deemed necessary, permitting the use of finer pitches and thus securing smoother operation and better gearing efficiency. However, the following schedule of face widths conforms to good commercial practice:

FW (spur gears) = 3 or 4 times circular pitch
FW (minimum for helical gears) = $4 \times CP$
FW (minimum for herringbone gears) = $6 \times CP$

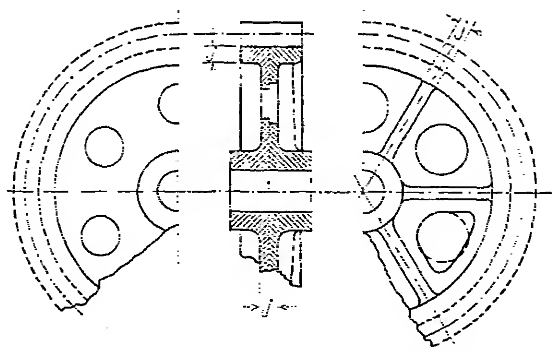


FIG. 31.—Webbed-gear designs.

In cases of webbed gears, which are usually of comparatively small size, the thickness of the web is advisably made equal to the thickness of the rim j and cored holes in cast gears, not only furnish convenient means of securing the gears for machining operations, but make for sounder castings. When these holes are large or are shaped to conform to the outline of the gear, stiffening ribs should be provided between the holes and these

ribs should be of about the same thickness as the web. Not only does this provide greater gear rigidity, but the danger of serious shrinkage strains developed by rapid cooling of the relatively thin web section is considerably lessened.

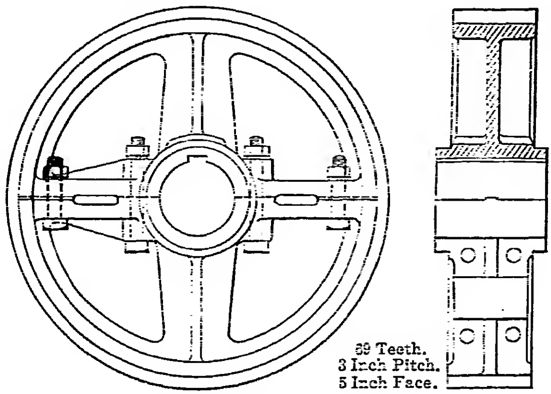


FIG. 32.—Split railway gear.

Split gears should be parted through opposite arms, and it is good practice to spline the adjacent surfaces (see Fig. 32) rather than to employ fitted bolts or depend upon dowel pins. However, when it is necessary to split a gear between arms, the outer attachment bolts should be placed as close to the rim as possible, making dimension b (Fig. 33) equal to or somewhat greater than dimension a. Unless this precaution is taken, the attachment bolts are subjected to eccentric strains that tend to spread the gear.

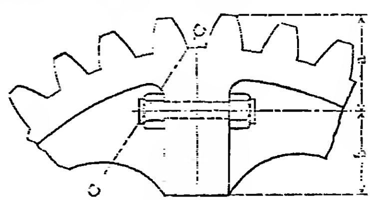


FIG. 33.—Gear split between arms.

Section CC (Fig. 33) should also be made stiff enough to resist the strains that tend to bend the bolt lugs, and the bolts should be heavy enough to carry the maximum applied load, as well as to resist the initial stress set-up in tightening the bolt nuts. For these obvious reasons, it is always advisable to locate the outer attachment bolts as close to the rim of the gear as is feasible, a practice that has the added merit of permitting the use of short bolt lugs, so minimizing the localization of extra weight on the gear rim. This latter point may be of considerable importance

if the gear is run at high speed, for, should the safe speed of the gear be exceeded, the liability of fracture at or near these points of localized weight is much greater than when the gear rotates at lower speed.

NONMETALLIC GEARING

The prevalent tendency in modern industrial establishments to increase volume of production through the use of high-speed machinery has created a pressing demand for some practical method of deadening the noise that results when running all-metal gear trains at high rates of speed and for a curtailment of the incidental vibration that greatly increases maintenance expenses and causes rapid deterioration of machines and gears. This has been accomplished successfully by the introduction, into the offending gear trains, of pinions made of nonmetallic materials, such as compressed fabric, rawhide, bakelite, and other substances, in the manufacture of which a synthetic resin of phenol variety is used as a binder.

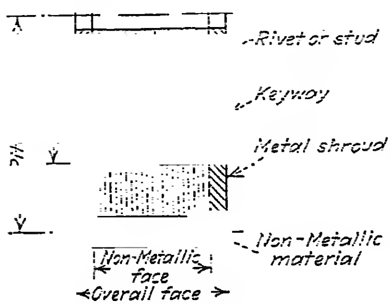


FIG. 34.—Nonmetallic pinion.

Owing to the low moduli of elasticity of these materials, most of the effects of any slight errors of tooth form and spacing are absorbed at the surface of the metal gear teeth with which the nonmetallic pinions engage. The result is that the profiles of the pinion teeth tend to conform quickly to the conjugate form of the metal gear with which they mate. Hence, it is an advantage to have the number of teeth on the engaging metal gear, or gears, some even multiple of the number of teeth on the nonmetallic pinion, for if this relationship exists the form of the pinion teeth need conform only to a minimum number of mating gear teeth.

In the usual construction of these pinions, in which the nonmetallic material is held under compression between metal shrouds by threaded-through stud bolts or rivets, there are certain mechanical limitations that may, if not properly provided for, cause considerable complication. For one thing, there must

be sufficient nonmetallic stock between the bottom of the teeth and the binding studs and at least $\frac{1}{16}$ in. of stock, as a minimum, between the bore and the studs. Approved practice is to have the stock between the bottom of the teeth and the studs equal to the full thickness of the teeth at the pitch line, for diametral pitches of 5 and less, and to two-thirds of the tooth thickness at the pitch line for all coarser pitches. The keyway, being generally cut between the studs, does not affect these proportions, provided the thickness of the stock between the bottom of the teeth and the top of the keyway is equal to at least the thickness of the nonmetallic pinion teeth on the pitch line. When these nonmetallic pinions are employed as idler pulleys, however, they should preferably be suitably bushed.

KEYS AND KEYWAYS

The keyseat standard (see Table 13) is based upon the employment of square keys, half embedded in the shaft and half in the

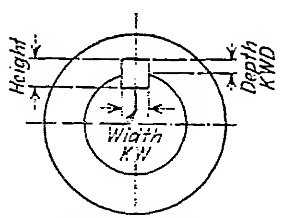


FIG. 35.—A.G.M.A. standard keyseat.

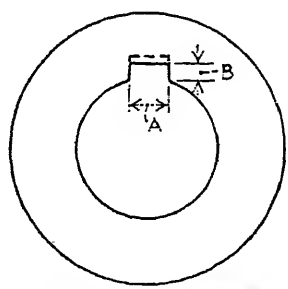


FIG. 36.—Taper keyway.

gear hub, the height and width of which approximate one-quarter of the shaft diameter. When conditions are such as to make it necessary or advisable to deviate from these proportions, the dimensions given in Table 13*a* are recommended.

As gears fastened with standard straight keys seldom work loose when properly fitted to their shafts, the use of taper keys (Table 14) is not ordinarily to be recommended. Taper keys have a tendency, when driven home, to throw the gear out of alignment, and their use should consequently be restricted to heavy work, where the mass of metal affords certain protection against such distortion.

TABLE 13.—A.G.M.A. KEY AND KEYWAY STANDARDS
(Dimensions in inches)

Diameter of holes inclusive	Keyways		Keystock
	Width	Depth	
5/16 – 7/16	3/32	3/64	3/32 × 3/32
1/2 – 9/16	1/8	1/16	1/8 × 1/8
5/8 – 7/8	3/16	3/32	3/16 × 3/16
15/16 – 1 1/4	1/4	1/8	1/4 × 1/4
1 3/16 – 1 3/8	5/16	5/32	5/16 × 5/16
1 7/16 – 1 3/4	3/8	3/16	3/8 × 3/8
1 13/16 – 2 1/4	1/2	1/4	1/2 × 1/2
2 5/16 – 2 3/4	5/8	5/16	5/8 × 5/8
2 13/16 – 3 1/4	3/4	3/8	3/4 × 3/4
3 5/16 – 3 3/4	7/8	7/16	7/8 × 7/8
3 13/16 – 4 1/2	1	1/2	1 × 1
4 9/16 – 5 1/2	1 1/4	7/16	1 1/4 × 7/8
5 9/16 – 6 1/2	1 1/2	1/2	1 1/2 × 1
6 9/16 – 7 1/2	1 3/4	5/8	1 3/4 × 1 1/4
7 9/16 – 9	2	11/16	2 × 1 3/8
9 1/16 – 11	2 1/2	13/16	2 1/2 × 1 5/8
11 1/16 – 13	3	1	3 × 2

TABLE 13a.—A.G.M.A. RECOMMENDATIONS FOR SPECIAL KEYWAYS
(Dimensions in inches)

Keyways		Keystock
Width	Depth	
1/8	3/64	1/8 × 3/32
3/16	1/16	3/16 × 1/8
1/4	3/32	1/4 × 3/16
5/16	3/32	5/16 × 3/16
3/8	1/8	3/8 × 1/4
1/2	3/16	1/2 × 3/8
5/8	7/32	5/8 × 7/16
3/4	1/4	3/4 × 1/2
7/8	5/16	7/8 × 5/8
1	3/8	1 × 3/4

TABLE 14.—TAPER-KEY PROPORTIONS
(Dimensions in inches)

Diameter of holes, incl.	Keyseat taper, width A	$\frac{1}{8}$ in. per foot, height B
1 –$1\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{32}$
$1\frac{3}{16}$–$1\frac{3}{8}$	$\frac{5}{16}$	$\frac{7}{64}$
$1\frac{7}{16}$–$1\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
$1\frac{11}{16}$–$1\frac{7}{8}$	$\frac{7}{16}$	$\frac{9}{64}$
$1\frac{15}{16}$–$2\frac{1}{8}$	$\frac{1}{2}$	$\frac{11}{64}$
$2\frac{3}{16}$–$2\frac{3}{8}$	$\frac{9}{16}$	$\frac{3}{16}$
$2\frac{7}{16}$–$2\frac{5}{8}$	$\frac{5}{8}$	$\frac{13}{64}$
$2\frac{11}{16}$–$2\frac{7}{8}$	$\frac{11}{16}$	$\frac{15}{64}$
$2\frac{15}{16}$–$3\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{4}$
$3\frac{3}{16}$–$3\frac{3}{8}$	$\frac{13}{16}$	$\frac{17}{63}$
$3\frac{7}{16}$–$3\frac{5}{8}$	$\frac{7}{8}$	$\frac{19}{64}$
$3\frac{11}{16}$–$3\frac{7}{8}$	$\frac{15}{16}$	$\frac{5}{16}$
$3\frac{15}{16}$–$4\frac{1}{8}$	1	$\frac{11}{32}$
$4\frac{3}{16}$–$4\frac{3}{8}$	$1\frac{1}{16}$	$\frac{13}{32}$
$4\frac{7}{16}$–$4\frac{3}{4}$	$1\frac{1}{8}$	$\frac{3}{8}$
$4\frac{7}{8}$ –$5\frac{1}{4}$	$1\frac{1}{4}$	$\frac{27}{64}$
$5\frac{3}{8}$ –$5\frac{3}{4}$	$1\frac{3}{8}$	$\frac{29}{64}$
$5\frac{7}{8}$ –$6\frac{1}{4}$	$1\frac{1}{2}$	$\frac{1}{2}$
$6\frac{3}{8}$ –$6\frac{3}{4}$	$1\frac{5}{8}$	$\frac{17}{32}$
$6\frac{7}{8}$ –$7\frac{1}{4}$	$1\frac{3}{4}$	$\frac{19}{32}$
$7\frac{3}{8}$ –$7\frac{3}{4}$	$1\frac{7}{8}$	$\frac{5}{8}$
$7\frac{7}{8}$ –8	2	$\frac{21}{32}$

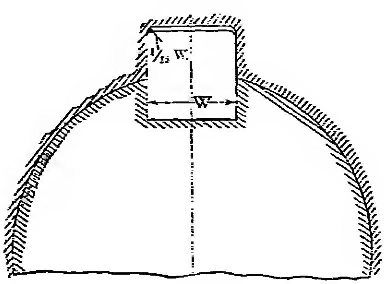


FIG. 37.—Filleted keyway.

For gears that are to be hardened, the corners of the keyway should be rounded, as a precaution against the development of cracks at the corners of the keyway. The radius of the fillet

is usually made equal to about one-sixteenth of the width of the keyway, and the top of the key is correspondingly beveled.

Semicircular Woodruff keys (Figs. 38 and 39) make satisfactory connections for machine-tool gears and numerous other

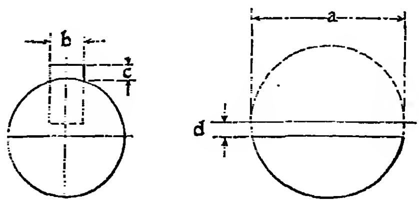


FIG. 38.—Standard Woodruff key.

gear assemblages, while for heavier service, for which a single standard straight key would be inadequate, two keys are frequently employed on opposite sides of the bore. For situations where still greater strains are involved, as characteristic of much rolling-mill work, the Kennedy key (Fig. 40) is well suited.

TABLE 15.—STANDARD WOODRUFF KEYS
(Dimensions in inches)

Number of key	Diameter of key	Thickness of key	Depth of keyway	Center of stock, from which key is made, to top of key	Number of key	Diameter of key	Thickness of key	Depth of keyway	Center of stock, from which key is made, to top of key
	a	*b*	*c*	*d*		*a*	*b*	*c*	*d*
1	1/2	1/16	3/64	3/64	*B*	1	5/16	5/32	1/16
2	1/2	3/32	3/64	3/64	16	1 1/8	3/16	3/32	5/64
3	1/2	1/8	1/16	3/64	17	1 1/8	7/32	7/64	5/64
4	5/8	3/32	3/64	1/16	18	1 1/8	1/4	1/8	5/64
5	5/8	1/8	1/16	1/16	*C*	1 1/8	5/16	5/32	5/64
6	5/8	5/32	5/64	1/16	19	1 1/4	3/16	3/32	5/64
7	3/4	1/8	1/16	1/16	20	1 1/4	7/32	7/64	5/64
8	3/4	5/32	5/64	1/16	21	1 1/4	1/4	1/8	5/64
9	3/4	3/16	3/32	1/16	*D*	1 1/4	5/16	5/32	5/64
10	7/8	5/32	5/64	1/16	*E*	1 1/4	3/8	3/16	5/64
11	7/8	3/16	3/32	1/16	22	1 3/8	1/4	1/8	3/32
12	7/8	7/32	7/64	1/16	23	1 3/8	5/16	5/32	3/32
A	7/8	1/4	1/8	1/16	*F*	1 3/8	3/8	3/16	3/32
13	1	3/16	3/32	1/16	24	1 1/2	1/4	1/8	7/64
14	1	7/32	7/64	1/16	25	1 1/2	5/16	5/32	7/64
15	1	1/4	1/8	1/16	*G*	1 1/2	3/8	3/16	7/64

These keys, like all standard straight and taper keys, are approximately one-quarter of the shaft diameter in width and have a top taper of 1/8 in. per foot, for a driving fit, and parallel sides. The inner corners of the keys project into the bore, as shown, the gear

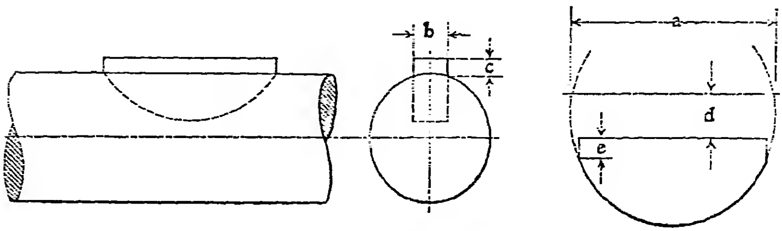


FIG. 39.—Special Woodruff key.

hubs being first bored for a press fit and then rebored slightly off center, an eccentricity of about one-sixteenth the shaft diameter being customary. The keyways are cut on the eccentric side, while the opposite side of the hole remains as concentrically

TABLE 15a.—SPECIAL WOODRUFF KEYS
(Dimensions in inches)

Number of key	Diameter of key	Thickness of key	Depth of keyway	Center of stock, from which key is made, to top of key	Width of flat	Number of key	Diameter of key	Thickness of key	Depth of keyway	Center of stock, from which key is made, to top of key	Width of flat
	a	*b*	*c*	*d*	*e*		*a*	*b*	*c*	*d*	*e*
26	2 1/8	3/16	3/32	1 7/32	3/32	31	3 1/2	7/16	7/32	1 3/16	3/16
27	2 1/8	1/4	1/8	1 7/32	3/32	32	3 1/2	1/2	1/4	1 3/16	3/16
28	2 1/8	5/16	5/32	1 7/32	3/32	33	3 1/2	9/16	9/32	1 3/16	3/16
29	2 1/8	3/8	3/16	1 7/32	3/32	34	3 1/2	5/8	5/16	1 3/16	3/16
30	3 1/2	3/8	3/16	1 3/16	3/16						

TABLE 15b.—WOODRUFF KEYS FOR SHAFT DIAMETERS
(Dimensions in inches)

Diameter of shaft	Number of keys	Diameter of shaft	Number of keys	Diameter of shaft	Number of keys
5/16–3/8	1	7/8 – 15/16	6, 8, 10	1 3/8 –1 7/16	14, 17, 20
7/16–1/2	2, 4	1	9, 11, 13	1 1/2 –1 5/8	15, 18, 21, 24
9/16–5/8	3, 5	1 1/16–1 1/8	9, 11, 13, 16	1 11/16–1 3/4	18, 21, 24
11/16–3/4	3, 5, 7	1 3/16	11, 13, 16	1 13/16–2	23, 25
13/16	6, 8	1 1/4 –1 5/16	12, 14, 17, 20	2 1/16–2 1/2	25

bored. These keys are especially desirable where it may be necessary to move heavy gears a considerable distance on the shaft before securing them in place.

For sliding gears in heavy service, the practice of using three or more keys with radial sides (Fig. 41), often employed for vertical roll mountings, is to be recommended, despite the comparatively high cost of the construction, and the arrangement

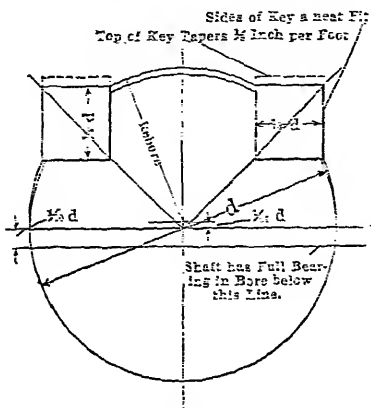


FIG. 40.—Kennedy keys.

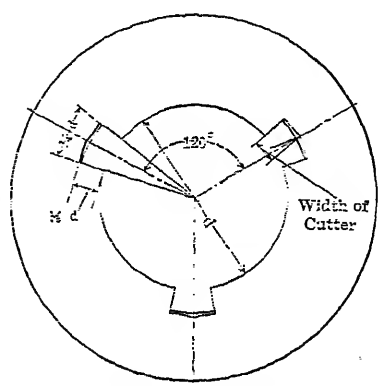


FIG. 41.—Keyways for heavy sliding gear.

is superior in some respect to the multiple-spline constructions more widely employed. The latter, however, lend themselves readily to more economical formation by a generating process.

BORE FITS AND TOLERANCES

It is upon the hole, or bore, of a gear that its *fit* on the carrying shaft depends, and in high-class commercial gear production there are limits of *allowance* and limits of *tolerance* within which the diameter of the finished bore must fall for the gear to function satisfactorily. The allowance depends, naturally, upon the kind of fit required; whether

Force fits, where hydraulic pressure is used.

Shrink fits, using heat.

Driving fits, using reasonable bumping force.

Snug fits or *wringing fits*, using manual force.

Running fits of the various kinds, close or easy, etc.

These different fits can be best secured by gages made to meet the individual requirements of the particular shop, for not only

do the proper fits vary for different materials and equipment, but changes are also to be expected with evolution in machine-shop processes. However, as there is a constant and unavoidable variation from the exact size or allowance necessary to produce a given fit in virtually all shop work, due to wear of cutting tools and gages, to errors of workmanship and, especially when setting tools to gages, to the human element, certain definite standards of limiting permissible tolerances have been established. For considerations of economy in gear production, it is furthermore advisable to tolerate as large a variation as possible for given classes of work.

Recommendations to this effect by the American Gear Manufacturers' Association—for precision gears, as used in aircraft, printing machinery, etc. (Class 1); gears for automobile transmissions, machine tools, etc. (Class 2); and gears for pumps, hoisting machines, general jobbing requirements, etc. (Class 3)—are given in Table 16. It will be noted that the advocated practice is to make the tolerance for Class 2 about double that for Class 1, and that of Class 3 twice that of Class 2, or about four times that of Class 1.

SPLINE FITTINGS

Leadership in the standardization of spline fittings (Tables 17, 18, and 19) has been taken by the Society of Automotive

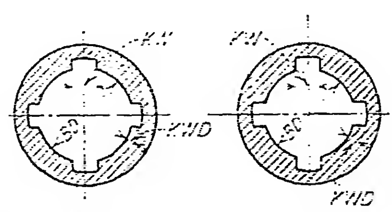


FIG. 42.—S.A.E. standard four-spline fittings.

Engineers (S.A.E.), which organization bases its recommendations upon dimensions that apply only to soft-broached holes and the stipulation that allowance must be made for machining the shaft to secure the desired fit. These standards make no allowance for radii on corners of splines, which radii should not exceed 0.015 in., or for clearances. The torque (T) capacity is expressed in terms of inch-pound units per inch width of bearing length at 1,000 lb. per square-inch pressure on sides of splines.

Table 16.—A.G.M.A. Standard Gear-hole Tolerances

	Nominal size	To ½ in., incl.	To 1 in., incl.	To 2 in., incl.	To 3 in., incl.	To 4 in., incl.	To 5 in., incl.	To 6 in., incl.	To 7 in., incl.	To 8 in., incl.	To 9 in., incl.	To 10 in., incl.	To 11 in., incl.	To 12 in., incl.
	Class 1 Precision Gears for Aircraft, Printing Machinery, Etc.													
Not go	+ Limit	0.000	0.000	0.000	0.00025	0.0005	0.0005	0.0005	......	......	0.00075	......	......	0.001
Go	− Limit	.00025	.0005	.00075	.00075	.00075	.001	.001	......	......	.001	......	......	.001
	Tolerance	.00025	.0005	.00075	.001	.000125	.0015	.0015	......	......	.00175	......	......	.002
	Class 2 Automobile Transmission, Machine Tool, Etc., Gears													
Not go	+ Limit	0.00025	0.0005	0.00075	0.001	0.00125	0.0013	0.00175	......	......	0.002	......	......	0.0025
Go	− Limit	.00025	.0005	.00075	.001	.001	.0012	.00125	......	......	.002	......	......	.0025
	Tolerance	.0005	.001	.0015	.002	.00225	.0025	.003	......	......	.004	......	......	.005
	Class 3 Standard Jobbing Gears													
Not go	+ Limit	0.0005	0.00075	0.001	0.00125	0.0015	0.00175	0.002	......	......	0.003	......	......	0.004
Go	− Limit	.0005	.00075	.001	.00125	.0015	.00175	.002	......	......	.003	......	......	.004
	Tolerance	.001	.0015	.002	.0025	.003	.0035	.004	......	......	.006	......	......	.008

TABLE 17.—S.A.E. STANDARD FOUR-SPLINE FITTINGS
(Dimensions in inches)

Nominal diameter	D		d		W		h		T
	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	
				Permanent Fit					
3/4	0.750	0.749	0.637	0.636	0.181	0.179	0.056	0.055	78
7/8	.875	.874	.744	.743	.211	.209	.066	.065	107
1	1.000	.999	.850	.849	.241	.239	.075	.074	139
1 1/8	1.125	1.124	.956	.955	.271	.269	.084	.083	175
1 1/4	1.250	1.249	1.062	1.061	.301	.299	.094	.093	217
1 3/8	1.375	1.374	1.169	1.168	.331	.329	.103	.102	262
1 1/2	1.500	1.499	1.275	1.274	.361	.359	.112	.111	311
1 5/8	1.625	1.624	1.381	1.380	.391	.389	.122	.121	367
1 3/4	1.750	1.749	1.487	1.486	.422	.420	.131	.130	424
2	2.000	1.998	1.700	1.698	.482	.479	.150	.148	555
2 1/4	2.250	2.248	1.912	1.910	.542	.539	.169	.167	703
2 1/2	2.500	2.498	2.125	2.123	.602	.599	.187	.185	865
3	3.000	2.998	2.550	2.548	.723	.720	.225	.223	1,249
				To Slide When Not under Load					
3/4	0.750	0.749	0.562	0.561	0.181	0.179	0.094	0.093	123
7/8	.875	.874	.656	.655	.211	.209	.109	.108	167
1	1.000	.999	.750	.749	.241	.239	.125	.124	219
1 1/8	1.125	1.124	.844	.843	.271	.269	.141	.140	277
1 1/4	1.250	1.249	.937	.936	.301	.299	.156	.155	341
1 3/8	1.375	1.374	1.031	1.030	.331	.329	.172	.171	414
1 1/2	1.500	1.499	1.125	1.124	.361	.359	.187	.186	491
1 5/8	1.625	1.624	1.219	1.218	.391	.389	.203	.202	577
1 3/4	1.750	1.749	1.312	1.311	.422	.420	.219	.218	670
2	2.000	1.998	1.500	1.498	.482	.479	.250	.248	875
2 1/4	2.250	2.248	1.687	1.685	.542	.539	.281	.279	1,106
2 1/2	2.500	2.498	1.875	1.873	.602	.599	.312	.310	1,365
3	3.000	2.998	2.250	2.248	.723	.720	.375	.373	1,969

Fig. 43.—S.A.E. standard six-spline fittings.

Table 18.—S.A.E. Standard Six-spline Fittings

(Dimensions in inches)

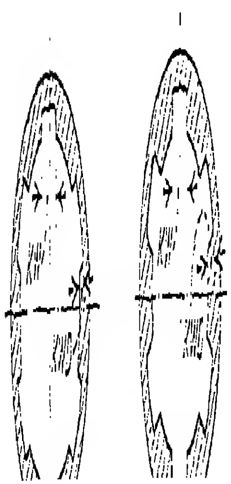
Nominal diameter	D Max.	D Min.	d Max.	d Min.	W Max.	W Min.	T
			Permanent Fit				
3/4	0.750	0.749	0.675	0.674	0.188	0.186	80
7/8	.875	.874	.788	.787	.219	.217	109
1	1.000	.999	.900	.899	.250	.248	143
1 1/8	1.125	1.124	1.013	1.012	.281	.279	180
1 1/4	1.250	1.249	1.125	1.124	.313	.311	223
1 3/8	1.375	1.374	1.238	1.237	.344	.342	269
1 1/2	1.500	1.499	1.350	1.349	.375	.373	321
1 5/8	1.625	1.624	1.463	1.462	.406	.404	376
1 3/4	1.750	1.749	1.575	1.574	.438	.436	436
2	2.000	1.998	1.800	1.798	.500	.497	570
2 1/4	2.250	2.248	2.025	2.023	.563	.560	721
2 1/2	2.500	2.498	2.250	2.248	.625	.622	891
3	3.000	2.998	2.700	2.698	.750	.747	1,283
			To Slide When Not under Load				
3/4	0.750	0.749	0.638	0.637	0.188	0.186	117
7/8	.875	.874	.744	.743	.219	.217	159
1	1.000	.999	.850	.849	.250	.248	208
1 1/8	1.125	1.124	.956	.955	.281	.279	263
1 1/4	1.250	1.249	1.063	1.062	.313	.311	325
1 3/8	1.375	1.374	1.169	1.168	.344	.342	393
1 1/2	1.500	1.499	1.275	1.274	.375	.373	468
1 5/8	1.625	1.624	1.381	1.380	.406	.404	550
1 3/4	1.750	1.749	1.488	1.487	.438	.436	637
2	2.000	1.998	1.700	1.698	.500	.497	833
2 1/4	2.250	2.248	1.913	1.911	.563	.560	1,052
2 1/2	2.500	2.498	2.125	2.123	.625	.622	1,300
3	3.000	2.998	2.550	2.548	.750	.747	1,873
			To Slide When under Load				
3/4	0.750	0.749	0.600	0.599	0.188	0.186	152
7/8	.875	.874	.700	.699	.219	.217	207
1	1.000	.999	.800	.799	.250	.248	270
1 1/8	1.125	1.124	.900	.899	.281	.279	342
1 1/4	1.250	1.249	1.000	.999	.313	.311	421
1 3/8	1.375	1.374	1.100	1.099	.344	.342	510
1 1/2	1.500	1.499	1.200	1.199	.375	.373	608
1 5/8	1.625	1.624	1.300	1.299	.406	.404	713
1 3/4	1.750	1.749	1.400	1.399	.438	.436	827
2	2.000	1.998	1.600	1.598	.500	.497	1,080
2 1/4	2.250	2.248	1.800	1.798	.563	.560	1,367
2 1/2	2.500	2.498	2.000	1.998	.625	.622	1,688
3	3.000	2.998	2.400	2.398	.750	.747	2,430

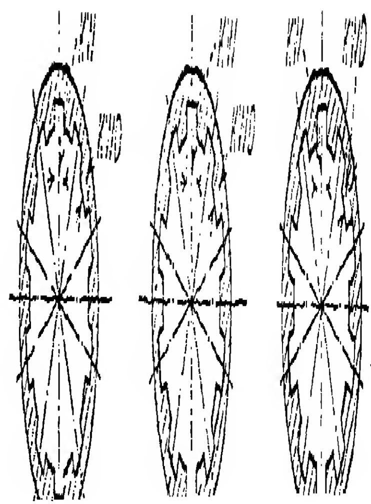
Fig. 44.—S.A.E. standard ten-spline fittings.

Table 19.—S.A.E. Standard Ten-spline Fittings

(Dimensions in inches)

Nominal diameter	D Max.	D Min.	d Max.	d Min.	W Max.	W Min.	T
			Permanent Fit				
3/4	0.750	0.749	0.683	0.682	0.117	0.115	120
7/8	.875	.874	.796	.795	.137	.135	165
1	1.000	.999	.910	.909	.156	.154	215
1 1/8	1.125	1.124	1.024	1.023	.176	.174	271
1 1/4	1.250	1.249	1.138	1.137	.195	.193	336
1 3/8	1.375	1.374	1.251	1.250	.215	.213	406
1 1/2	1.500	1.499	1.365	1.364	.234	.232	483
1 5/8	1.625	1.624	1.479	1.478	.254	.252	566
1 3/4	1.750	1.749	1.593	1.592	.273	.271	658
2	2.000	1.998	1.820	1.818	.312	.309	860
2 1/4	2.250	2.248	2.048	2.046	.351	.348	1,088
2 1/2	2.500	2.498	2.275	2.273	.390	.387	1,343
3	3.000	2.998	2.730	2.728	.468	.465	1,934
3 1/2	3.500	3.497	3.185	3.182	.546	.543	2,632
4	4.000	3.997	3.640	3.637	.624	.621	3,438
4 1/2	4.500	4.497	4.095	4.092	.702	.699	4,351
5	5.000	4.997	4.550	4.547	.780	.777	5,371
5 1/2	5.500	5.497	5.005	5.002	.858	.855	6,500
6	6.000	5.997	5.460	5.457	.936	.933	7,735
			To Slide When Not under Load				
3/4	0.750	0.749	0.645	0.644	0.117	0.115	183
7/8	.875	.874	.753	.752	.137	.135	248
1	1.000	.999	.860	.859	.156	.154	326
1 1/8	1.125	1.124	.968	.967	.176	.174	412
1 1/4	1.250	1.249	1.075	1.074	.195	.193	508
1 3/8	1.375	1.374	1.183	1.182	.215	.213	614
1 1/2	1.500	1.499	1.290	1.289	.234	.232	732
1 5/8	1.625	1.624	1.398	1.397	.254	.252	860
1 3/4	1.750	1.749	1.505	1.504	.273	.271	997
2	2.000	1.998	1.720	1.718	.312	.309	1,302
2 1/4	2.250	2.248	1.935	1.933	.351	.348	1,647
2 1/2	2.500	2.498	2.150	2.148	.390	.387	2,034
3	3.000	2.998	2.580	2.578	.468	.465	2,929
3 1/2	3.500	3.497	3.010	3.007	.546	.543	3,987
4	4.000	3.997	3.440	3.437	.624	.621	5,208
4 1/2	4.500	4.497	3.870	3.867	.702	.699	6,591
5	5.000	4.997	4.300	4.297	.780	.777	8,137
5 1/2	5.500	5.497	4.730	4.727	.858	.855	9,846
6	6.000	5.997	5.160	5.157	.936	.933	11,718
			To Slide When under Load				
3/4	0.750	0.749	0.608	0.607	0.117	0.115	241
7/8	.875	.874	.709	.708	.137	.135	329
1	1.000	.999	.810	.809	.156	.154	430
1 1/8	1.125	1.124	.911	.910	.176	.174	545
1 1/4	1.250	1.249	1.013	1.012	.195	.193	672
1 3/8	1.375	1.374	1.114	1.113	.215	.213	813
1 1/2	1.500	1.499	1.215	1.214	.234	.232	967
1 5/8	1.625	1.624	1.316	1.315	.254	.252	1,135
1 3/4	1.750	1.749	1.418	1.417	.273	.271	1,316
2	2.000	1.998	1.620	1.618	.312	.309	1,720
2 1/4	2.250	2.248	1.823	1.821	.351	.348	2,176
2 1/2	2.500	2.498	2.025	2.023	.390	.387	2,688
3	3.000	2.998	2.430	2.428	.468	.465	3,869
3 1/2	3.500	3.497	2.835	2.832	.546	.543	5,266
4	4.000	3.997	3.240	3.237	.624	.621	6,878
4 1/2	4.500	4.497	3.645	3.642	.702	.699	8,705
5	5.000	4.997	4.050	4.047	.780	.777	10,746
5 1/2	5.500	5.497	4.455	4.452	.858	.855	13,003
6	6.000	5.997	4.860	4.857	.936	.933	15,475





SECTION IV

SPUR-GEAR CALCULATIONS

The general physical characteristics and common dimensions of toothed-gear members include: number of teeth; pitch, either circular or diametral; pitch diameter; pitch circle; face; bore; outside diameter; root diameter; diameter of hub; keyway; backlash; tooth forms (see Sec. I); materials; and customary practice divides the gear members in a train of two or more engaging gears into *gears* and *pinions*. The latter are the smaller gears,

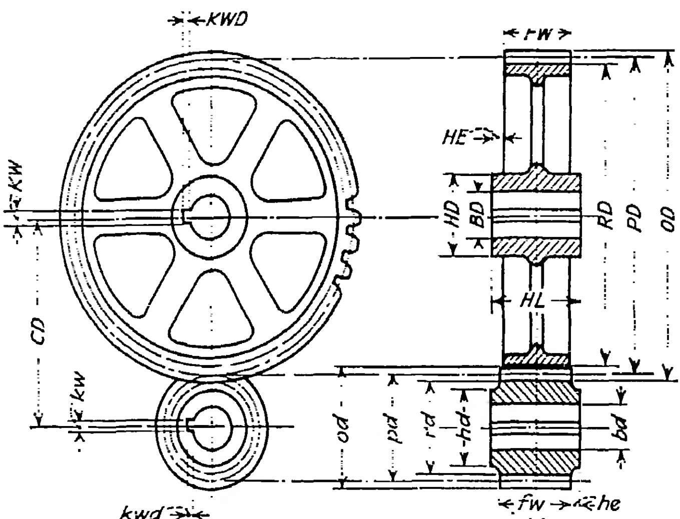


FIG. 45.—Spur gear and pinion.

usually the driving members of a gear train; and the former, the larger, and usually driven, members.

In the case of spur gears, the most commonly employed type of toothed gearing, the gear teeth are parallel with the axis or bore of the gear. These gears are used to transmit motion between parallel shafts, and high-grade cut spur gears, when correctly proportioned and mounted, have an average power transmitting efficiency of about 98 per cent at normal speeds and loads.

SPUR-GEAR NOMENCLATURE*

Dimension	Symbol	Dimension	Symbol
Addendum	*A*	Hub extension	*HE*
Angle, pressure	*VP*	Hub length	*HL*
Arc of action	*AA*	Keyway depth	*KWD*
Arc of approach	*AP*	Keyway width	*KW*
Arc of recession	*AR*	Normal involute pitch	*NIP*
Backlash	*B*	Number of teeth	*N*
Base-circle diameter	*BCD*	Number of teeth in contact	*NTC*
Base-circle radius	*BCR*	Outside diameter	*OD*
Bore diameter	*BD*	Outside radius	*OR*
Center distance	*CD*	Pitch-circle circumference	*PCC*
Chordal addendum	*CA*	Pitch diameter	*PD*
Circular pitch	*CP*	Pitch radius	*PR*
Dedendum	*D*	Root diameter	*RD*
Diametral pitch	*DP*	Root radius	*RR*
Face width	*FW*	Tooth bearing	*TB*
Gear ratio	*GR*	Whole depth (tooth)	*WD*
Hub diameter	*HB*		

* Symbols for pinions are customarily distinguished from corresponding symbols for gears by the use of small, instead of capital, letters.

GENERAL RELATIONSHIPS

There are certain definite relationships existing between certain physical characteristics and common dimensions of all standard types of engaging spur-tooth gears, irrespective of the form or arrangement of the teeth, which are based upon the diameters of the gears' respective pitch circles and, hence, upon the pitches, circular and diametral, of the gear teeth. These, together with the distance between the centers of engaging gears, form the bases of all spur-gear calculations and are

$$CP = \frac{3.1416}{DP} = \frac{3.1416PD}{N} \tag{13}$$

$$DP = \frac{3.1416}{CP} = \frac{N}{PD} \tag{14}$$

$$PD = \frac{N}{DP} = 0.3183N \times CP \tag{15}$$

$$CD = \frac{PD + pd}{2} = \frac{N + n}{2DP} \tag{16}$$

Formulas for Standard-depth Spur Gears

(A.G.M.A. 14½-deg. composite, 14½- and 20-deg. involute full-depth systems)

$$A = \frac{1}{DP} = \frac{CP}{3.1416} = \frac{PD}{N} = \frac{OD}{N + 2} \tag{17}$$

$$D = \frac{1.157}{DP} = 0.3683CP \tag{18}$$

$$WD = A + D = \frac{2.157}{DP} = 0.6866CP \tag{19}$$

$$C = \frac{0.157}{PD} = 0.05CP \tag{20}$$

$$OD = \frac{N + 2}{DP} = PD + \frac{2}{DP} = \frac{(N + 2)PD}{N} = (N + 2)A$$
$$= 0.3183(N + 2)CP = PD + 0.6366CP \tag{21}$$

$$RD = OD - 2WD \tag{22}$$

$$N = PD \times DP = \frac{3.1416PD}{CP} \tag{23}$$

Formulas for A.G.M.A. 20-deg. stub-tooth gears

$$A = \frac{0.8}{DP} = 0.2546CP \tag{24}$$

$$D = \frac{1}{DP} = 0.3183CP \tag{25}$$

$$WD = \frac{1.8}{DP} = 0.5729CP \tag{26}$$

$$C = \frac{0.2}{DP} = 0.0636CP \tag{27}$$

$$OD = \frac{N + 1.6}{DP} = 0.5092PD \times CP \tag{28}$$

SECTION V

STRAIGHT-TOOTH BEVEL GEARS

The design of straight-tooth bevel gearing is complicated, not only by the fact that the proportions of the teeth vary over the face width of the gears, but by the fact that the rolling pitch surface of a bevel gear is, at each point, of a somewhat longer radius of curvature than is the pitch circle passing through the same point. This materially modifies the form of the teeth that can be successfully employed and makes the machining operations entailed in the commercial production of bevel gearing more involved than those in the production of ordinary spur gears.

STRAIGHT-TOOTH BEVEL-GEAR NOMENCLATURE*
Number of teeth, N

Angle	Symbol	Angle	Symbol
Angle of axes	*VA*	Decrement angle	*VD*
Back angle	*VBk*	Face angle	*VF*
Bottom (cutting) angle	*VB*	Increment angle	*VI*
Center angle	*VC*	Pressure angle	*VP*
Dimension		Dimension	
Addendum	*A*	Diametral pitch	*DP*
Apex distance	*AD*	Diametral pitch, inner	*DPs*
Backing (pitch line)	*X*	Face width	*FW*
Bore diameter	*BD*	Hub length	*HL*
Circular pitch	*CP*	Mounting distance	*MD*
Circular thickness (tooth)	*CTh*	Outer diameter	*OD*
Crown backing	*CB*	Pitch diameter	*PD*
Dedendum	*D*	Whole depth (tooth)	*WD*
Diameter increment	*DI*		

* Symbols for pinions are customarily distinguished from corresponding symbols for gears by the use of small, instead of capital, letters.

Prior to the development of the modern bevel-gear generating machine, by which the gear teeth with correctly varying profile curvature at all points are accurately generated by a process reproducing the smooth rolling action of a pair of bevel gears in mesh, two general systems of bevel gearing were in vogue and

are still employed to some extent. In one of these systems, the gear teeth, machined with formed gear cutters, are of the tapering type and follow in general the proportions adopted for standard-spur gear teeth; in the other (parallel-depth bevel gears), the teeth are of uniform depth and of constant profile curvature.

Thus, three distinct basic systems of bevel gearing (standard-depth bevel gears, parallel-depth bevel gears, and generated-bevel gears) command consideration, although the quite general adoption of the A.G.M.A. (Gleason Works) standard for straight-tooth bevel gears makes it now of chief interest. However, the various systems of bevel gearing may properly be taken up in the order of their development.

STANDARD-DEPTH BEVEL GEARS

In the so-termed *standard-depth bevel-gear system*, having straight, radial, tapering teeth machined with formed cutters, the profile of the teeth as cut, which is at best simply an approximation of the desired tooth contour, is of the correct form only at the outer, or larger, ends of the teeth, for which the cutters are selected. An excess of metal is left outside the pitch line elsewhere, increasing in thickness toward the inner ends of the teeth, and this surplus is subsequently removed by filing (see Fig. 46); *i.e.*, the small ends of the teeth are rounded by filing. As there is nothing gained by having an excessively long tooth face, necessitating much filing, approved practice is to limit the face width of bevel gears to not over one-third the apex distance or to 2½ times the circular pitch of the gears.

The cutter selected for the machining operations, while governed as to form by the required contour of the teeth at their outer ends, must also be proportioned so as to clear the small, inner ends of the gear teeth. A central cut, leaving the teeth somewhat heavy, is first taken, in which the tooth profiles are, of necessity, left normal to the path of the cutting tool. Consequently, in the finishing cuts for the respective sides of the teeth, the gear blank has to be indexed over, so as to bring the pitch elements of the gear teeth at the outer pitch circle parallel to the path of the cutting tool. The finishing cuts are then taken and the teeth finished to correct outer-end thickness, leaving the surplus metal to be removed by filing toward the inner end of the gear teeth and outside the pitch line.

The pitch, all tooth dimensions, pitch and outer diameters are measured at the larger, outer ends of the gear teeth, as is quite general for bevel gearing, and the conical pitch surfaces of the mating gears establish the critical center, face, and bottom, or cutting, angles of the respective gear members. For bevel gears meshing at right angles, the natural tangent of the center angle of either gear is found by dividing the number of its teeth by the number of teeth on the meshing gear. For example, the tangent of the center angle of a 16-tooth bevel gear meshing with a 12-tooth bevel pinion is 1.333, and the tangent of the center angle of the pinion is 0.750.

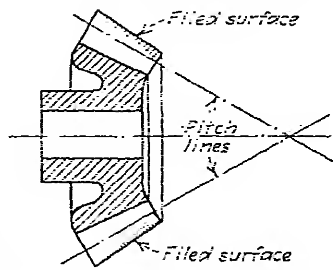


FIG. 46.—Filed surfaces of standard-depth bevel gears.

A comparatively small increment angle added to the center angle establishes the face angle and a slightly larger decre-

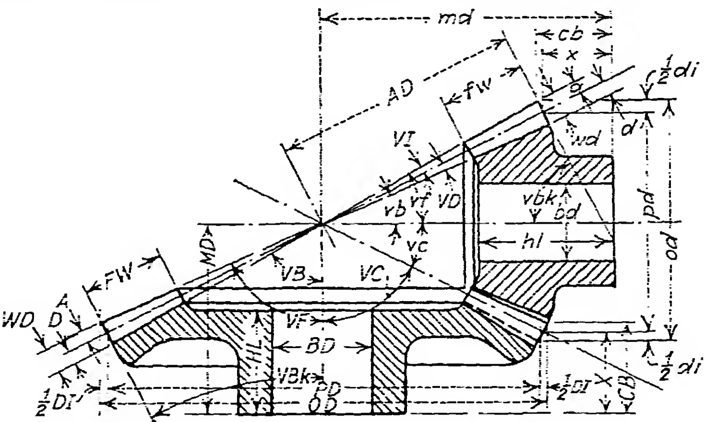


FIG. 47.—Standard-depth bevel gears.

ment angle deducted from the center angle the bottom, or cutting, angle, such small angles being governed by the addendum and dedendum dimensions, respectively, of the bevel-gear teeth. The back angle of a bevel gear is the complement of its center angle.

Formulas for Standard-depth Bevel Gears—Axes at Right Angles

$$\tan VC = \frac{N}{n} \text{ or } \tan vc = \frac{n}{N} \tag{29}$$

$$\tan VI = \frac{2 \sin VC}{N} \quad \frac{A}{AP} \tag{30}$$

$$\tan VD = \frac{2.314 \sin VC}{N} = \frac{D}{AP} \tag{31}$$

$$VF = VC + VI \tag{32}$$

$$VB = VC - VD \tag{33}$$

$$AD \quad \frac{PD}{2 \sin VC} = \frac{N}{2DP \sin VC} \tag{34}$$

$$CP \quad \frac{3.1416}{DP} = \frac{3.1416PD}{N} \tag{13}$$

$$DP = \frac{N}{PD} = \frac{3.1416}{CP} \tag{14}$$

$$PD = \frac{N}{DP} = 0.3183N \times CP \tag{15}$$

$$DI \quad 2A \cos VC \tag{35}$$

$$OD \quad PD + DI \tag{36}$$

$$FW = \frac{AD}{3} \text{ or } \frac{5CP}{2} \text{ (whichever is smaller)} \tag{37}$$

Formulas for Standard-depth Bevel Gears—Axes at Odd Angles

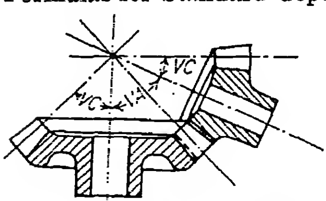


FIG. 48.—Bevel gears with shafts at less than 90 deg.

$$\tan VC = \frac{\sin VA}{\frac{n}{N} + \cos VA} \tag{38}$$

$$vc = VA - VC$$

$$\tan VC = \frac{\sin (180 - VA)}{\frac{n}{N} - \cos (180 - VA)} \tag{39}$$

$$vc = VA - VC$$

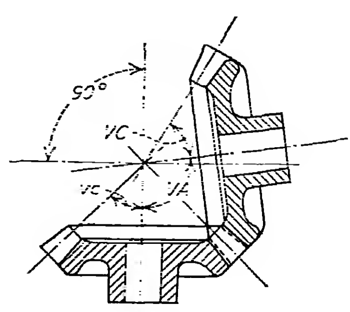


FIG. 49.—Bevel gears with shafts at more than 90 deg.

$$VC = 90 \text{ deg.}$$
$$vc = VA - 90 \text{ deg.}$$

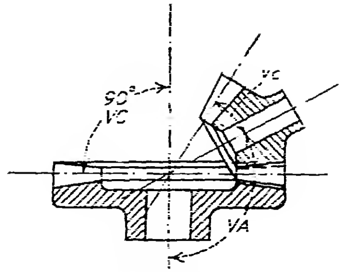


FIG. 50.—Crown gears, 90-deg. center angle.

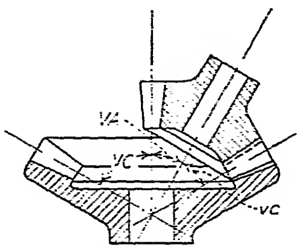


FIG. 51.—Internal bevel gears.

$$\tan VC = \frac{\sin VA}{\sin VA - \dfrac{n}{N}} \tag{40}$$
$$vc = VC - VA$$

PARALLEL-DEPTH BEVEL GEARS

In the parallel-depth bevel-gear system, developed to avoid the necessity of file finishing the inner ends of standard-depth bevel gears machined with formed cutters, the depth of the teeth,

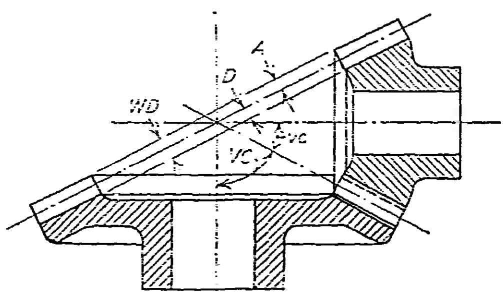


FIG. 52.—Parallel-depth bevel gears.

which are finished with standard spur gear cutters, is constant from end to end and, consequently, the profile curvature of the teeth as well. There are no decrement or increment angles and, which is unusual, the diametral pitch at the inner end of the bevel-gear tooth governs the tooth proportions of parallel-depth

bevel gears. The resulting teeth are of a distinctive stubby appearance at their outer edge, and the form of tooth is considerably easier to machine with formed cutters than the tapering form of tooth.

Formulas for Parallel-depth Bevel Gears[1]

$$DPs = \frac{DP \times AD}{AD - FW} \tag{41}$$

$$A = \frac{1}{DPs} \tag{42}$$

$$D = \frac{1.157}{DPs} \tag{43}$$

$$WD = A + D = \frac{2.157}{DPs} \tag{44}$$

GENERATED BEVEL GEARS

The foregoing systems of bevel gearing, the standard-depth bevel gears machined with formed cutters and the parallel-

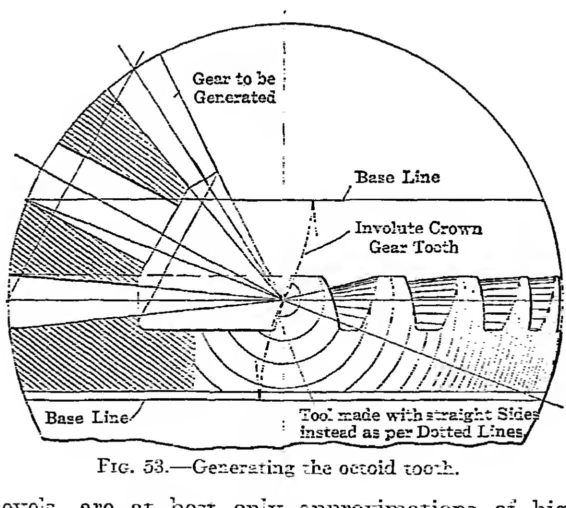


FIG. 53.—Generating the octoid tooth.

depth bevels, are at best only approximations of high-grade gearing, and, while fairly satisfactory for gears that are to run at low speeds, for gears of fine pitch and for gears that are to carry only light loads, they do not meet today's requirements in the automotive field, those of high-speed machine-tool gears and

[1] For dimensions of parallel-depth bevel gears other than those of tooth-depth proportions, use formulas for standard-depth bevel gears.

where the use of high-quality bevel gearing is virtually essential. Furthermore, the question of expense in commercial production of high-grade gearing is an important consideration, for obviously it is a costly, exacting, and time-consuming task to file accurately the teeth of standard-depth bevel gears machined with formed cutters, especially the teeth of coarse-pitch gears.

In fact, bevel gears with correctly shaped teeth can be machined economically only by a generating process and with teeth of the octoid form, rather than of the involute type. The reason for this special shape of tooth is that, while ordinary spur gearing can be generated by the action of a reciprocating tool representing the basic straight-sided rack tooth of the involute spur-gear system, a true involute bevel-gear crown tooth does not, like a spur-gear rack-tooth, have straight sides but has profiles of distinctive double curvature (see Fig. 53) that cannot be readily duplicated for a cutting generating tool. Consequently, the basic crown gear tooth in generated bevel gearing is made with straight sides, like a spur-gear rack-tooth, and the profile curvature of generated bevel-gear teeth is of octoid and not involute form (see Fig. 54).

Involute Tooth

Octoid Tooth

FIG. 54.—Comparison of involute and octoid bevel gear tooth.

A.G.M.A. STANDARD FOR STRAIGHT-TOOTH BEVEL GEARS

(Gleason Works System)

With the idea of developing a thoroughly practical system of strong, quiet, and wear-resisting bevel gears, well suited to production by generating process, the lead was taken and much pioneering work conducted by the Gleason Works of Rochester, N. Y. This progressive organization developed a comprehensive system, applicable to any pair of generated bevel gears operating at right angles when the pinion member is the driver and has 10 or more teeth, that has now been adopted by the A.G.M.A. and has become the approved system of straight-tooth bevel gearing.

The principal qualities considered in arriving at this system, arranged in the order of their recognized importance, are quietness, strength, and durability, all of which it was found could be best secured by the use of the lowest pressure angle that can be employed without sacrifice of tooth strength through excessive undercutting. Furthermore, as the system has been laid out,

the strength of the pinion teeth is about on a par with that of the mating gear teeth, when the pinions and gears are made of the same material.

The greater arc of action secured when the lower pressure angles are used tends to quiet gear operation, any eccentricity has less effect and the radial component of the tooth load is minimized. This avoidance, so far as possible, of the thrust forces that are common to all systems of bevel gearing is especially desirable, for the majority of bevel gears are overhung from their supports and the total load should, consequently, be kept as low as possible. The greater arc of action of gears with low-pressure angles also has the desirable effect of offsetting in large measure any strength advantage possessed by the heavier section of a higher pressure angle tooth that is handicapped by a smaller arc of action.

Durability is also traceable to a considerable extent to the greater arc of action characteristic of low-pressure angle gearing, since the greatest amount of wear tends to take place near the pitch point where the unit pressure is the greatest. When the point of contact between engaging gear teeth is near the pitch point, all the load is borne by a single tooth, while at the beginning and end of action the load is distributed over two teeth with a consequent reduction in unit pressure. This means in substance that gear-tooth wear tends to be greatest where there is actually a minimum of sliding and a maximum of rolling action and demonstrates that the chief cause of gear-tooth wear is a matter of unit pressure, rather than of sliding action.

A.G.M.A. Proportions for Generated Straight-tooth Bevel Gears Operating at Right Angles, Where the Pinion Is the Driver and Has 10 or More Teeth

(Gleason Works System)

Pressure angles.

Ratios having 14 or more teeth in pinion	14½
13–13 to 13–24	17½
13–25 and higher	14½
12–12 and higher	17½
11–11 to 11–14	20
11–15 and higher	17½
10–10 and higher	20

TABLE 20.—ADDENDUM VALUES FOR ONE DIAMETRAL PITCH FOR DIFFERENT RATIOS

(Gleason Works System)

$$\text{Gear ratio} = \frac{N}{n} = \frac{\text{number of teeth in gear}}{\text{number of teeth in pinion}}$$

Ratios		A	Ratios		A	Ratios		A	Ratios		A
From	To		From	To		From	To		From	To	
1.00	1.01	1.000	1.15	1.17	0.880	1.42	1.45	0.760	2.06	2.16	0.640
1.01	1.02	.990	1.17	1.19	.870	1.45	1.48	.750	2.16	2.27	.630
1.02	1.03	.980	1.19	1.21	.860	1.48	1.52	.740	2.27	2.41	.620
1.03	1.04	.970	1.21	1.23	.850	1.52	1.56	.730	2.41	2.58	.610
1.04	1.05	.960	1.23	1.25	.840	1.56	1.60	.720	2.58	2.78	.600
1.05	1.06	.950	1.25	1.27	.830	1.60	1.65	.710	2.78	3.05	.590
1.06	1.08	.940	1.27	1.29	.820	1.65	1.70	.700	3.05	3.41	.580
1.08	1.09	.930	1.29	1.31	.810	1.70	1.76	.690	3.41	3.94	.570
1.09	1.11	.920	1.31	1.33	.800	1.76	1.82	.680	3.94	4.82	.560
1.11	1.12	.910	1.33	1.36	.790	1.82	1.89	.670	4.82	6.81	.550
1.12	1.14	.900	1.36	1.39	.780	1.89	1.97	.660	6.81	∞	.540
1.14	1.15	.890	1.39	1.42	.770	1.97	2.06	.650			

Addendum:

(Gear)

$$A = \frac{\text{value, Table 20}}{DP} \tag{45a}$$

(Pinion)

$$a = \frac{2.000}{DP} - A \tag{45b}$$

Dedendum:

(Gear)

$$D = \frac{2.188}{DP} - A \tag{46a}$$

(Pinion)

$$d = \frac{2.188}{DP} - a \tag{46b}$$

Whole Depth:

$$WD \quad \frac{2.188}{DP} \tag{47}$$

Circular Thickness of Teeth:

(Gear—14½ deg. *VP*)

$$CTh = \frac{1.071}{DP} + 0.5A - \frac{K\ (\text{Table 21})}{DP} \qquad (48a)$$

(Gear—17½ deg. *VP*)

$$CTh = \frac{0.971}{DP} + 0.6A - \frac{K}{DP} \qquad (48b)$$

(Gear—20 deg. *VP*)

$$CTh = \frac{0.871}{DP} + 0.7A - \frac{K}{DP} \qquad (48c)$$

(Pinions—14½ deg., 17½ deg., or 20 deg. *VP*)

$$cth = \frac{3.142}{DP} - CTh \qquad (48d)$$

The Gleason Works also prepared a table giving computed values of the form factor Y appearing in the Lewis formulas for the strength of gear teeth, obtained, as in previous determinations of this important factor, by considering the load as applied at the outer end of the recommended form of generated bevel-gear tooth (see Table 22). Actually, in the Gleason Works system, as in all efficiently operating gear assemblages, the point of application of load should be determined for each separate combination of gears, for the number of teeth in contact at the same time exerts considerable influence upon the actual strength of the gearing, which, as a rule, is a good deal higher than the values arrived at by the Lewis method of computation.

"MASTER-FORM" BEVEL GEARING

Another meritorious contribution to the art of bevel gearing was that made by the late Harvey D. Williams, in which marked reductions in the expenses of bevel-gear production are secured by taking advantage of both the low cost of cutting parallel-depth bevel-gear teeth and of the economies of gear-tooth generation as well. The teeth of the larger gear, in this system, are machined with simple, rack-tooth-shaped, high-speed milling cutters, while the less numerous teeth of the mating bevel pinion are cheaply and accurately produced by modern gear-generating machines with cutting tools that are the conjugate of the simple, straight-sided tools employed for gashing out the tooth spaces in the "master-form" gears.

TABLE 21.—VALUES OF K FOR CIRCULAR THICKNESS OF TOOTH FORMULAS
(Gleason Works System)

Number of teeth in pinion (n)	Ratios														
	1.00 to 1.25	1.25 to 1.50	1.50 to 1.75	1.75 to 2.00	2.00 to 2.25	2.25 to 2.50	2.50 to 2.75	2.75 to 3.00	3.00 to 3.25	3.25 to 3.50	3.50 to 3.75	3.75 to 4.00	4.00 to 4.50	4.50 to 5.00	5.00 to ∞
10	0.025	0.070	0.100	0.120	0.140	0.160	0.175	0.190	0.205	0.215	0.225	0.230	0.240	0.250	0.255
11	.010	.015	.050	.080	.105	.125	.145	.160	.170	.180	.190	.195	.200	.210	.220
12	.000	.040	.070	.100	.120	.140	.155	.170	.180	.185	.190	.195	.205	.210	.215
13	.000	.015	.040	.045	.050	.060	.070	.080	.090	.100	.110	.120	.135	.150	.165
14	.000	.015	.030	.050	.065	.080	.090	.100	.110	.120	.125	.130	.140	.150	.160
15–17	.000	.000	.010	.020	.030	.045	.060	.070	.080	.090	.095	.100	.110	.115	.120
18–21	.000	.000	.000	.000	.010	.030	.045	.060	.070	.080	.085	.090	.095	.100	.100
22–29	.000	.000	.000	.000	.010	.030	.040	.050	.060	.065	.070	.070	.080	.085	.085
30 up	.000	.000	.000	.000	.010	.025	.035	.040	.045	.050	.055	.060	.065	.070	.070

TABLE 22.—FORM FACTOR Y (LEWIS FORMULAS) FOR GENERATED BEVEL-GEAR TEETH
(Gleason Works System)

Number of teeth in pinion (n)	Ratios														
	1.00 to 1.25	1.25 to 1.50	1.50 to 1.75	1.75 to 2.00	2.00 to 2.25	2.25 to 2.50	2.50 to 2.75	2.75 to 3.00	3.00 to 3.25	3.25 to 3.50	3.50 to 3.75	3.75 to 4.00	4.00 to 4.50	4.50 to 5.00	5.00 to ∞
10	0.231	0.260	0.280	0.294	0.305	0.315	0.324	0.332	0.340	0.347	0.353	0.358	0.365	0.371	0.377
11	.238	.264	.273	.286	.296	.303	.309	.315	.320	.324	.328	.332	.336	.340	.342
12	.248	.265	.281	.295	.308	.318	.328	.335	.341	.345	.348	.351	.353	.355	.356
13	.264	.278	.291	.280	.278	.286	.291	.295	.298	.299	.301	.303	.305	.307	.310
14	.242	.254	.263	.272	.281	.288	.294	.299	.304	.307	.310	.313	.316	.318	.319
15	.248	.258	.266	.274	.283	.290	.296	.301	.305	.308	.312	.315	.318	.319	.320
16	.252	.261	.269	.277	.285	.292	.298	.304	.308	.312	.314	.317	.319	.321	.323
17 18	.257	.265	.273	.281	.288	.295	.302	.307	.311	.315	.318	.320	.322	.325	.326
19 21	.265	.272	.279	.286	.294	.300	.307	.312	.317	.320	.324	.326	.328	.330	.332
22 25	.274	.281	.288	.295	.301	.307	.314	.319	.324	.327	.331	.332	.335	.337	.338
26 30	.284	.291	.297	.304	.310	.317	.322	.327	.332	.336	.339	.342	.344	.346	.347

This radical departure from the usual design of bevel gearing (see Fig. 55) is particularly noteworthy in that it is a reversion to and adaptation of fundamental mechanical principles which appear to have been overlooked until quite recently in the evolution of gearing, in spur gearing as well as bevel gearing. Toothed gears will roll together smoothly and efficiently, provided their teeth and tooth spaces are conjugately related. This is the one essential requirement, and the form and shape of the teeth are of quite secondary importance.

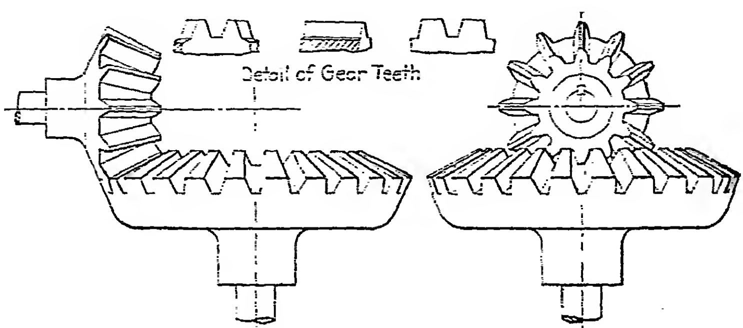


FIG. 55.—"Master-form" bevel gearing.

The development of the almost universal octoid system for bevel gearing, with its varying tooth profile, not only from end to end of tooth, but for every size of gear, is based on the requirement that each and every bevel gear be conjugately related to a crown gear with flat-sided teeth. This is in line with the simple rack-tooth base for the involute system of spur gearing, and, though there may be some advantage to such a basis in spur gearing, although in commercial gear production in large quantities this is now recognized as more or less of a fallacy, there is none whatever in the case of bevel gearing.

In the master-form bevel gear, the teeth are almost as readily cut as those of an ordinary spur rack and more expeditiously and cheaply than they can be generated. Each gear cut with these master-form teeth becomes the standard to which its mating pinion or pinions are proportioned. As these pinions can be generated as readily as those with octoid teeth, no added difficulty of expense is entailed in the manufacture of the pinions.

PRODUCTION ECONOMIES

The cost of cutting master-form bevel gears depends largely upon the number of teeth on the gears, and it has been estimated that about 80 per cent, of the combined number of teeth on the gear and pinions of automobile transmissions, for which mechanisms these distinctive gears are well suited, are on the gear, or larger wheel. The master-form system materially reduces the cost of cutting the teeth on the gear, resulting in a considerable reduction in the cost of manufacturing the complete transmission, as the cost of generating the pinion is no greater than that of generating an octoid pinion of the same size.

The economy in manufacturing cost is probably the chief advantage of this system of bevel gearing, but it also possesses other features which have considerable merit. The peculiar shape of the teeth makes the gears exceedingly smooth running, and they are said to develop unusually high efficiency in operation. Another advantage of the gears—one which is of particular value in automobile drives—is the distinctive capacity of the gears to discount disalignment.

EFFECT OF SPRUNG SHAFTS

The strains and wrenches to which the transmission of an automobile is frequently unavoidably subjected have a tendency to

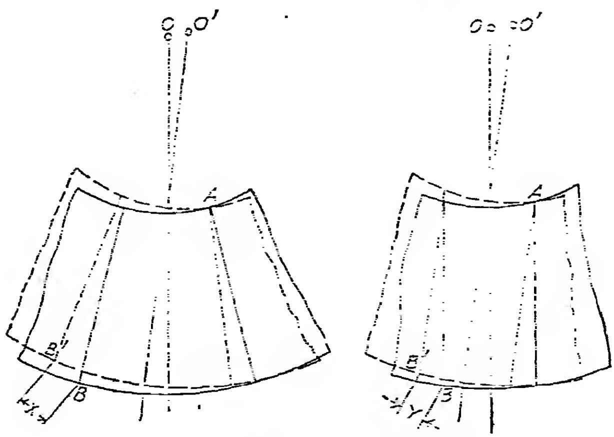


FIG. 56.—Effects of shaft disalignment.

spring the driving shaft out of alignment, seriously interfering with the satisfactory operation of the transmission, which depends

largely upon the exactitude of shaft alignment. The teeth of the gear may remain in mesh with those of the pinion, but the sprung shaft throws a heavy unbalanced pressure on the gears, causing noisy operation and undue tooth wear. This is well-nigh unavoidable whatever system of gearing is employed, but with master-form gearing the troubles produced by sprung shafts are greatly discounted.

Figure 56 shows diagrammatically what might be expected to occur should the transmission shaft of an automobile be sprung out of alignment, from O to O'. The normal position of the gears with their centers at O is depicted in full lines and in sprung position with centers at O'. For the gears to remain in mesh, the inner corner of the tooth A remains in the same relative position, but, if it were not for the restraining influence of the meshing teeth, the diagonally opposite corner would move from B to B' and the strain to which the gearing is subjected in preventing such movement is measured by the distance between B and B'. In the case of the ordinary octoid form of tooth, the disalignment strain is measured by the distance X and in the case of the master form of tooth by the distance Y, and Y is very appreciably less than X. The difference in intensity of strain, furthermore, is much more than the respective differences between the points B and B' and, if the gears are under considerable load, may produce fracture in one case and not in the other.

EFFICIENCY OF BEVEL GEARING

The chief cause of decrease in efficiency in bevel gearing is due to inaccuracies in the shape and finish of the teeth, whereby a lateral thrust is produced that has a tendency to force the gears out of mesh. This is especially noticeable in bevel gears produced with formed cutters, unless the gears have teeth of the parallel-depth variety, owing to the fact that the center angles of such gears and pinions seldom coincide exactly as they roll together. Under such circumstances, the tooth pressure cannot be normal at all points of engaging teeth, and a decided lateral thrust tending to separate the teeth is produced. Furthermore, the pinion, and usually the gear member as well, has to be overhung, so that the thrust is greatly increased at the bearing by leverage, materially increasing friction, twisting the shaft, etc.

Nevertheless, the average efficiency of modern, high-quality bevel gearing is about 96 per cent under favorable conditions,

although speed ratios in excess of about 6 to 1 are not recommended. The most efficient center angle is that of 45 deg. and

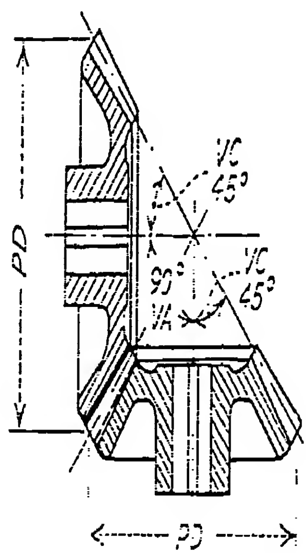


Fig. 57.—Miter gears.

miter gears, *i.e.*, bevels of 1 to 1 ratio transmitting power between shafts at right angles, the most satisfactory of bevel-gearing assemblages when accurately proportioned and properly mounted.

SECTION VI

HELICAL AND HERRINGBONE SPUR GEARS

That silence and smoothness in operation, long gear life, and high operating efficiency demanded of modern, quality, gearing assemblages are well served by the expediency of having the gear teeth extend helically across the face of the gears, instead of

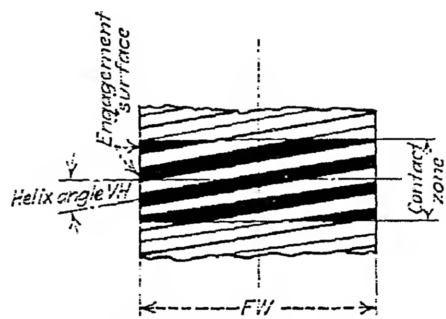


FIG. 58.—Helical arrangement of spur-gear teeth.

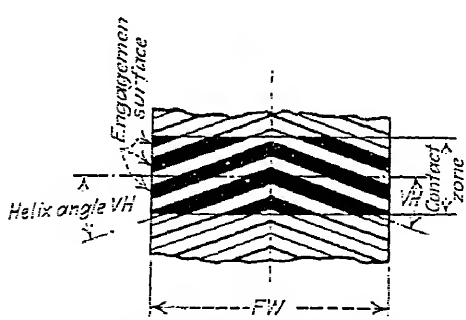


FIG. 59.—Herringbone arrangement of spur-gear teeth.

axially, as in ordinary spur gearing. The obliquity thus given to the teeth keeps each meshing pair in engagement until one or more of the following sets, or pairs, of teeth are well engaged, so that there is at no time any sudden transference of load from tooth to tooth or shock of sudden tooth impact. The load is gradually put on a tooth and as gradually taken off. Furthermore, in the development of this type of gearing, full advantage has been taken quite generally of long- and short-tooth addenda,

relatively high-pressure angles, and of a certain amount of stubbing of the involute form of gear tooth by manufacturers of both helical and herringbone types of gear, the latter variety being in effect simple forms of double-helical gears with the teeth-changing obliquity at mid-face.

The characteristic helical and herringbone gear tooth is not only considerably stronger than that of ordinary standard spur gearing, but it also secures much more rolling action between engaging gear teeth, and, consequently, the more rugged teeth are subjected to less destructive sliding wear. Also, the tooth overlap that distributes the load more uniformly over the points of tooth contact serves to prevent undue wear on any localized portion of the teeth, still further prolonging the useful life of the gears.

HELIX-ANGLE LIMITATIONS

In the ordinary type of helical gearing for connecting parallel shafts, the obliquity of the teeth across the full face of the gears sets up an unavoidable end, or axial, thrust, making it desirable to use two gears of opposing but similar obliquity on the respective shafts, so that the axial thrusts of the gears are counterbalanced, or else to employ intermediate gears by which a partial thrust balance is effected. However, while it is always advisable to employ duplicate helical gears of opposing tooth obliquity in this manner, limitations of space or other considerations frequently prohibit such constructions, making it necessary to rely upon the use of suitable thrust collars, or thrust-resisting bearings, to hold the gears in mesh. This, naturally, places an arbitrary limitation upon the degree of feasible helix angle in customary applications of helical gearing; in ordinary commercial practice, the helix angle of helical gears is advisably kept within values that will hold the developed end thrust to a maximum of 10 to 13 per cent of the transmitted load. Such amount of created axial pressure has proved not to be excessive in industrial practice and well within the range of efficient control by standard thrust-resisting accessories.

Where duplicate helical gears of opposing obliquity are employed on the same shaft for effecting a single stage in power transmissions, the construction is analogous to the use of herringbone gears, which by virtue of their distinctive tooth arrangement are free from unbalanced axial thrusts. The helix angle of

herringbone gears, consequently, is not so sharply curtailed, being usually about double that which is feasible in comparable helical gearing.

The primary object of the helical arrangement of the gear teeth being to secure continuity of tooth engagement, however, and, as when this is once secured further obliquity in the arrangement of the teeth tends simply to dissipate the applied power, the helix angle for herringbone gears is also arbitrarily limited. The maximum angle recommended by the A.G.M.A. is one of 45 deg., and this should only be employed in the case of high-speed ratios. When the engaging gears are more nearly of a size, the minimum advisable helix angle has been set by the same organization at 20 deg., the active face width of herringbone gears being usually about 50 per cent greater than the gear's diametral pitch in the plane of rotation. In the case of helical gears, while a helix angle of half that recommended for herringbone gears is required for gears of similar face width, smaller helix angles suffice for helical gears with face widths exceeding 1½ times the diametral pitch of the gears. The helix angle should be such as will assure simultaneous tooth engagement between at least two sets of meshing teeth.

DOUBLE-PITCH AND PRESSURE-ANGLE RELATIONSHIPS

The helical arrangement of helical and herringbone gear teeth introduces a double set of pitch values: one in the plane of the gear's rotation, which governs the pitch diameter of the gears and the number of gear teeth; and a normal tooth-section pitch, which governs the proportions of the gear teeth. As a result, the pressure angle of the gear teeth in the plane of rotation is considerably greater than the pressure angle of the normal tooth profile, upon which the outer and root diameters of the gears are dependent. The diametral pitch in the plane of the gear's rotation is equal to the normal

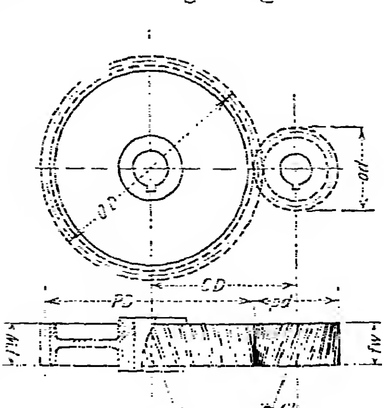


FIG. 60.—Helical spur gear and pinion.

diametral pitch multiplied by the cosine of the helix angle; and the tangent of the pressure angle in the plane of gear rotation is equal to the tangent of the normal pressure angle divided by the cosine of the helix angle.

The recommendations for minimum and maximum helix angles for herringbone gears are such that the minimum pressure angle in planes normal to the face of the teeth is approximately 14½ deg., with the minimum recommended helix angle; and the maximum normal pressure angle is approximately 18¼ deg., with the maximum recommended helix angle. These pressure angles in planes normal to the helical angle, it will be noted, are quite similar to the pressure angles customarily employed for ordinary straight-tooth spur gears.

MODIFICATIONS OF GEAR-TOOTH PROPORTIONS

As no tooth action can take place within the base circles of gears with teeth of involute form, since the involute profiles originate at the base circles, it may happen that for certain speed, or gear, ratios and pitches the well established tooth proportions for standard helical and herringbone gears do not provide sufficient radial distance between the pitch and base circles to accommodate adequate involute curvature for the dedendum portion of a well-proportioned pinion tooth. This will cause objectionable undercutting and weakening of the pinion tooth. Under these circumstances, while retaining the standard working depth of the tooth, the addenda of the pinion teeth may be increased somewhat and, consequently, the outside diameter of the pinions correspondingly enlarged. This has the effect of making use of a portion of the involute curve that is farther removed from the curve's origin for the working profile of the pinion tooth.

When it is desired to eliminate the undercutting and maintain the standard whole-tooth depth, the enlargement of the outside diameter of the pinion is

$$od \text{ enlarged} = od + \left(2D - \frac{(\sin VP)^2 \times n}{2DP}\right) \qquad (49a)$$

er, if it is desired simply to obtain full involute action the working depth of teeth, between the pinion and its mating gear, the outside diameter of the pinion, while maintaining the whole-tooth depth standard, should be made

$$od \text{ enlarged} = od + \left(2A - \frac{(\sin VP)^2 \times n}{2DP}\right) \qquad (493)$$

When the outside diameter of the pinion is enlarged in either of these ways, a maintenance of the center distance between gear and pinion necessitates that the outside diameter of the mating gear, to maintain standard whole-tooth depth, be contracted a similar amount. However, if the center distance can be increased, it is advisable to increase the distance between the gear and pinion centers by an amount equal to one-half the enlargement in the outer diameter of the mating pinion and make no contraction in the gear-member diameters.

VARIETIES OF HERRINGBONE GEARS

There are at present three distinct varieties of herringbone gears in general use: gears with sets, or pairs, of half teeth con-

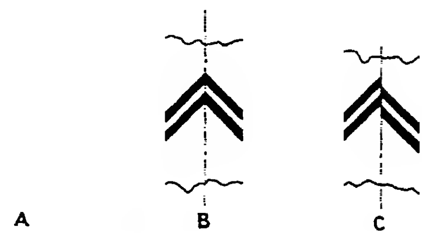


FIG. 61.—Varieties of herringbone gear teeth. A, parted tooth; B, continuous tooth; C, staggered tooth.

verging toward common mid-face apexes that are parted by a central groove, or channel, provided for the clearance of the generating tool employed for cutting the gear teeth; gears in which the half, right and left, helix portions of the teeth of opposing obliquity meet at mid-face apexes (continuous herringbones); and gears with half teeth of opposing obliquity in staggered arrangement, to provide for the necessary tool clearance.

These various arrangements of herringbone-gear teeth are simply the result of different methods of gear-tooth generation and have little, if any, effect upon the efficiency of the gearing. The design, tooth proportions, and general dimensions of the finished gears are much alike, except for the arrangement of the teeth, and have been fairly well standardized. In fact, one set of working formulas will suffice for all three varieties of herring-

bone gears and for plain helical spur gearing as well. The various relationships and standard terminology as adopted by the A.G.M.A. are given in the table below.

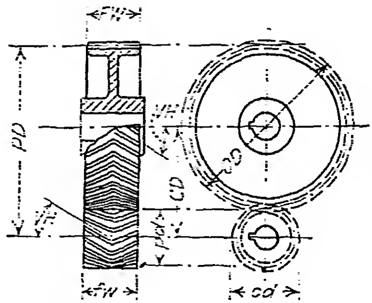


FIG. 62.—Herringbone spur gearing.

HERRINGBONE AND HELICAL SPUR-GEAR TERMINOLOGY[1]

Dimension	Symbol	Dimension	Symbol
Addendum	A	Number of teeth	N
Active face	AF	Outside diameter	OD
Backlash	B	Pitch diameter	PD
Clearance	C	Face width	FW
Center distance	CD	Tooth load	TL
Dedendum	D	Pressure angles,	
Diametral pitches,		in plane of rotation	VP
in plane of rotation	DP	in normal profile plane	VPn
in normal profile plane	DPn	Helix angle	VH
Groove depth	GD	Whole depth (tooth)	WD
Groove width	GW		

[1] Symbols for pinion members are customarily distinguished from corresponding symbols for gear members by the use of small, instead of capital, letters.

Formulas for Helical and Herringbone Spur Gears

$$PD = \frac{N}{DP} \tag{15}$$

VP (in plane of rotation)

Maximum 25 deg. 0 min. (helical)

Minimum 15 deg. 23 min. (helical)

$$\tan VPn \text{ (in normal profile plane)} = \tan VP \times \cos VH \tag{50a}$$

$$\tan VP = \frac{\tan VPn}{\cos VH} \tag{50b}$$

VH (herringbone gears only)
Maximum 45 deg. 0 min.
Minimum 20 deg. 0 min.

$$A = \frac{1}{DP} \text{ (max.)}$$

$$= \frac{0.7}{DP} \text{ (min.)}$$

$$C = \frac{0.3}{DP} \text{ (max.)}$$

$$= \frac{0.157}{DP} \text{ (min.)}$$

$$D = A + C$$

$$WD = 2A + C$$

$$DP = DPn \times \cos VH \quad (51a)$$

$$DPn = \frac{DP}{\cos VH} \quad (51b)$$

$$OD = \frac{N}{DP} + 2A$$

$$AF \text{ (min.)} = \frac{7.22568}{DP \times \tan VH} \quad (52)$$

$$FW \text{ (parted tooth)} = AF + GW \quad (53)$$

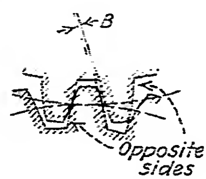


Fig. 63.—Backlash.

Table 23.—Backlash Limitations
(A.G.M.A. Recommendations)

Minimum industrial gears B, inch	DP	Minimum high-speed gears B, inch
0.002	24	0.003
.002	16	.003
.003	12	.004
.003	10	.004
.004	8	.005
.005	6	.007
.006	5	.008
.008	4	.010
.010	3	.013
.012	2½	
.015	2	
.020	1½	
.030	1	

MAXIMUM HERRINGBONE-TOOTH LOAD

The progressive, silent meshing of the modern helical gear tooth employed for high-quality herringbone-spur gearing, the avoidance of sudden application of tooth load and the strong, rugged, wear-resisting form of the gear tooth now generally standard for such gearing have created a need for a dependable formula for ascertaining the maximum tooth load the gearing can safely carry, a formula that takes into account the questions of tooth wear and suitable tooth lubrication, as well as that of tooth strength. Both the A.G.M.A. and certain manufacturers of herringbone gearing have devoted much attention to the derivation of such a formula and marked progress has been made.

The A.G.M.A. has adopted a simple equation for determining the maximum tooth load in which the controlling factors are a tooth proportion factor, allowable static stress of the gear material and a velocity factor, as well as the diametral pitch of the gear in the plane of rotation and an empirical factor pertaining to tooth wear and lubrication.

$$TL = \frac{Y \times S \times K}{DP \times P} \tag{54}$$

where TL = maximum tooth load per inch active gear face.
Y = tooth proportion factor.
S = allowable static stress of gear material.
K = velocity factor.
DP = diametral pitch in plane of gear rotation.
P = wear and lubrication factor.

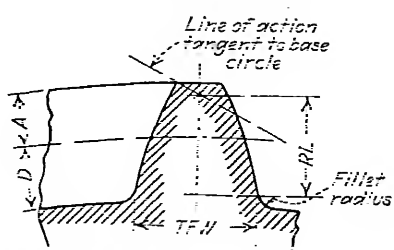


Fig. 64.—Section of herringbone gear tooth in plane of rotation.

The tooth proportion factor Y, which has been established by experience, is equal to the square of the tooth-flank width in the plane of gear rotation, measured just above the fillets, divided by six times the radial leverage of the gear tooth. This latter

is the radial distance from the intersection of the line of action and the center line of the tooth to the tooth-flank-width plane.

$$Y = \frac{TFW^2}{6RL} \tag{54a}$$

The allowable static stresses S of certain suitable herringbone-gear materials are given in Table 24. Materials of higher physical characteristics than those listed for steels are not considered well suited for herringbone gears, as such gears are customarily cut *after* the gear blanks have been heat treated, and harder materials would present undue machining difficulties.

TABLE 24.—ALLOWABLE STATIC STRESS OF MATERIAL

Material	S
High carbon or alloy steels heat treated to an elastic limit of approximately 60,000 lb. per square inch	15,000
0.40 to 0.50 carbon steel heat treated to an elastic limit of approximately 50,000 lb. per square inch	12,500
0.40 to 0.50 carbon steel untreated with an elastic limit of approximately 40,000 lb. per square inch	10,000
Cast steel A.S.T.M. Class B. Elastic limit approximately 36,000 lb. per square inch	7,500
Cast iron. Tensile strength approximately 24,000 lb. per square inch	4,000
Bronze 88-10-2 Tensile strength approximately 27,000 lb. per square inch	4,000

The velocity factor K, in feet per minute, is computed by dividing the constant 78 by the sum of 78 and the square root of the pitch-line velocity of the gear, in feet per minute.

$$K = \frac{78}{78 + \sqrt{PLV}} \tag{54b}$$

For enclosed gearing for which the viscosity and characteristics of the lubricant are correctly chosen, for both the type of gear assemblage and the service, a wear and lubrication factor P of 1.15 is recommended.

COMMERCIAL HORSEPOWER FORMULA

(Farrel-Birmingham Company)

Commercially, the approach to this important question of herringbone-gear tooth strength has been somewhat different, the problem having been resolved to the derivation of a practical horsepower formula that takes into account the considerations of

character of load and of duration of service, as well as the influencing factors accounted for in the A.G.M.A. formula for maximum tooth load. One of the most accurate and useful of these horsepower formulas is

$$HP = \frac{S \times FW \times PD^2 \times Q \times C \times RPM}{1{,}260} \tag{55}$$

where S = material factor (see Tables 25*a* and 25*b*).
FW = face width (active) of gear, inches.
PD = pitch diameter in plane of gear rotation, inches.
Q = installation factor (see Table 25*c*).
C = character of loads factor (see Table 25*d*).
RPM = revolutions per minute of pinion member.

TABLE 25*a*.—GEAR-MATERIAL SPECIFICATIONS

Number	Material	Hardness, Brinell
	Pinion Steel	
1*P*	0.40–0.50 per cent carbon	175–200
2*P*	.50– .60 per cent carbon	175–200
3*P*	.50– .60 per cent carbon	200–225
4*P*	.50– .60 per cent carbon	225–250
5*P*	S.A.E. 3240 or equivalent	225–250
6*P*	S.A.E. 3240 or equivalent	250–275
7*P*	S.A.E. 3240 or equivalent	275–300
8*P*	S.A.E. 2320 case-hardened	450–500
	Gear Steel	
1*G*	Approximately 0.30 per cent carbon	
2*G*	Approximately 0.40 per cent carbon	
3*G*	Special	

If the assumption is made that the face width of the pinion member, in a herringbone-gear drive, should be 50 per cent greater than its pitch diameter (*pd*), which is quite common practice in many installations of herringbone gearing, the correct pitch diameter is readily computed by the formula

$$pd = \sqrt[3]{\frac{HP \times 840}{S \times Q \times C \times RPM}} \tag{55a}$$

TABLE 25b.—MATERIAL FACTOR S

Material factor S	Gear-material specifications	
	Pinion member	Gear member
1.0	$1P$	$1G$
1.1	$2P$	$1G$
1.2	$3P$	$1G$
1.3	$4P$	$1G$
1.4	$5P$	$2G$
1.5	$6P$	$2G$
1.6	$7P$	$2G$
2.0	$8P$	$3G$

TABLE 25c.—INSTALLATION FACTOR Q

Type of installation	Tooth velocity, feet per minute		
	1,000 to 2,000	500 to 1,000	Under 500
Enclosed gears..........	1.0	1.2	1.4
Open gearing...........	.8	1.0	1.2

TABLE 25d.—CHARACTER OF LOADS C

Character of loads	Conditions of daily service			
	Continuous 24 hr.	Continuous 10 hr.	Intermittent over 5 hr.	Intermittent under 5 hr.
Full-load rating with shutdowns only for repairs...	0.45	0.60	0.80	1.10
Friction loads to full motor overloads with frequent power fluctuations.......	0.60	0.80	1.10	1.50
Friction loads to part full-load rating, average running 75% full-load rating	0.80	1.10	1.50	2.00
Friction loads to part full-load rating with majority under 50% full-load rating..................	1.10	1.50	2.00	2.70

NOISE AND VIBRATION

While herringbone and helical spur-gear assemblages are inherently quiet in operation, this highly desirable characteristic is attained only if the gearing is accurately formed and certain definite precautions are taken. Even then, external causes may create disturbances which sometimes have disagreeable results. For this reason, it is important to have a clear appreciation of the causes of objectionable noise in gearing, which may mean anything from a crushing rumble to an ear-splitting scream.

As a general rule, the cause of noisy gearing is the recurrent separation and contact of the engaging gear teeth, and this takes place only when there is a variation in the instantaneous angular velocities of the respective gear members. Ordinarily, the disturbance is due to some imperfection in the tooth profiles or to eccentricities in the rotating gears, though occasionally outside conditions, which have little or nothing to do with the accuracy of the gear cutting, are responsible.

The chief machining, and hence controllable, errors in helical- and herringbone-gear teeth productive of variations in angular velocity are either in the division of the teeth around the gear axis or the fact that some point, or points, on the profile surface of one or more of the gear teeth lies outside the involute curve passing through the pitch point of the tooth. The correction for these evils is, obviously, greater care and accuracy in generating the gear teeth.

Also, high-speed gears must be accurately balanced, the gear shafts true, round, and parallel to each other, the helix angles correct and equal, and the gear teeth properly lubricated. Proper bearing support, likewise, plays an important part.

A set of gears may rattle when running light, for no other reason than because the larger gear is slightly out of balance and periodically overruns the pinion, yet, when the gearing is placed under load, the noise may largely disappear. However, this occurs only if the gear teeth are accurately generated and the gear is well mounted. In other cases, as slight an excess in weight as a short piece of key protruding behind a pinion coupling may cause a high-speed steam-turbine drive to be excessively noisy when under load.

Noise disturbances traceable to external causes, to trying service conditions, are naturally apt to be the most troublesome;

still, in many instances, even these can be much reduced, and the deterioration the noise usually indicates considerably retarded. Typical of this situation are the roar and violent vibration commonly developed when synchronous motors are used to drive ball-grinding mills through helical or herringbone gearing of relatively high-gear, or speed, ratio. Such objectionable disturbances apparently have nothing to do with the quality of the gearing employed. Nevertheless, they are occasioned by the repeated make-and-break contact of the gear teeth which is attributable to variations in the angular velocity of the respective gear members.

The ball mill, because of the sliding and rolling of its heavy charge, is subjected to rapid changes in the angular velocity of its revolving cylinder, to which the gear member of the drive transmission is connected. These changes in rotary speed may not be large, but they are relatively rapid and vibratory in character. The motor and the driving pinion, on the other hand, endeavor to maintain a constant rotary speed, with the result, since a certain backlash between the engaging teeth is essential, that the meshing pinion and gear teeth are continuously separating and reengaging.

No matter how accurately the gears may be cut, balanced and mounted, there is a decided unbalance in momentum between the gear and its load, on the one hand, and the pinion with its power supply, on the other. Variation in the angular velocity of the respective gear members is the inevitable result.

In such situations, which, incidentally, are far from rare, a certain amount of relief is usually possible by reducing the momentum (flywheel effect) of the driving motor and pinion by the interposition of a resilient connection, such as a suitable form of flexible coupling, between the motor and the pinion. This will have the effect of permitting the pinion, which of itself is comparatively light and develops little momentum, to follow up the rapid changes in the angular velocity of the gear and thus maintain more intimate and constant tooth contact. Not only will the noise be greatly reduced, but the gearing will not wear out so rapidly.

EFFICIENCY OF HELICAL AND HERRINGBONE GEARING

The accuracy attainable in form and divisional spacing of teeth around the axes of helical and herringbone gears, made

possible by modern methods of approved gear-tooth generation, is not only the basic reason for the comparatively silent and vibrationless operation of such gearing but, together with the constancy of engagement of helically arranged gear teeth and absence of all shock of tooth impact, explains the high mechanical efficiency secured with the gearing. This is assuming, of course, that the gears are correctly designed, proportioned, and mounted in the first place, that the gear teeth are properly lubricated, and that the original accuracy of tooth form is not impaired by stresses incidental to motion and transmission of power, or to marked changes of temperature.

In well-proportioned steam-turbine drives, enclosed helical gearing has developed transmission efficiencies of 99 per cent and even higher, while an efficiency of 98½ per cent is held to be quite a conservative figure. For gear ratios up to 10:1, the efficiency of accurately generated herringbone gears is better than 98 per cent and, not infrequently, by a point or more, leaving little attainable improvement.

Not only are the gears remarkably efficient, but the sturdy, well-proportioned tooth standardized upon for helical and herringbone gears resists wear well and maintains its original accuracy of form over long periods of service—if the gears are accurately mounted and well balanced. Good alignment, concentricity, and equalization of all journal pressures are, however, prerequisites.

SECTION VII

SPIRAL GEARING

"Spiral gearing" is the term somewhat erroneously applied to that large group of helical-type gears employed for transmitting motion between shafts that are neither parallel nor in the same plane. The teeth of these gears are arranged helically, like the threads of a screw, or worm, not spirally, and the axes of the gears do not intersect. This combination of tooth and shaft angularities tends naturally to complicate both the design and production of this form of gearing.

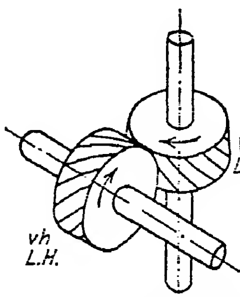


FIG. 65.—Right-angle spiral gearing.

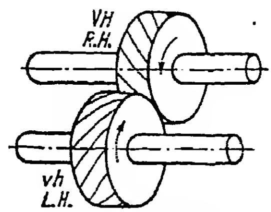


FIG. 66.—Helical-spur gearing.

The simplest arrangement of these distinctive gears is, obviously, that in which the axes of the respective gear members lie in normal planes, *i.e.*, when the axes of the mating gears lie in planes that are at right angles. Under these conditions, the helix angles of the engaging gears, commonly mistermed "spiral angles," are of the same direction of obliquity, right or left hand, though not necessarily of the same angular measure. However, the sum of the two helix angles are, in right-angle spiral gearing, always equal to 90 deg.

In helical-spur gearing connecting parallel shafts (see Sec. VI), the obliquity of the helix angles of the respective gear members, while the same in angular measure, is always opposed, right hand and left. The sum of the helix angles in such gearing is not fixed but is governed by the need of keeping the axial

thrust developed by the oblique arrangement of teeth within reasonable limits.

When the gear axes in spiral gearing are at any angle between 90 deg., when they lie in normal planes, and 0 deg., when they are parallel, the helix angles of the respective gear members driver and follower, may be of the same directional obliquity or one may be right hand and the other left hand. Should the helix angle of one of the gear members be the same as the acute angle of normally intersecting gear-axis planes, however,

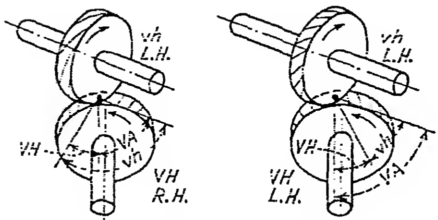


Fig. 67.—Spiral gearing with helix angles of opposing and similar hands.

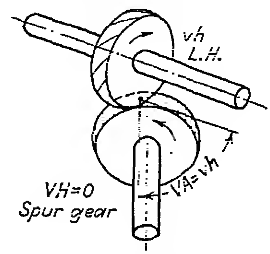


Fig. 68.—Spiral-gear and spur-gear assemblage.

i.e., equal to the shaft angle, the mating gear member takes the form of an ordinary spur gear with teeth parallel to its axis.

The relationship between the helix and shaft angles of spiral gears, when the axes of the engaging gears are not parallel or at right angles, is such that, when the helices are of the same hand, the sum of the helix angles is equal to the shaft angle; and, when the helices are of dissimilar obliquity, one right and the other left, the difference of the helix angles is equal to the shaft angle. Consequently, when one of the mating gears is a plain spur gear, with a helix angle of zero, the helix angle of the spiral-gear member is the same as the shaft, or axis, angle of the combination.

SPEED-RATIO RELATIONSHIPS

The gear, or speed, ratio of engaging spiral gears can be modified in two ways: by changing the pitch diameters of the respective gear members, as is customary in other systems of gearing, and, also, by changing the helix angles of the driver and follower members. The latter method of changing the speed ratio, while it changes the number of teeth and so alters the pitch-line velocities of the respective gear members, need not

necessarily change the pitch diameters of the respective gears. In fact, the pitch-line velocities of the engaging gears in spiral gearing are never the same unless the helix angles of both the driver and follower are the same and equal to half the shaft angle.

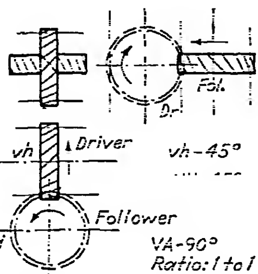


FIG. 69.—Right-angle spiral gears of common pitch diameter and helix angle.

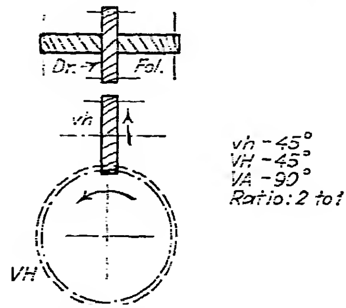


FIG. 70.—Right-angle spiral gears of 2 to 1 ratio with common helix

The right-angle spiral gears, shown in three-view diagram form, with 45-deg. helix angles and common pitch diameters, have naturally a speed ratio of 1 to 1, both the driver and follower having the same number of teeth. The spiral gears (Fig. 70) also have 45-deg. helix angles, but the follower has twice as many teeth as the driver and so the speed ratio is 2 to 1.

In the case of the gearing illustrated in Fig. 71, the helix angle of the driver is considerably greater than 45 deg. and that of the follower correspondingly less, their combined measure equaling 90 deg. The result is that, while the normal pitches of the gear members are necessarily the same, their circular pitches in the planes of rotation are quite dissimilar —hence the pitch-line velocities of the two gears are different. The follower revolves at only half the speed of the driver and a 2-to-1 speed ratio is secured with spiral-gear members of like pitch diameters. This ability of modifying the speed ratio in spiral gearing by simply changing the helix angles of the respective gear members (which can be done with or without alteration in their pitch diameters) makes it possible to secure

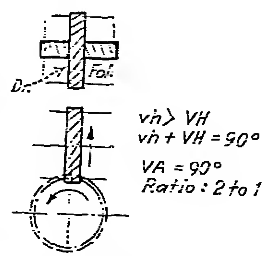


FIG. 71.—Right-angle spiral gears of 2 to 1 ratio with dissimilar helix angles.

any desired speed, or gear, ratio with a pair of spiral gears having combined pitch radii equal to the distance between the axes of the gears; *i.e.*, equal to the center distance of the gearing.

HELIX-ANGLE SELECTION

While from the viewpoint simply of gear design, there is thus a limitless range of choice in respect to the helix angles of spiral gears, the most favorable helix angle, so far as durability and minimum tooth wear are concerned, is one of 45 deg. for right-angle spiral gearing or one equal to half the shaft angle when the axes of the mating gears are at other angles. Naturally, it is advisable to employ such helix angles when possible, but as there is really little increase in the amount of tooth wear with helix angles of 30 to 60 deg. and no serious developments over a helix-angle range of 20 to 70 deg., there is also a wide practical, as well as theoretical, permissible variation in the selection of helix angles.

If the axes of the gear members are at right angles and the required speed ratio can be made equal to the direct ratio of the pitch diameters of the gears, the same ratio applies to the number of teeth and the helix angles of the engaging gear teeth are both advisably 45 deg. In such cases, the circular pitches of the gears in the planes of rotation are also alike, a condition that is only possible when the helix angles of the mating gears are both 45 deg. However, if the ratio of the diameters determined upon should be larger or smaller than the required speed ratio, the helix angles for the respective gear members, driver and follower, should advisably be:

$$\tan vh = \frac{pd \times N}{PD \times n} \quad \text{and} \quad \tan VH = \frac{PD \times n}{pd \times N} \qquad (56)$$

In such combinations of spiral gearing, the speed, or gear, ratio is measured by the number of teeth and not by the pitch diameters of the gears.

HELIX LEADS

To determine the circular pitch of spiral gears in the plane of gear rotation, which may or may not be the same for mating spiral gears, it is only necessary to divide the pitch circumference by the number of teeth, but the normal circular pitch, which must be the same for the two gears, is that portion of the helix

normal to the pitch-line element of the gear teeth that lies between consecutive gear teeth and is at right angles to the tooth helix (see Fig. 72). It is equal to the circular pitch of the gears in the plane of rotation multiplied by the cosine of the helix angle of the gear teeth.

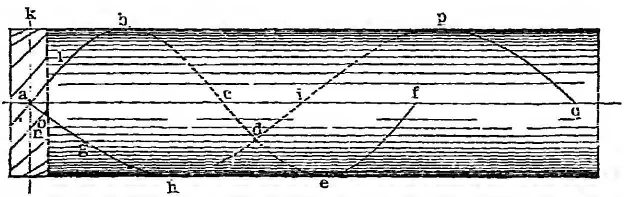


FIG. 72.—Spiral gear helices on pitch cylinder. Tooth helix, *abcdef;* normal helix, *aghdipq.*

When the helix angles of mating spiral-gear teeth are the same, when they are both 45 deg., the normal pitches of the gears are equal and, if the gears are of the same pitch diameter, they will have the same number of teeth and also the same helix lead. This latter measure is the distance that the *normal* helix advances in one complete wrap of the pitch cylinder. If the number of teeth in one of the gear members is larger or smaller than in the other, the pitch diameters of the gears and the leads of their respective normal helices are proportional to the number of teeth in the gears, provided both gears have 45-deg. tooth-helix angles.

The normal pitch, which governs the tooth proportions and for which the cutting tools in gear production are selected, when multiplied by the number of teeth gives the length *aghd* (Fig. 72) of the normal helix between consecutive intersections of the normal helix with its mating tooth helix. This is shown somewhat more clearly in Fig. 73, where the tooth-helix angle *daq* is a small angle. In this latter illustration, the portion of the normal helix *ad,* which corresponds to *aghd* in Fig. 72, takes in, or cuts, all the teeth, and the product of the normal pitch *ao* multiplied by the number of teeth equals the length of the partial normal-helix wrap *ahpd.*

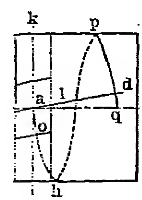


FIG. 73.—Pitch cylinder for large-angle spiral gear.

The portion of the normal-helix wrap intersected by the helix of a single tooth, *i.e.,* by the portion of the normal-helix wrap

that cuts all the teeth, grows shorter as the tooth-helix angle increases and may form only a small part of a complete wrap of the normal helix. Diagram *A* (Fig. 74) is a development of Fig. 73, *ad* being the developed length of the portion of the normal helix cutting all the teeth and the angle *akd* the tooth-helix angle. In diagram *B*, the tooth-helix angle is considerably larger and the portion of the normal helix *ad* correspondingly shorter, while in the third diagram the tooth-helix angle is much larger and *ad* consequently much shorter.

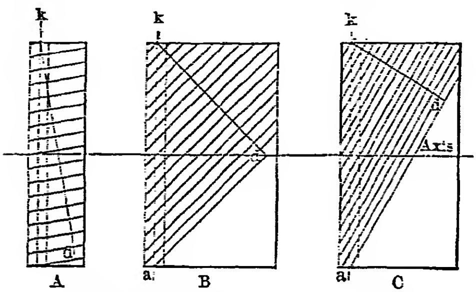


FIG. 74.—Developed spiral-gear helices.

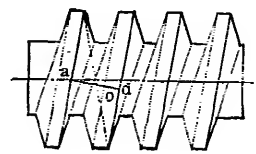


FIG. 75.—Single thread worm, or spiral gear.

In an extreme case, where the tooth-helix angle of a spiral gear is so large that the gear becomes a single-thread worm (see Fig. 75), the portion of the normal-helix wrap *ad* is subtended by the normal pitch *ao*, points *d* and *o* coinciding. As is common, the normal pitch multiplied by the number of teeth, 1 in this case, equals the length of the portion of the normal helix cut off between two consecutive intersections by the tooth helix.

SPIRAL-GEAR TERMINOLOGY*

Dimension	Symbol	Dimension	Symbol
Helix angle	*VH*	Circular pitch	*CP*
Pressure angle	*VP*	Diametral pitch	*DP*
Shaft angle (axes)	*VA*	Diametral pitch, normal	*NDP*
Addendum	*A*	Normal pitch (circular)	*NP*
Dedendum	*D*	Outer diameter	*OD*
Clearance	*C*	Pitch diameter	*PD*
Circular thickness	*CTh*	Lead (normal helix)	*L*
Whole-tooth depth	*WD*	Number of teeth	*N*

* Symbols for driver members are customarily distinguished from corresponding symbols for follower members by the use of small, instead of capital, letters.

General Formulas for Spiral Gearing

$$VA = vh + VH \quad \text{(tooth helices of same hand)} \tag{57a}$$

$$VA = vh - VH \quad \text{(tooth helices of opposite hand)} \tag{57b}$$

$$PD = \frac{CP \times N}{3.1416} = \frac{NP \times N}{3.1416 \cos VH} = \frac{N}{NDP \cos VH} \tag{58}$$

$$OD = PD + 2A = PD + \frac{2}{NDP} \tag{59}$$

$$CP = \frac{3.1416PD}{N} = \frac{NP}{\cos VH} \tag{60}$$

$$NP = CP \cos VH \tag{61}$$

$$NDP = \frac{3.1416}{NP} \tag{62}$$

$$A = \frac{NP}{3.1416} = \frac{1}{NDP} = 0.5(OD - PD) \tag{63}$$

$$D = A + 0.1CTh \tag{64}$$

$$C = 0.1CTh \tag{65}$$

$$CTh = 0.5NP \tag{66}$$

$$WD = 2A + 0.1CTh \tag{67}$$

$$L = \frac{N \times CP}{\tan VH} = \frac{3.1416}{DP \tan VH} \tag{68}$$

APPLICATION OF FORMULAS

Ordinarily the specifications for a pair of spiral gears give the required speed, or gear, ratio, center distance, and the angularity of the gear axes, or shafts. For instance, the requirements might call for a pair of spiral gears of 2 to 1 ratio operating on 6-in. centers on shafts at an angle of 30 deg. In such a case, either the number of teeth for the gears, or their common normal pitch would first have to be established, preferably the latter. The pitch is favored for the reason that a normal pitch can be selected for which standard working tools are available, while, if the number of teeth on the respective gears should first be arbitrarily fixed, the normal pitch that would result would almost certainly involve the use of special cutting tools, the resulting pitch seldom being of standard measure.

With a shaft angle of 30 deg., it is evident that, if the helix angles for the teeth of the gears are not to be less than 20 or greater than 70 deg., the tooth helices of the respective gear members must either be of opposite hand, or one of the gears have teeth at a helix angle of 30 deg. and the other have

ordinary, axial, spur-gear teeth. It may be assumed, furthermore, that a normal diametral pitch of 4 would be satisfactory.

Problem in Spiral-gear Design

Specifications: Required a pair of spiral gears: ratio 2 to 1, to operate on 6-in. centers between shafts at 30 deg.

Pitch diameter ratio (spur gears) for 2 to 1 ratio on 6-in. centers.......... 4 in. and 8 in.

Assume 4 diametral pitch

Respective number of teeth of 4DP for 4-in. gear......................16

for 8-in. gear......................32

Trial Computations:

VH = 30 deg.: 30-tooth spiral-gear follower..................4NDP
vh = 0 deg.: 15-tooth spur-gear driver........................4DP

$$PD = \frac{30}{4 \times \cos 30 \text{ deg.}} = 8.661 \text{ in.} \qquad (58)$$

$$pd = {}^{15}\!/_{4} = 3.750 \text{ in.}$$

$$CD = \frac{8.661 + 3.75}{2} = 6.206 \text{ in.}$$

VH = 0 deg.: 30-tooth spur-gear follower......................4DP
vh = 30 deg.: 15-tooth spiral-gear driver....................4NDP

$$PD = {}^{30}\!/_{4} = 7.500 \text{ in.}$$

$$pd = \frac{15}{4 \times \cos 30 \text{ deg.}} = 4.331 \text{ in.} \qquad (58)$$

$$CD = \frac{7.50 + 4.331}{2} = 5.915 \text{ in.}$$

VH = 20 deg.: 30-tooth spiral-gear follower..................4NDP
vh = 50 deg.: 15-tooth spiral-gear driver....................4NDP

$$PD = \frac{30}{4 \times \cos 20 \text{ deg.}} = 7.981 \text{ in.} \qquad (58)$$

$$pd = \frac{15}{4 \times \cos 50 \text{ deg.}} = 4.895 \text{ in.} \qquad (58)$$

$$CD = \frac{7.981 + 4.895}{2} = 6.438 \text{ in.}$$

VH = 20 deg.: 28-tooth spiral-gear follower..................4NDP
vh = 50 deg.: 14-tooth spiral-gear driver....................4NDP

$$PD = \frac{28}{4 \times \cos 20 \text{ deg.}} = 7.449 \text{ in. (O.K.)} \qquad (58)$$

$$pd = \frac{14}{4 \times \cos 50 \text{ deg.}} = 4.568 \text{ in. (O.K.)}$$

$$CD = \frac{7.449 + 4.568}{2} = 6.008 \text{ in. (O.K.)}$$

Normal pitch:

$$NP \quad 3.1416 = 0.7854 \text{ in.} \tag{62}$$

Circular thickness:

$$CTh \quad 0.5 \times 0.8754 = 0.3927 \text{ in.} \tag{66}$$

Clearance:

$$C \quad 0.1 \times 0.3927 = 0.0393 \text{ in.} \tag{65}$$

Addendum:

$$A \quad \frac{0.7854}{3.1416} = \frac{1}{4} = 0.2500 \text{ in.} \tag{63}$$

Dedendum:

$$D = 0.25 + 0.0393 = 0.2893 \text{ in.} \tag{64}$$

Whole depth (tooth):

$$WD = 2 \times 0.25 + 0.0383 = 0.5393 \text{ in.} \tag{67}$$

Circular pitch:
(Follower)

$$CP \quad \frac{0.7854}{\cos 20 \text{ deg.}} = 0.8358 \text{ in.} \tag{60}$$

(Driver)

$$cp \quad \frac{0.7854}{\cos 50 \text{ deg.}} = 1.0253 \text{ in.} \tag{60}$$

Outside diameter:
(Follower)

$$OD \quad 7.449 + 2 \times 0.25 = 7.949 \text{ in.} \tag{59}$$

(Driver)

$$od \quad 4.568 + 2 \times 0.25 = 5.068 \text{ in.} \tag{59}$$

If the gear axes were parallel and the gear assemblage had consisted of a pair of ordinary spur gears, the pitch diameters of the respective gears for a 2-to-1 ratio would have been 4 and 8 in. With a diametral pitch of 4, one gear would then have had 16 teeth and the other 32 teeth. While these considerations have no direct connection with the specific problem, they do nevertheless limit the scope of the investigations needed for a solution of the problem. As an effect of the oblique arrangement of the teeth in spiral gearing is to increase the pitch diameters of the gears, the circumferential pitch of a spiral gear is always larger than its normal pitch. It is quite obvious then that the respective spiral-gear members must have fewer teeth than the gears of ordinary spur type for a given speed ratio on fixed centers, if the normal pitch of the spiral gears, for which their working tools are selected, is to be the same as the circumferential pitch of spur gears of like ratio operating on a similar center distance.

The fact that the specified shaft angle is an angle that would be suitable for the helix angle of the teeth of a spiral gear is of interest, for such a helix angle could be employed under the proper conditions for one of the spiral gears and an ordinary spur gear used for its mate. The combination would be almost as efficient as an all-spiral gear drive and somewhat cheaper.

To ascertain whether such a spiral-spur-gear combination could be used with gears of the pitch selected on the prescribed centers, it is first necessary to select gears with fewer teeth than would be required for spur gears operating on the same center distance. The smallest reduction would be two teeth in the larger gear, the follower member, and one tooth in the smaller, or driver member. That is, the spiral-spur-gear combination might have 30 teeth in the follower and 15 teeth in the driver.

Considering first making the follower the spiral-gear member with 30 teeth of 4 normal diametral pitch at 30-deg. obliquity, the pitch diameter of such a gear is found by formula (58) to be 8.661 in. The driver, being an ordinary 15-tooth, 4-diametral-pitch spur gear would have a pitch diameter of 3.75 in. The combined pitch radii of the two gears would then be 6.206 in., or slightly more than 1/5 in. greater than the prescribed center distance.

Making the follower the spur-gear member and the driver the spiral gear would give somewhat different results, the combined pitch radii in such case being 5.915 in. This is almost 1/10 in. less than the prescribed center distance and, while this gearing and the other spiral-spur-gear combination would involve only slight modifications in the center distance to operate efficiently, neither would represent the highest type of gearing called for by the specifications. Better results could be secured with the helical arrangement of teeth for both gears.

The limitations placed upon spiral gearing by excessive and by inadequate obliquities in helix angles narrow the range of suitable angles for the driver to between 50 and 70 deg. and for the follower to between 20 and 40 deg. [formula 57*b*)]. Selecting the smallest suitable tooth obliquity for the follower, the pitch diameter of a 30-tooth spiral gear with 4-normal-diametral pitch teeth at 20-deg. obliquity would be 7.981 in. That for a mating driver with 15 teeth of similar normal-diametral pitch and necessary 50-deg. tooth helix would be 4.895 in. Half the sum of these pitch diameters is 6.438 in., or nearly 1/2 in. greater than the prescribed center distance.

Any greater obliquity to the follower teeth would result in a larger follower-pitch diameter, the increase being more than the decrease in driver-pitch diameter resulting from the corresponding decrease in the driver's tooth-helix angle. Consequently, spiral gears either with still fewer teeth or with teeth of finer pitch are required.

The pitch diameter of a 28-tooth, 4-normal-diametral pitch, 20-deg. tooth-helix spiral gear is 7.449 in. and the pitch diameter of its mating 14-tooth, 4-normal-diametral pitch, 50-deg. tooth-helix driver is 4.568 in., making the combined pitch radii of this combination 6.008 in. This dimension agrees so closely with the prescribed center distance of 6 in. that the gear combination may be said to conform to the specifications. The difference of 8/1,000 in. in the spacing of the gears might be shaded by some other combination, but it is sufficiently small to be disregarded with safety, even in high-class work, especially as spiral gearing permits of somewhat greater center adjustment than does ordinary spur gearing.

SPIRAL-GEAR EFFICIENCY

Spiral gearing, while it has certain inherent weaknesses, is generally smoother in operation than comparable spur gearing. Tooth contact takes place first at one side of a gear, passes across the gear face, and ceases at the other side. This action tends to cover up any minor defects in the profiles and form of the teeth and maladjustments of centers. Accurately generated spiral gears, well mounted and properly cared for, have efficiencies varying from about 50 up to as high as 90 per cent, depending upon the helix angles of the teeth.

Ordinarily, spiral gears are operated at right angles with a common tooth obliquity of 45 deg., and, while this helix angle is desirable from the viewpoint of holding down tooth wear, the shaft angle of 90 deg. sharply limits the duration of tooth contact and tends to concentrate the load on the mid-tooth section. This is especially marked when the gears differ considerably in size, for which reason gear ratios of more than 10 or 12 to 1 are not usually to be recommended.

DIRECTION OF ROTATION AND THRUST

In common with all other types of helical gearing, spiral gears develop axial thrusts that necessitate the provision of

suitable antifriction-thrust collars or bearings, which, naturally, must be correctly located. In this connection, both the direction of the tooth helix, right or left hand, and the direction of gear rotation have to be taken into account, for reversing the direction

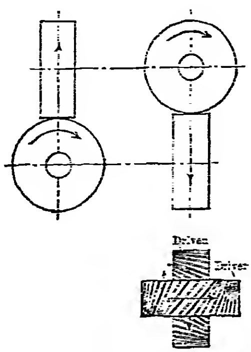


FIG. 76.—Right-hand spiral gears—clockwise rotation.

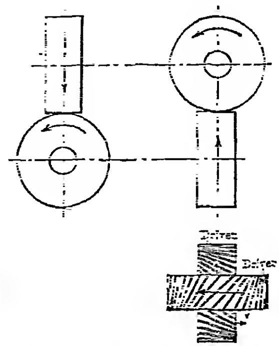


FIG. 77.—Right-hand spiral gears—counterclockwise rotation.

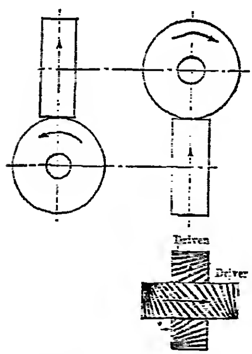


FIG. 78.—Left-hand spiral gears—clockwise rotation.

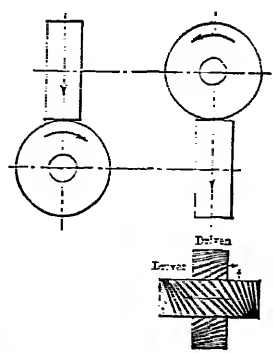


FIG. 79.—Left-hand spiral gears—counterclockwise rotation.

of spiral-gear drivers changes the direction of the axial thrusts. Also, if the driven gears, or followers, are made the drivers, the axial thrusts are also reversed.

The diagrams (Figs. 76 to 79) show by the small arrows protruding from the sides of the respective gears the directions of the thrusts in the several cases. Figures 76 and 77 are depictions of right-hand spiral gears, while Figs. 78 and 79 illustrate left-hand gear combinations.

SECTION VIII

WORM GEARING

Advance in the design of toothed gearing has been most pronounced in the case of worms and worm gears, owing to the fact that, while the principle of the worm and gear as a means of transmitting power is old, it has only been comparatively recently that the essentials underlying the approved design of worm gearing, embodying many modern features of high demonstrated merit, have been generally appreciated. From the old screw-type form of worm with a low helix angle, or lead, and a mechanical efficiency of 30 or 40 per cent to the high-helix, glass-hard, modern, ground worm developing an efficiency of frequently better than 97 per cent is a long step, one that has upset many of the previously accredited theories in reference to worm gearing.

For this reason, it is advisable first to take up the recommendations of the A.G.M.A. for formulas and specifications covering so-termed "standard" worm and worm gears suitable for general commercial purposes, where the gearing is furnished without bearings or housings. This group includes only right-angle gearing of ordinary pitches, multiplicity of threads, and gear ratios, and is not intended to cover cases where the duty imposed on the gearing justifies a further refinement of design than that used for general-purpose worms and gears.

GENERAL INDUSTRIAL RANGE

The standard linear pitches, *i.e.*, the axial distance between worm threads and the circumferential distance between consecutive engaging teeth on the worm gear, covered by this general-purpose range are: $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$ and 2 in. Single-, double-, triple-, and quadruple-thread worms and worm gears are included and the standard gear ratios range from 10 to 1 to 100 to 1.

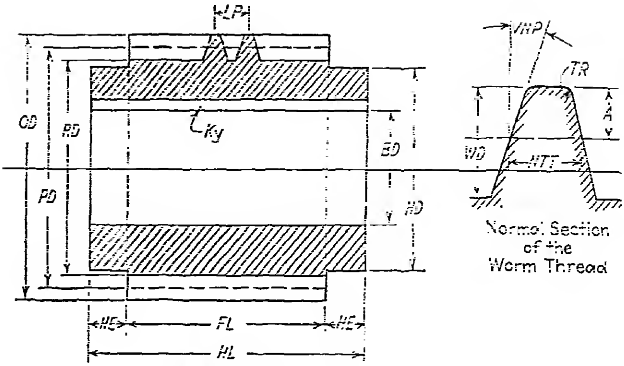


FIG. 80.—Diagram of standard worm.

NOTATION FOR STANDARD WORMS

Dimension	Symbol	Dimension	Symbol
Linear pitch..............	LP	Keyway.................	Ky
Pitch diameter............	PD	Addendum..............	A
Outside diameter..........	OD	Whole depth (tooth)......	WD
Root diameter............	RD	Normal tooth thickness...	NTT
Hub diameter.............	HD	Top round..............	TR
Bore (maximum)..........	BD	Lead angle..............	VL
Face length..............	FL	Lead....................	L
Hub extensions...........	HE	Normal pressure angle....	VNP
Hub length..............	HL	Number of teeth (worm gear)	N

FORMULAS FOR STANDARD WORMS

Symbol	Single and double thread	Triple and quadruple thread	Formula
PD	$2.4 \times LP + 1.1$	$2.4 \times LP + 1.1$	(69)
OD	$3.036 \times LP + 1.1$	$2.972 \times LP + 1.1$	(70)
RD	$1.664 \times LP + 1.1$	$1.726 \times LP + 1.1$	(71)
HD	$1.664 \times LP + 1$	$1.726 \times LP + 1$	(72)
BD	$LP + 0.625$	$LP + 0.625$	(73)
FL	$LP \times (4.5 + 0.02N)$	$LP \times (4.5 + 0.02N)$	(74)
HE	LP	LP	(75)
HL	$FL + 2 \times LP$	$FL + 2 \times LP$	(76)
Ky	A.G.M.A. Standard	A.G.M.A. Standard	
A	$0.318 \times LP$	$0.286 \times LP$	(77)
WD	$0.686 \times LP$	$0.623 \times LP$	(78)
VL	$\cot VL = \frac{PD \times 3.1416}{L}$	$\cot VL = \frac{PD \times 3.1416}{L}$	(79)
NTT	$0.5 \times LP \times \cos VL$	$0.5 \times LP \times \cos VL$	(80)
TR	$0.05 \times LP$	$0.05 \times LP$	(81)
VNP	14½ deg.	20 deg.	

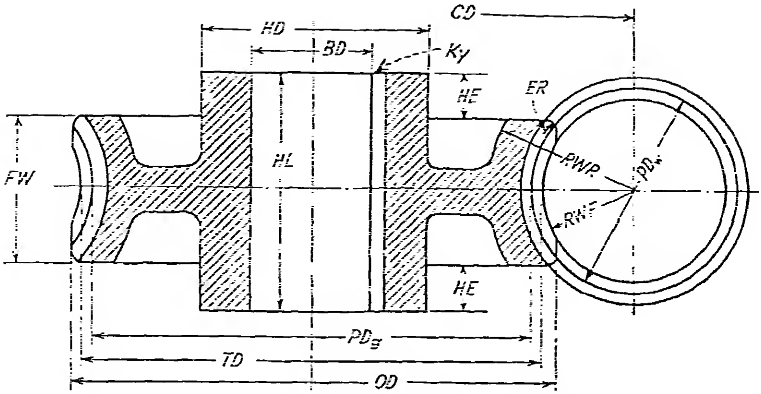


Fig. 81.—Diagram of standard worm gear.

Additional Notation for Standard Worm Gears

Dimension	Symbol	Dimension	Symbol
Circular pitch (linear pitch)...............	*LP*	Radius of wheel face...	*RWF*
Throat diameter........	*TD*	Radius of wheel rim...	*RWR*
Face width............	*FW*	Edge round...........	*ER*
		Center distance.......	*CD*

Formulas for Standard Worm Gears

Symbol	Single and double thread	Triple and quadruple thread	Formula
PD	$N \times 0.3183 \times LP$	$N \times 0.3183 \times LP$	(82)
TD	$PD + 0.636LP$	$PD + 0.572LP$	(83)
OD	$TD + 0.4775LP$	$TD + 0.3183LP$	(84)
HD	$1.875BD$	$1.875BD$	(85)
Ky	A.G.M.A. Standard	A.G.M.A. Standard	
FW	$2.38LP + 0.25$	$2.15LP + 0.2$	(86)
HE	$0.25BD$	$0.25BD$	(87)
HL	$FW + 0.5BD$	$FW + 0.5BD$	(88)
RWF	$0.882LP + 0.55$	$0.914LP + 0.55$	(89)
RWR	$2.2LP + 0.55$	$2.1LP + 0.55$	(90)
ER	$0.25LP$	$0.25LP$	(91)
CD	$(PD_g + PD_w) \times 0.5$	$(PD_g + PD_w) \times 0.5$	(92)
FNP	14½ deg.	20 deg.	

The standard form of worm thread is that produced by a straight-sided milling cutter having a diameter equal to not less than the outside diameter of the worm, nor greater than 1¼ times the outside diameter of the worm. For single- and

double-thread worms, the obliquity of the sides of the thread cutter should be $14\frac{1}{2}$ deg., while for triple and quadruple worms this obliquity should be 20 deg.

The hobs for cutting single- and double-thread worms may be fluted parallel to their axes, but the hobs for cutting triple- and quadruple-thread worms should be fluted normally to the thread angle measured from the outside diameter of the hob.

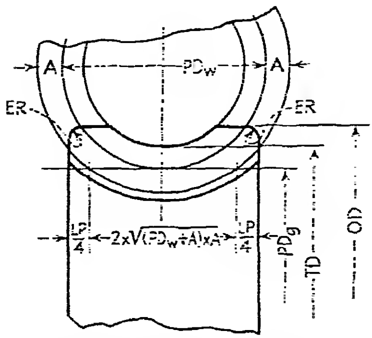


FIG. 82.—Diagram of single- and double-thread worm gearing not conforming to standard worm gearing—A.G.M.A. recommendations.

In cases where the hobs employed do not have pitch diameters corresponding to the pitch diameters of A.G.M.A. standard worms, the recommendation for face widths and outer diameters of worm gears, to mesh with worms cut with such hobs, are for single- and double-thread worms:

$$FW = 2\sqrt{(PD_w + A) \times A} + 0.50LP \tag{93}$$
$$OD = PD_g + 3.5A \tag{94}$$

For worm gears meshing with triple- or quadruple-thread worms, further modifications are recommended, as follows:

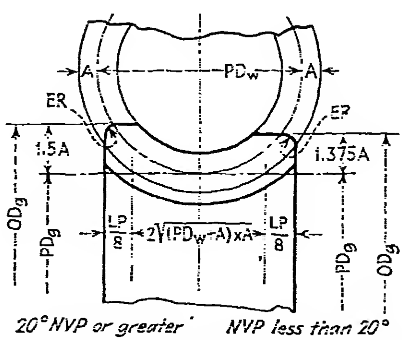


FIG. 83.—Diagram of triple- and quadruple-thread worm gearing not conforming to standard worm gearing—A.G.M.A. recommendations.

$$FW = 2\sqrt{(PD_w + A) \times A} + 0.25LP \tag{93a}$$
$$OD(\text{20-deg. } NVP \text{ or greater}) = PD_g + 2.75A \tag{94a}$$
$$OD(NVP \text{ less than 20 deg.}) = PD_g + 3A \tag{94b}$$

Example in Standard Worm-gearing Design

Worm

Required: Dimensions of 1-in. linear pitch, double-thread worm to engage 50-tooth worm gear

Symbol	Formula	Computation	Dimension
LP	....	Predetermined	1.000 in.
PD	(69)	2.4 × 1 + 1.1	3.50
OD	(70)	3.036 × 1 + 1.1	4.136
RD	(71)	1.664 × 1 + 1.1	2.764
HD	(72)	1.664 × 1 + 1	2.664
BD	(73)(maximum)	1 + 0.625	1.625
FL	(74)	1 × (4.5 + 0.02 × 50)	5.50
HE	(75)		1
HL	(76)	5.5 + 2 × 1	7.50
A	(77)	0.318 × 1	0.318
WD	(78)	0.686 × 1	0.686
cot *VL*	(79)	$\frac{3.5 \times 3.1416}{2}$	0.49785 *VL* = 10 deg 19 min.
NTT	(80)	0.5 × 1 × 0.9838	0.4919 in.
TR	(81)	0.05 × 1	0.05
L	*N* × *LP*	2 × 1	2
VNP	....	Predetermined	14½ deg.

Worm Gear

Symbol	Formula	Computation	Dimension
LP	....	Predetermined	1.000 in.
PD	(82)	50 × 0.3183 × 1	15.915
TD	(83)	15.915 + 0.636 × 1	16.551
OD	(84)	16.551 + 0.4775 × 1	17.028
BD	....	Predetermined	2
HD	(85)	1.875 × 2	3.75
Ky	-	(A.G.M.A. Standard)	½ × ¼
FW	(86)	2.38 × 1 + 0.25	2.63
HE	(87)	0.25 × 2	0.50
HL	(88)	2.63 + 0.5 × 2	3.63
RWF	(89)	0.882 × 1 + 0.55	1.43
RWR	(90)	2.2 × 1 + 0.55	2.75
ER	(91)	0.25 × 1	0.25
CD	(92)	(15.915 + 3.5) × 0.5	9.7075
VNP	....	Predetermined	14½ deg.

DOMINATING CONSIDERATIONS

This standard worm gearing for general commercial applications is not truly representative, however, of the marked advances that have been made in the development of this type of gearing. These advantages are far more apparent in the modern gear units, or speed reducers, of the worm-gear variety, in which not only are the niceties of greater refinements in gear production to be found, but where the bearings and housings that are not such important considerations in the general industrial range of worm gearing play an exceedingly vital part. To secure the high efficiency, compact design, sturdy construction, quietness in operation, freedom from vibration, improved wearing qualities, and long life under continuous and severe operating conditions demanded of modern worm gearing, the gear designer is confronted with mechanical problems that, while they may not be peculiar only to worm gearing, are distinctly more complex in that form of transmission.

The dominating considerations tending to high mechanical efficiency in worm gearing are a large gear-tooth helix angle with correspondingly coarse worm thread pitch and a low peripheral worm speed. The high-helix angle tends to the development of greater transmission efficiency and the low peripheral speed of the worm to a reduction in abrasive wear and in the generation of the resulting frictional heat that has constituted one of the chief obstacles in the development of worm gearing.

The high-helix angle of the gear teeth, the lead angle of the worm (VL), while it is the chief factor in the development of high transmission efficiency, also has the effect of aggravating the amount of sliding taking place between engaging worm threads and gear teeth and, in addition, is productive of heavy axial thrusts. The low peripheral speed of the worm, however, combats these objectionable effects and serves to curtail the intensity of their consequences. At lower relative sliding speeds between the worm threads and the gear teeth, less abrasive wear occurs and less frictional heat is generated. The thrust arm is reduced, *i.e.*, the line of thrust is brought closer to the axis of the worm, and suitable provisions for taking care of this thrust are simplified.

HELIX ANGLE AND EFFICIENCY

The effect of increasing the helix angle of the worm gear, thus increasing the pitch of the worm by a corresponding enlargement in the lead angle, is depicted graphically in Fig. 84, where the diagrams *A*, *B*, and *C* represent, respectively, worms with 15-, 30-, and 45-deg. lead angles. The pressure of a tooth of a mating worm gear upon the worm threads, the load on the worm, is considered equal and is indicated in each case by the arrow *P*. This pressure may be resolved into two components: *ax*, normal to the worm thread, and *ao*, parallel to the worm-thread surface. The perpendicular component produces friction between the gear tooth and worm thread, while the component parallel to the worm thread is that which causes the axial thrust on the worm gear.

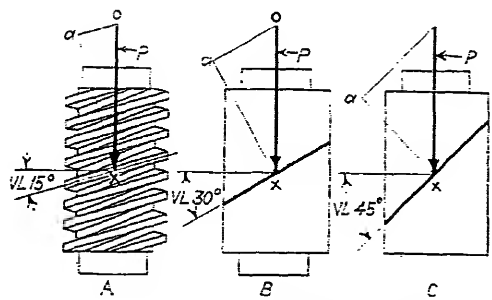


FIG. 84.—Efficiency and lead-angle increase.

The useful work performed by the gearing during each revolution of the worm is proportional to the load *P* multiplied by the pitch of the worm, while the work lost in friction is measured by the product of the normal component *ax*, the coefficient of friction, and the distance traversed in a revolution of the worm, the last being the length of one complete turn of the worm thread. The normal component *ax*, it will be noted, becomes progressively less as the lead angle is increased, and, though the length of a complete turn of the worm thread increases slightly, the work lost by friction does not actually vary much with the change in worm pitch. On the other hand, the useful work performed increases directly with the increase in pitch, being twice as great in the case of the 30-deg. worm as in the case of the 15-deg. worm and three times as great in the case of the 45-deg. worm.

The useless worm-gear axial thrust, which is proportional to *ao*, increases rapidly at the same time, so the net gain in transmission efficiency is not in any way directly proportional to the increase in pitch, but it is nevertheless quite substantial. This is clearly shown in Fig. 85, where the relationship existing between the lead angle and the efficiency of transmission is charted for a given set of conditions.

For a given amount of useful work, the work lost through friction is reduced, as the lead angle of the worm increases, and, consequently, the objectionable heating and abrasive wear.

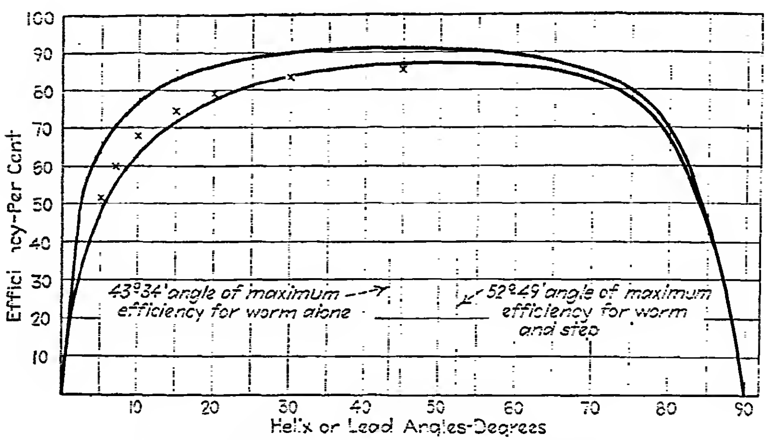


FIG. 85.—Relationship between lead angle and efficiency.

For fine pitches, the change is rapid and becomes gradually less, until a maximum efficiency is attained when the helix angle is about 45 deg. Conditions then reverse themselves, and a further increase in the linear pitch of the worm produces an accelerated decline in efficiency of transmission. However, between the critical helix angles of about 30 and 50 deg. there is no great amount of change in attainable efficiency, and it is within this prescribed range of helix angles for worm gears and lead angles for worms that the pitch for most of the modern high-efficiency worm gearing is selected.

Expressed algebraically, the average efficiency of worm gearing, disregarding the frictional resistance imposed by the worm's thrust bearings, is

$$E = \frac{\tan VL(1 - f \tan VL)}{f + \tan VL} \tag{95}$$

where VL = helix angle, or lead angle (angle of worm thread).
f = coefficient of friction.

DIAMETERS AND STRENGTH OF WORMS

Other conditions being the same, the best results are secured, *i.e.*, the highest transmission efficiency attained, with worms of minimum feasible pitch diameters. Not only is the lowest practical peripheral worm speed obtained, but a helix, or lead, angle of maximum value is assured. The pitch diameter of the worm gear is likewise at a minimum, and the lead of the worm is the shortest for the particular pitch.[1] The result of all of which is: there is less work lost in friction, the heating annoyance is decreased, and the abrasive wear of worm thread and gear teeth is kept as low as possible.

Such practice makes possible higher rotary speeds, with still higher transmission-efficiency attainment, but, as it also customarily entails the use of worms with a multiplicity of fin threads, the strength and load-carrying capacity, especiall of the worm-gear teeth, become considerations of prime impor tance. Ordinarily some modification of the Lewis equatio for the strength of gear teeth (see Sec. II) is employed in thi connection, the most convenient of which is

$$HP = \frac{W \times PLV}{33,000} \tag{5}$$

The total transmitted load W, which is the maximum safe tangential load in pounds at the pitch circle of the worm gear, is the product of the allowable unit stress for the gear material, face width of worm gear, and a form factor Y divided by diametral pitch [formula (96)], the allowable unit stress be the safe working unit stress multiplied by a velocity fac The values of this form factor found proper in commer practice, together with empirically determined allowable u stresses for worm-gear drives having worms of single, dou triple or quadruple threads, are given in Table 26.

$$W = \frac{600AUS(FW \times Y)}{DP(600 + PLV)} \tag{96}$$

[1] The lead is the distance that any one thread advances in one revolution of the worm, while the pitch is the linear distance between the centers of two adjacent threads. The lead and pitch are equal for a single-threaded worm; for a double-threaded worm the lead is twice the linear pitch; for a triple-threaded worm, three times the pitch; etc.

Table 26.—Values of Form Factor Y for Worm Gears*

Pitch-line velocity (PLV), ft. per min.	Form factor Y	Allowable unit stress (AUS), lb. per sq. in.	
		Cast iron	Phosphor bronze
0	1.000	5.300	8.000
100	.857	4.550	6.800
200	.750	4.000	6.000
300	.667	3.550	5.350
450	.571	3.000	4.500
600	.500	2.650	4.000

* Compiled by Foote Bros. Gear and Machine Co.

CENTER DISTANCE AND GEAR RATIOS

Since, owing to the helical arrangement of the worm threads and gear teeth, the relative angular velocities of the worm and its gear depend, as in spiral gearing, upon the number of teeth (threads in the case of the worm) in the respective rotating members and not upon the pitch diameters of the gears, the center distance for a given gear, or speed, ratio is subject to variation. The gear ratio, moreover, is measured by the number of worm-gear teeth per thread of worm, each worm thread being in itself a complete worm. For instance, a 30-tooth worm gear mating with a single-thread worm has a gear ratio of 30 to 1; a similar gear meshing with a double-thread worm, a ratio of 15 to 1; a 30-tooth gear running with a triple-thread worm, one of 10 to 1; etc. The velocity of a worm gear meshing with any multiple-thread worm is, consequently, inversely proportional to the number of threads on the worm.

Great speed reductions are thus possible with worm gearing, and the form of transmission, incidentally, is one of the most powerful known. Gear ratios of as high as 500 to 1 have been successfully employed with a single worm and gear combination, but ordinarily an arbitrary top limit for a single worm and gear assemblage of 50 or, at most, 100 to 1 is set, larger ratios being more efficiently effected by compounding the gearing, *i.e.*, employing two sets of worms and gears in series.

The advantage of employing this double reduction for high ratios in worm gearing, which may be several thousand to one, is due to the fact that with large ratios it is usually necessary,

not only to increase the number of worm-gear teeth, but to decrease the linear pitch, *i.e.*, the circular pitch and lead, modifications which are productive of losses in transmission efficiency. There is a critical point as the ratios increase at which, unless the advantages of high-helix angle and small-diameter worm are sacrificed (the reason for the drop in efficiency), the gear teeth become too small for the proper proportioning of the worm and gear.

With a gear ratio of 100 to 1, for example, a single-worm gear drive might have a transmission efficiency of between 60 and 67 per cent, while a 10-to-1 assemblage, owing to better proportioning of the worm and gear, has an efficiency of 90 to 96 per cent. Under these circumstances, the substitution of two 10-to-1 ratio units in series, so securing a 100-to-1 ratio, would effect the transmission at an efficiency of from 81 to 92 per cent.

LOW WORM-GEARING RATIOS

The lower gear-ratio limit, while quite commonly set at 5 or 10 to 1 for worm gearing, is also far from fixed, for some excellent worm drives have been designed with ratios of less than 1.5 to 1. These low-ratio worm-gearing combinations are meeting with considerable favor as substitutes for less efficient spiral-gear applications, spiral gearing having inherently substantially lower load-carrying capacity.

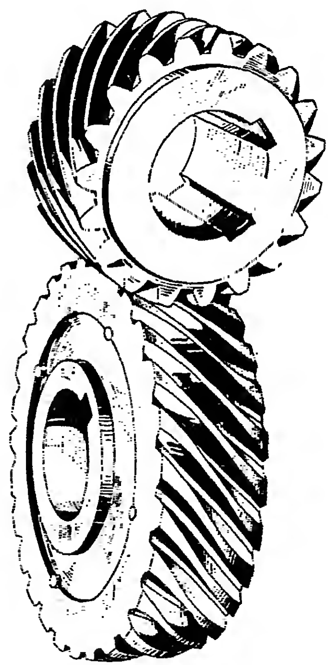


FIG. High efficiency 28 to 19 worm gearing.

The intimacy of worm-thread and worm-gear tooth contact, effected by the concave face of the gear conforming over its entire width to the convex curvature of the engaging worm, serves to distribute the load over the full face of the driven worm gear, while in the case of spiral gearing, especially when the connected shafts are at right angles, the contact of engaging spiral-gear teeth is sharply limited. The load is concentrated on the mid-

face section of the teeth, localizing wear and failing either to develop or utilize the full strength of the gear teeth.

The successful employment of worm gearing in this class of service is made possible only by the ability of the modern hardened-steel worm, accurately ground and polished, to withstand the sliding engagement of worm thread and teeth, which is characteristic of the class of gearing, without appreciable wear or generation of excessive frictional heat. A specific example of such worm gearing is one making use of a 19-thread worm to drive a 28-tooth worm gear, the gears being of 3-normal diametral pitch and 45-deg. helix angle, with teeth of 20-deg. pressure angle.

NORMAL PITCHES AND UNDERCUT TEETH

In multiple-threaded worms and their mating worm gears, the dimensions of the threads and teeth should be proportioned, as for spiral gears, normal to the tooth profiles when the helix, lead, or thread angle of the worm is more than 18 deg. The formulas for spiral gears may be used, bearing in mind that the axial pitch for worms is the same as the circular pitch for spiral gears.

In worm gears of less than 30 teeth, the thread of the worm, when the thread angle (twice the pressure angle) is the usual 29 deg., interferes with the flank of the meshing gear teeth, if the throat diameter is made equal, as is customary, to the pitch diameter of the worm gear plus twice the standard tooth-addendum dimension. Under such circumstances it is necessary, in order to avoid the undercutting of the worm-gear teeth, either to increase the throat diameter of the gear or to employ a heavier pressure angle for the gear teeth. Both methods are employed in the commercial production of worm gearing.

The first method has the advantage of effecting the required modification without the need of changing the hob employed to generate the worm wheel and is of especial interest in that it draws attention to the important fact that the addenda of the worm-thread and worm-gear teeth need not necessarily be exactly the same. In fact, it is advocated by some of the leading worm-gear manufacturers that it is better to decrease the addendum on the worm gear slightly and increase the addendum of the worm thread correspondingly, provided the pressure angle of the gear teeth and the number of teeth in the gear will permit such practice without undercutting the gear teeth. Where

there are only a comparatively few teeth in the gear, less than 30, the reverse operation is involved, naturally the addendum of the gear teeth is increased and the addendum of the worm thread correspondingly decreased. In worm gearing, this is analogous to increasing the throat diameter of the worm gear and overcomes the undercutting of the worm-gear teeth.

For worm gears with 14½-deg. teeth, the throat diameter of the worm gear can be increased by an amount equal to the difference between twice the addendum and about one-sixteenth of the pitch diameter, provided this enlargement does not entail too great a reduction in the outside diameter of the worm, or, expressed in the form of an equation,

$$TD \text{ enlarged } (14\tfrac{1}{2} \text{ deg. } VNP) = 0.937PD + 4A \qquad (97)$$

where $A = 1/NDP$.

The second method of avoiding undercutting the worm-gear teeth, by increasing the pressure angle of the teeth, makes no changes in the diameters of either the worm gear or worm, but it does necessitate the use of a multiplicity of hobs if depended upon exclusively for the avoidance of all worm-gear tooth undercutting, one for each number of teeth in the worm gear. A formula for ascertaining the required pressure angle is

$$\cos VNP = \qquad (98)$$

In commercial practice, it is customary to select a limited number of these pressure angles, such as 29, 40, 45, and 50 deg., and employ each angle over a range of teeth in the worm gear. Still greater precision, if required, may then be secured by some slight modification of the throat diameter, one way or the other, of the worm gear and corresponding alteration in worm diameter.

WORM-DIMENSION LIMITATIONS

Too great a reduction in the worm diameter, however, must be guarded against, for, while the diameter of the worm may be reduced, that of the hob employed for generating the worm-gear teeth remains the same, and any considerable difference between the diameters of the worm and hob results in a variation in the helix angles of the gear teeth and worm threads, preventing the worm fitting properly with the worm gear generated by the

hob. The pitch diameter of the worm must correspond to the pitch diameter of the hob in order to secure proper contact between the engaging worm threads and gear teeth.

When possible, it is desirable to make the pitch diameter for shell-type worms in strict accord with formula (69), the rule adopted by the A.G.M.A. for standard worms. However, when the worm is cut integral with its shaft, a somewhat greater reduction in diameter is permissible, a desirable rule being

$$PD = 2.35 \times LP + 0.4 \tag{99}$$

Recommendations for the length of the worm to give satisfactory contact and the advisable length of the generating hob are

$$LW = LP\left(4.5 + \frac{N}{50}\right) \tag{100}$$

$$LH = LW + LP \tag{101}$$

TYPES OF WORM GEARS

While the foregoing presentation on worm gearing has dealt exclusively with worm gears of the standard type, or form, *i.e.*,

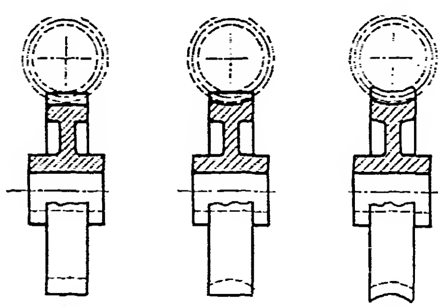


FIG. 87.—Diagrams of worm gearing. *A*, straight face; *B*, hobbed straight face; *C*, concave face.

worm gears of concave-face and curved-tooth elements that partially enwrap the engaging worm, there are two other forms of worm gears in commercial use, the straight face and the hobbed straight face. There is much less demand for these latter and less efficient gears; nevertheless, they are employed to some extent and represent practical forms of gearing with which the gear designer should be fully conversant.

Straight-face worm gears are in reality plain helical gears mating with worms and are not nearly so efficient as standard, concave-face worm gears. They have been employed in the past chiefly to replace spiral gearing, but with the development of the modern ground worm and the greater familiarity gained with the principles underlying high-efficiency worm gearing even this limited field is rapidly being usurped by the more efficient worm gear with concave face. Nevertheless, the straight-face worm gear possesses one distinctive peculiarity of merit, so far as worm gearing is concerned, and that is that side, or axial, adjustment can be made. Also, the construction has a tendency to discount to some extent the bad effects of vibration by permitting a certain weave between the worm and gear.

The hobbed straight-face type of worm gear, however, has retained a better hold upon its particular field and is used generally for indexing purposes and where light loads only are encountered. These gears are usually generated on hobbing machines, and the face of the gears is then turned straight. The tooth contact with these gears, while not so good or so efficient as that of the standard form of worm gear, is considerably better than that of the straight-face type of worm gear.

SELF-LOCKING WORMS AND GEARS

One of the features peculiar to worm gearing, one that is of high value in certain services, is that worm drives of low-helix angle can be made self-locking, *i.e.*, so proportioned that while the worm can drive the worm wheel the worm wheel cannot drive the worm. In designing worm gearing, to make use of this peculiarity, it is essential to keep the helix angle of the gearing under 5 deg. Such a fine-pitched worm is not especially efficient, the over-all efficiency of the worm and gear being less than 50 per cent, but any betterment in transmission ability defeats the object of the construction.

The starting friction, which is invariably greater than the running friction, is a consideration, for the worm and gear combination may have an efficiency of slightly over 50 per cent when running and still be self-locking when at rest. An ill-fitting worm drive with a helix angle of greater than the critical 5-deg. limit may apparently be self-locking, but, when the worm and worm gear wear in, and the drive is subjected to a live load or to vibrations, the quality is lost.

SECTION IX

SPIRAL-BEVEL, SKEW-BEVEL, AND HYPOID GEARS

Those decided operating advantages that are attained by the helical arrangement of gear teeth in cylindrical pitch surface gearing are secured in bevel gearing by a quite similar arrangement of teeth curved on the arc of a circle, such gearing being popularly termed *spiral-bevel gearing.* The use of a circular curve in place of a spiral, not only makes the commercial production of these gears at relatively low cost possible, but the slight difference in the radii of curvature for the convex and concave sides of the gear teeth proves of considerable advantage from the standpoints of both gear assemblage and operation.

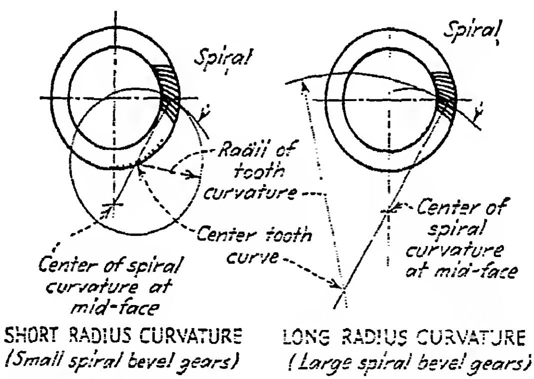


FIG. 5.—Similarity of circular arcs and face with section of spiral curves.

A circular cutter of the face-mill type, with straight-blade edges corresponding in obliquity to the profile, or pressure, angle of a crown gear, the straight-sided teeth of which constitute the basis of the octoid form of bevel-gear teeth, is used for the generation of the smaller sizes of spiral-bevel gears. The larger gears are generated by a single planing tool, with similar inclined cutting edges, operating on a continuously rotating gear blank, the tool entering a different tooth space on each successive stroke. By either method, employing the short radius of curvature for generating the teeth of the smaller spiral-bevel

gears or the long radius of curvature for the teeth of larger spiral gears, the circular tooth curve varies only a negligible degree from the curvature of the section of a spiral embraced by the face width of the gear (see Fig. 88).

ADVANTAGES OF SPIRAL-BEVEL GEARING

The essential difference between these spiral-bevel gears and ordinary straight-tooth bevel gears is that they have a continuous pitch-line contact and a larger number of teeth are in mesh at the same time. The teeth of spiral bevels, instead of meshing with full line contact engage one another gradually with a distinctive progressive action. Due to this overlapping feature, the load is transmitted from tooth to tooth without shock or sudden change in tooth pressure intensity. It is the presence of these shocks and rapid changes in tooth pressure in straight-tooth bevel gearing which is productive of noise and vibration, especially noticeable and objectionable at high peripheral speeds.

Owing to the fact that the radius of curvature on the convex side of spiral-bevel gear teeth is a trifle shorter than the corresponding radius of curvature for the concave side of the teeth, the longitudinal contact of the teeth is a shade the heaviest over the central section of the tooth face and tends to ease off slightly toward the ends of the teeth. The greater radius of curvature on the concave side of the teeth is not sufficiently marked to cause any appreciable concentration of the load but is just enough to allow some little displacement of the gears under operating loads, in this manner providing a somewhat greater range of axial adjustment than can be secured with well-proportioned straight-tooth bevel gearing.

STRENGTH, WEARING QUALITIES, AND EFFICIENCY

The tooth load on spiral-bevel gears is a resultant of the transmitted load and the thrust load, which for a 30-deg. spiral angle, the average value, amounts to 15 per cent more than the transmitted load. The load per tooth is less, however, than with straight-tooth bevel gears, owing to the fact that a greater number of teeth are in contact at the same time. Since the sectional area of the teeth is the same for both straight- and spiral-bevel gears, the strength of the latter exceeds that of comparable straight-tooth bevels. Consequently, spiral-bevel

gears proportioned for strength on the same basis as straight-tooth bevel gears will have a larger margin, or factor, of safety.

While the normal sliding action between meshing gear teeth is the same in spiral-bevel gearing as in any other form of toothed gearing, there is no longitudinal sliding between engaging teeth, owing to the obliquity of the tooth arrangement or any other cause. The convex side of the curved tooth nestles into the concave recess of the curved tooth with which it meshes, and there can be, consequently, no sliding motion between the pitch cones of the engaging gears. Another peculiarity of the gearing, exerting influence upon the wearing qualities of spiral-bevel gears, is that there is a continuous pitch-line contact, owing to the gradual and overlapping action of the curved gear teeth, where there is no sliding action whatever. As a result, the average sliding action, which is a measure of the wear, is reduced, and spiral-bevel gears, ordinarily, have a longer life than comparable straight-bevel gears.

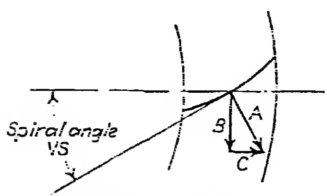


FIG. 89.—Diagram illustrating direction of forces acting on spiral-bevel gear teeth.

The continuous pitch-line contact of spiral bevels also tends to the development of higher transmission efficiencies and more than compensates for the additional thrust created by the oblique arrangement of the gear teeth. In actual practice, the distinctive bevel gears with curved teeth have shown themselves fully as efficient as the best of straight-tooth bevel gears, developing under favorable conditions efficiencies upward of 96 per cent. In common with all other types of bevel gears, however, the efficiency and wearing qualities of spiral-bevel gears are better with a large number of teeth than with smaller numbers. Consequently, for gear ratios of 4 to 1 and over, it is advisable to have not less than 12 teeth in the pinion member, 10 being held to be the practical minimum.

SPIRAL-ANGLE AND AXIAL-THRUST LOAD

The spiral angle (see Fig. 90), while usually made about 30 deg., should be large enough to make the arc of spiral AS, or pitch-diameter chord subtended by the pitch-cone elements

that embrace a full single curved tooth, not less than from 15 to 40 per cent greater than the circular pitch of the gear. It is important that this spiral angle be checked for each design of gear, to make certain of sufficient tooth overlap. Frequently, a somewhat greater overlap than the specified minimum may be found advantageous.

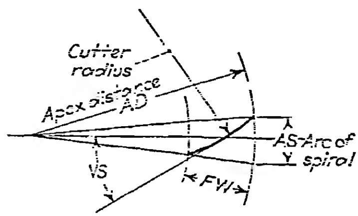


Fig. 90.—Spiral angle and arc of spiral.

The hand of the spiral, which is different for the two members of a gear-and-pinion combination, is a consideration of some importance. While it makes little or no difference so far as quietness and smoothness of operation are concerned or in the efficiency of the drive, whether the pinion or the gear member is the one with a right- or left-hand spiral, the hand of the spiral being the direction of tooth curvature viewed from the apex of the gear, there is a decided difference in the effect of the axial thrusts developed.

These thrust components of the tooth load are caused by the oblique arrangement of the spiral-gear teeth, and their direction is governed by the hand of spiral, direction of gear rotation, and whether the gear in question is a driving or driven member. The intensity of the thrusts is dependent upon the tooth load, spiral angle, pressure angle, and pitch angle. Formulas for determining the values, or amounts, of the unbalanced axial thrusts (*ATL*) in terms of the transmitted tangential tooth load (*TTL*), for all possible combinations of direction of rotation and hand of spiral, together with a guiding diagram (Fig. 91) indicating directions of thrust, are shown on page 130.

A right-hand spiral pinion, driving clockwise, tends to move toward the cone center, while a left-hand pinion tends to back away, *i.e.*, get out of mesh. Consequently, if there is any end play in the pinion shaft, the tendency of a right-hand pinion, driving clockwise, is to take up the essential backlash under heavy load and wedge the teeth of the gear and pinion members together; while a left-hand spiral pinion, under similar conditions, tends to back away and introduce additional backlash between the engaging teeth, a situation that still permits the gearing to function. In a one-way drive, therefore, it is generally advisable to select the hand of spiral that will tend to move the gear member

having the heavier axial thrust out of mesh when under load.

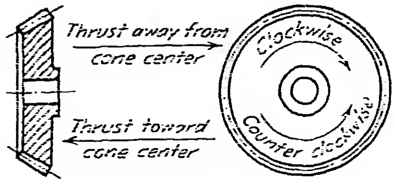


FIG. 91.—Direction of axial thrusts.

Driving member: Hand of spiral	Driving member: Rotation	Value of axial thrust. Arrow indicates thrust direction of each factor (Fig. 91). Resultant direction is that of larger factor	Formula
Right or left	Clockwise	$ATL = TTL\left(\overleftarrow{\tan VS \cos VPi} - \overrightarrow{\frac{\tan VP \sin VPi}{\cos VS}}\right)$	(102c)
Right or left	Counterclockwise	$ATL = TTL\left(\overrightarrow{\tan VS \cos VPi} + \overrightarrow{\frac{\tan VP \sin VPi}{\cos VS}}\right)$	(102b)

VP = pressure angle. VPi = pitch angle. VS = spiral angle.

SPIRAL-BEVEL GEAR SYSTEM

As the spiral angle, though ordinarily made 30 deg., is subject to a certain amount of variation, in order to secure the desired tooth overlap, the gear ratio of spiral-bevel gears is influenced by this fact as well as by the pitch diameters of the respective gear members. For this reason, as well as because a limited number of pressure angles is desirable from a manufacturing standpoint, and the further fact that the radii of tooth curvature (cutter radius), while of necessity the same for both gear members of a specific spiral-bevel drive, need not be exactly proportional either to the gear ratio or to the pitch diameters of the gears, the spiral-bevel gear system developed by the Gleason Works and adopted as a recommended standard by the A.G.M.A. is based upon a series of empirically determined rules rather than upon equations evaluating the numerous variable factors.

In formulating the various rules constituting the system, the governing consideration was to secure the utmost in the way of quietness in gear operation and requisite tooth strength

and durability, while making the system simple and practical. The system is a noninterchangeable one, the gears being intended to operate in pairs only.

In all but a few cases, a 14½-deg. pressure angle has been adopted for the system, such angle being the smallest that avoids excessive undercutting of gear teeth and, consequently, the most efficient and least liable to develop objectionable noise. The exceptions are chiefly where pinions of less than 10 teeth are employed and certain specific gear ratios.

The addenda of the gear teeth increase and the addenda of the pinion teeth decrease somewhat as the gear ratios increase, so that the arc of recession is greater than the arc of approach. The result is that the amount of sliding in the engagement of teeth is about the same as or slightly less during approach than it is during recession, a desirable arrangement since recess action is inherently quieter than approach action.

A.G.M.A. Proportions for Spiral-bevel Gears Operating at Right Angles, Where the Pinion Is the Driver

(Gleason Works System)

Pressure angle:

	VP, degrees
5, 6, or 7 teeth in pinion	20
8 or 9 teeth in pinion	17½
Ratios having 12 or more teeth in pinion	14½
11–11 to 11–19	17½
12–20 and higher	14½
10–10 to 10–24	17½
10–25 and higher	14½

Addendum, gear:

For 5 teeth in pinion,

$$A = \frac{\text{value, Table 27}}{DP} \times \frac{14}{17} \tag{103a}$$

For 6 teeth in pinion,

$$A = \frac{\text{value, Table 27}}{DP} \times \frac{15}{17} \tag{103b}$$

For 7 or 8 teeth in pinion,

$$A = \frac{\text{value, Table 27}}{DP} \times \frac{16}{17} \tag{103c}$$

For 9 or more teeth in pinion,

$$A = \frac{\text{value, Table 27}}{DP} \tag{103d}$$

Addendum, pinion:

5-tooth pinion,

$$a = \frac{1.4000}{DP} - A \tag{103e}$$

6-tooth pinion,

$$a = \frac{1.5000}{DP} - A \tag{103f}$$

7- or 8-tooth pinion,

$$a = \frac{1.6000}{DP} - A \tag{103g}$$

9 or more teeth in pinion,

$$a = \frac{1.7000}{DP} - A \tag{103h}$$

Dedendum, gear:

For 5 teeth in pinion,

$$D = \frac{1.557}{DP} - A \tag{104a}$$

For 6 teeth in pinion,

$$D = \frac{1.657}{DP} - A \tag{104b}$$

For 7 teeth in pinion,

$$D = \frac{1.757}{DP} - A \tag{104c}$$

For 8 teeth in pinion,

$$D = \frac{1.788}{DP} - A \tag{104d}$$

For 9 or more teeth in pinion,

$$D \quad \frac{1.}{DP} - A \tag{104e}$$

Dedendum, pinion:

5-tooth pinion,

$$d = \frac{1.557}{DP} - a \tag{104f}$$

6-tooth pinion,

$$d = \frac{1.657}{DP} - a \tag{104g}$$

7-tooth pinion,

$$d = \frac{1.757}{DP} - a \tag{104h}$$

8-tooth pinion,

$$d = \frac{1.788}{DP} - a \tag{104i}$$

9 or more teeth in pinion,

$$d = \frac{1.888}{DP} - a \tag{104j}$$

Whole depth (gear and pinion):

With 5 teeth in pinion,

$$WD = \frac{1.557}{DP} \tag{105a}$$

With 6 teeth in pinion,

$$WD = \frac{1.657}{DP} \tag{105b}$$

With 7 teeth in pinion,

$$WD = \frac{1.757}{DP} \tag{105c}$$

With 8 teeth in pinion,

$$WD = \frac{1.788}{DP} \tag{105d}$$

With 9 or more teeth in pinion,

$$WD = \frac{1.}{DP} \tag{105e}$$

Circular thickness of teeth, gear:

With 5 teeth in pinion,

$$CTh = \frac{1.011}{DP} + 0.84 - \frac{K \text{ (Table 28)}}{DP} \tag{106a}$$

With 6 teeth in pinion,

$$CTh = \frac{0.971}{DP} + 0.84 - \frac{K}{DP} \tag{106b}$$

With 7 teeth in pinion,

$$CTh = \frac{0.931}{DP} + 0.84 - \frac{K}{DP} \tag{106c}$$

With 8 teeth in pinion,

$$CTh = \frac{1.011}{DP} + 0.7A - \frac{K}{DP} \tag{106d}$$

With 9 teeth in pinion,

$$CTh = \frac{0.976}{DP} + 0.7A - \frac{K}{DP} \tag{106e}$$

With 10 or more teeth in pinion,

(14½ deg. VP)

$$CTh = \frac{1.061}{DP} + 0.6A - \frac{K}{DP} \tag{106f}$$

(17½ deg. VP)

$$CTh = \frac{0.976}{DP} + 0.7A - \frac{K}{DP} \tag{106g}$$

Circular thickness of teeth, pinion:

For all combinations,

$$cth = \frac{3.142}{DP} - CTh \tag{106h}$$

TABLE 27.—ADDENDUM VALUES FOR ONE DIAMETRAL PITCH FOR DIFFERENT RATIOS
(Gleason Works System)

Gear ratio $\frac{N}{n} = \frac{\text{number of teeth in gear}}{\text{number of teeth in pinion}}$

Ratios			Ratios			Ratios			Ratios		
From	To	A	From	To	A	From	To	A	From	To	A
1.00	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.01	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.02	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.03	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.05	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.06	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.08	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.09	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE 28.—VALUES OF K FOR CIRCULAR THICKNESS OF TOOTH FORMULAS
(Gleason Works System)

Number of teeth in pinion	Ratios														
	1.00 to 1.25	1.25 to 1.50	1.50 to 1.75	1.75 to 2.00	2.00 to 2.25	2.25 to 2.50	2.50 to 2.75	2.75 to 3.00	3.00 to 3.25	3.25 to 3.50	3.50 to 3.75	3.75 to 4.00	4.00 to 4.50	4.50 to 5.00	5.00 and higher
5	0.020	0.040	0.075	0.110	0.135	0.155	0.170	0.185	0.200	0.215	0.230	0.240	0.255	0.270	0.285
6	.010	.035	.060	.085	.105	.130	.150	.165	.180	.195	.210	.220	.235	.250	.265
7	.000	.025	.050	.075	.095	.115	.135	.155	.170	.185	.195	.205	.220	.235	.250
8	.000	.010	.030	.045	.065	.080	.095	.110	.125	.135	.145	.155	.170	.180	.195
9	.000	.010	.025	.040	.055	.070	.085	.095	.105	.115	.125	.135	.150	.165	.185
10	.020	.055	.085	.105	.125	.125	.110	.120	.130	.140	.150	.155	.160	.170	.180
11	.030	.075	.105	.070	.085	.095	.105	.115	.125	.135	.140	.145	.150	.155	.160
12 to 13	.005	.015	.025	.035	.045	.055	.065	.075	.085	.095	.105	.115	.125	.135	.135
14 to 16	.000	.005	.015	.025	.035	.050	.060	.075	.085	.095	.100	.105	.105	.105	.105
17 to 19	.000	.000	.005	.015	.025	.035	.050	.065	.075	.085	.090	.090	.090	.090	.090
20 up	.000	.000	.000	.005	.015	.025	.040	.050	.055	.060	.060	.060	.060	.060	.060

TABLE 29.—FORM FACTOR Y (LEWIS FORMULAS) FOR SPIRAL-BEVEL GEAR TEETH
(Gleason Works System)

Number of teeth in pinion (n)	Ratios														
	1.00 to 1.25	1.25 to 1.60	1.60 to 1.75	1.75 to 2.00	2.00 to 2.25	2.25 to 2.50	2.50 to 2.75	2.75 to 3.00	3.00 to 3.25	3.25 to 3.50	3.50 to 3.75	3.75 to 4.00	4.00 to 4.50	4.50 to 5.00	5.00 to ..
5	0.297	0.322	0.343	0.361	0.376	0.388	0.398	0.406	0.411	0.416	0.426	0.424	0.431	0.438	0.450
6	.316	.332	.353	.372	.386	.398	.406	.414	.418	.424	.428	.431	.436	.443	.452
7	.318	.333	.347	.360	.373	.384	.392	.398	.405	.410	.415	.419	.426	.432	.439
8	.298	.326	.336	.348	.357	.366	.373	.378	.384	.388	.392	.394	.397	.400	.405
9	.292	.313	.327	.338	.346	.352	.357	.363	.367	.370	.373	.376	.380	.384	.388
10	.315	.338	.353	.363	.371	.345	.326	.342	.351	.357	.363	.367	.371	.374	.377
11	.316	.335	.343	.325	.327	.333	.338	.344	.350	.356	.361	.367	.375	.384	.390
12	.298	.318	.333	.343	.351	.357	.363	.368	.372	.377	.373	.381	.384	.386	.388
13	.302	.326	.334	.343	.351	.358	.365	.371	.376	.381	.384	.386	.388	.391	.393
14	.306	.322	.334	.345	.354	.362	.368	.374	.378	.382	.386	.389	.391	.393	.395
15	.314	.336	.342	.352	.366	.368	.374	.386	.385	.389	.392	.394	.397	.399	.402
16	.322	.335	.347	.358	.367	.374	.381	.386	.390	.394	.397	.400	.402	.404	.406
17-18	.329	.343	.354	.364	.373	.382	.389	.394	.398	.400	.403	.406	.407	.409	.416
19-21	.339	.351	.362	.373	.382	.388	.396	.401	.405	.407	.410	.411	.412	.414	.415
22-25	.351	.363	.373	.382	.391	.398	.403	.407	.410	.412	.413	.414	.415	.417	.418
26-30	.364	.374	.384	.393	.399	.404	.407	.410	.412	.414	.415	.416	.417	.418	.419

A listing of values for the form factor Y (Lewis equations) for the strength of spiral-bevel gear teeth is given in Table 29, but the actual strength of well-proportioned overlapping teeth is almost always higher than the computed value.

BACKLASH AND MACHINING TOLERANCES

In the commercial production of spiral-bevel gearing, a predetermined amount of backlash (reduction in chordal tooth thickness) should be provided, the amount differing for soft gears and gears to be hardened, as well as for the pitch of the gearing. This change in tooth thickness should be divided between the gear and pinion teeth, making the total reduction in chordal tooth thicknesses the backlash provided:

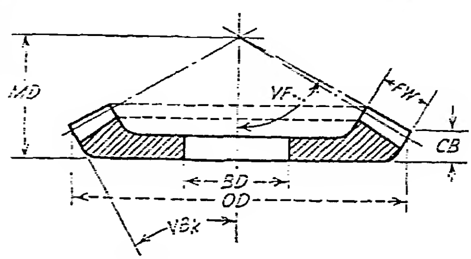


FIG. 92.—Diagram of spiral-bevel gear.

REDUCTION IN CHORDAL THICKNESS

Pitch	Soft gears, in.	Gears to be hardened, in.	Backlash, in.
1*DP*	0.010	0.012	0.020–0.024
2*DP*	.006	.008	.012– .016
3*DP*	.004	.005	.008– .010
4– 5*DP*	.003	.004	.006– .008
6–12*DP*	.002	.003	.004– .006

The following are suggested as practical limits for the machining of spiral-bevel gears:

Symbol	Dimension	Tolerance
OD	Outside diameter	+0.000 in. – 0.005 in.
CB	Crown backing	+ .000 in. – .002 in.
BD	Bore diameter	+ .001 in. – .000 in.
FW	Face width	+ .000 in. – .010 in.
VBk	Back angle	+ 15 min.– 15 min.
VF	Face angle	+ 8 min.– 0 min.

As practical limits after hardening and before grinding, it is suggested that the outside diameter of the gear should not run out more than 0.003 in., bores should not be out of round more than 0.003 in., and that the face angle should be held to a limit of minus 8 min. In internal-type pinions, the run-out on the shank at the head bearing should not exceed 0.001 in.

SKEW-BEVEL GEARS

Another variety of bevel gearing, in which the tooth arrangement also departs from the usual radial plan of straight-line

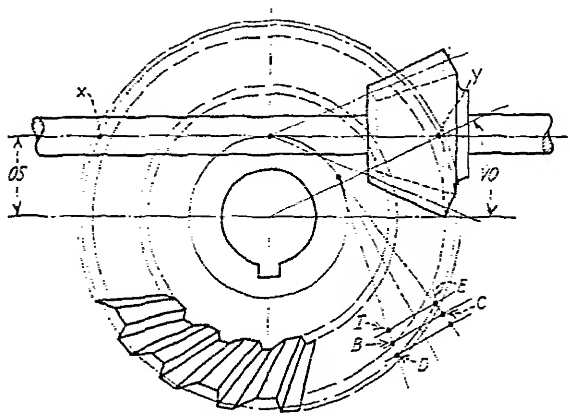


FIG. 93.—Plan of skew-bevel gear and straight-tooth pinion.

teeth, is that popularly known as *skew-bevel gears,* the axes of the respective gear members having no common plane. There are two types of these unorthodox gear drives: one in which the pinion member is of the ordinary straight-tooth pinion variety, with the oblique arrangement of teeth confined to the gear member; and the other in which the teeth in both the pinion and gear are askew. In either type, the pitch surface of the pinion is the frustum of a figure generated by the revolution of a straight line about a nonparallel axis, a *hyperboloid of revolution.*

The first and more common type is illustrated in Fig. 93, the second in Fig. 94, where the dimension *OS* measures the offset of the pinion. In either arrangement, the apex of the pinion member lies in the perpendicular axis plane of the gear.

In the case of the gear combination with a standard straight-tooth pinion (Fig. 93), it is obvious that not only do the pinion

tooth-face elements converge to the apex point in the perpendicular axis plane of the gear, but the face elements of the gear tooth in mesh with the pinion as well. As successive teeth of the gear engage with the teeth of the pinion, there is a similar common convergence of engaging gear-tooth elements, the apices of the successive gear-tooth elements lying on the circumference of a circle having a radius equal to the offset of the pinion.

The pinion member differing in no respect from a regular, standard, straight-tooth bevel gear, it is evident that, if the pinion shaft was not offset and the gear combination simply a set of ordinary bevels having the same gear ratio as the skew-bevel gear assemblage, the pitch diameter of the mating gear with straight radial teeth would then be equal to the distance xy, the length of the chord of the skew-bevel gear-pitch circle subtended by th epinion axis. This *equivalent pitch diameter*, consequently, governs the number of gear teeth, whether the gear member is a straight-tooth, standard bevel gear, or a skew-bevel gear.

In the case of the skew-bevel gear, the normal pitch BC must conform to the circular pitch of the pinion, and the number of teeth in the skew-bevel gear, it follows, equals the number in the standard bevel gear having a pitch diameter equal to xy. The circular pitches of the mating gears in the skew-bevel gear assemblage differ, however, the circular pitch DE of the skew-bevel gear being governed by the actual pitch diameter of that gear and not by its equivalent pitch diameter xy.

The longitudinal sliding between the engaging teeth present in this form of gearing also depends upon the amount of pinion-shaft offset. In the combination illustrated in Fig. 93, the longitudinal tooth sliding is measured by the distance ID, and, if the shaft offset were greater, this sliding would be further accentuated. The limiting condition would be when the offset is equal to half the pitch diameter of the skew-bevel gear, when there would be only sliding action between the teeth and no turning moment.

The various angles and tooth proportions of the pinion member in this form of skew-bevel gearing differ in no respect from those for similar standard, straight-tooth, bevel gears and are easily computed. The calculations for the skew-bevel gear are, however, somewhat special and require the application of other formulas.

Skew-bevel Gear Nomenclature*
(Straight-tooth Pinion)

Angle	Symbol	Angle	Symbol
Angle of offset	VO	Decrement angle	VD
Bottom (cutting) angle	VB	Face angle	VF
Center angle	VC	Increment angle	VI
Dimension		**Dimension**	
Addendum	A	Equivalent pitch diameter	PDe
Circular pitch	CP	Face width	FW
Dedendum	D	Normal pitch	NP
Diameter increment	DI	Offset	OS
Diametral pitch	DP	Outer diameter	OD
Pitch diameter	PD	Number of teeth	N

* Symbols for pinion members are customarily distinguished from corresponding symbols for gear members by the use of small, instead of capital, letters.

Formulas for Skew-bevel Gears—Straight-tooth Pinion

$$PDe = \frac{N}{DP} \tag{107}$$

$$\tan VO = \frac{2OS}{PDe} \tag{108}$$

$$PD = \frac{2OS}{\sin VO} \tag{109}$$

$$NP = \frac{3.1416PD}{N} \tag{110}$$

$$\tan vc = \frac{pd}{PDe} \tag{111}$$

$$\tan VI = \frac{2 \sin vc}{n} \tag{30}$$

$$\tan VD = \frac{2.314 \sin vc}{n} \tag{31}$$

$$vf = vc + VI \tag{32}$$

$$vb = vc - VD \tag{33}$$

$$A = \frac{pd}{n} \tag{17}$$

$$di = 2A \times \cos vc \tag{35}$$

$$OD = PD + \frac{di \times PDe}{PD} \tag{112}$$

$$VF = \frac{(VC + VI)PD}{PDe} \tag{113}$$

Example in Design of Skew-bevel Gearing—Straight-tooth Pinion

Required: Pair of skew-bevel gears, 10 diametral pitch, 85 teeth in gear, 13 teeth in pinion; pinion-shaft offset $1\frac{1}{2}$ in.

Symbol	Formula	Computation	Dimension
pd	(15)	$\frac{13}{10}$	1.30 in.
PDe	(107)	$\frac{85}{10}$	8.50 in.
$\tan VO$	(108)	$\frac{2 \times 1.5}{85}$	0.3529
		VO = 19 deg. 26 min.	
PD	(109)	$\frac{2 \times 1.5}{0.33271}$	9.01 in.
NP	(110)	$\frac{3.1416 \times 9}{85}$	0.33 in.
$\tan vc$	(111)	$\frac{1.3}{8.5}$	0.1529
		vc = 8 deg. 42 min.	
$\tan VI$	(30)	$\frac{2 \times 0.15126}{13}$	0.02327
		VI = 1 deg. 20 min.	
$\tan VD$	(31)	$\frac{2.314 \times 0.15126}{13}$	0.02692
		VD = 1 deg. 33 min.	
vf	(32)	(8 deg. 42 min.) + (1 deg. 20 min.)	10 deg. 2 min.
vb	(33)	(8 deg. 42 min.) − (1 deg. 33 min.)	7 deg. 9 min.
A	(17)	$\frac{1.30}{13}$	0.10 in.
di	(35)	$2 \times 0.10 \times 0.1513$	0.03026 in.
OD	(112)	$9 + \frac{0.03026 \times 8.5}{9}$	9.02858 in.
VC		90 deg. − (8 deg. 42 min.)	81 deg. 18 min.
VB		(81 deg. 18 min.) − (1 deg. 33 min.)	79 deg. 45 min
VF	(113)	$\frac{((81 \text{ deg. } 18 \text{ min.}) + (1 \text{ deg. } 20 \text{ min.}))9}{8.5}$	87 deg. 29 min.

The outside diameter of the skew-bevel gear is somewhat less than would be the outer diameter of a standard bevel gear of the same pitch diameter, because the effect of the pinion-shaft offset is to reduce the diameter increment of the gear slightly. Formula (112), consequently, is based on the assumption that the decrease in the diameter increment is directly proportional to the ratio of the equivalent pitch diameter to the pitch diameter of the skew-bevel gear. Actually the decrease is not quite so

uniform, but the error introduced by the assumption is so trivial as to be quite negligible in commercial gear production.

A slight inaccuracy also exists in connection with the use of formula (113), the equation for the face angle of the skew-bevel gear. There is a slight increase in the face angle owing to the offset of the pinion shaft, an increase that is assumed, in the derivation of the formula, to be directly proportional to the ratio existing between the actual and equivalent pitch diameters of the skew-bevel gear. The increase is not, however, quite so constant, but, again, for all practical shop requirements the formula as given can be used with safety. The inaccuracy simply affects the total depth of the tooth by a trivial amount and only at the small end of the teeth where the error is least noticeable or harmful.

A detail of vital importance, so far as the satisfactory operation of skew-bevel gearing is concerned, however, and one, which the formulas presented for the calculations involved in proportioning the gear member do not cover, is the fact that the tooth-face elements of the skew-bevel gear are not quite straight, being really of spiral curvature and slightly longer on one side of the teeth than on the other. The result is that the tooth faces on the opposite sides of the individual tooth are not identical in a properly proportioned skew-bevel gear. The amount of variation depends, of course, upon the amount of pinion-shaft offset, making it essential to cut the teeth in the skew-bevel gear with the gear in the exact offset position it will occupy when running with its mating member.

GEAR AND PINION ASKEW

The design of the other type of skew-bevel gear drive, that in which the spiral tooth arrangement is used for both gear and pinion members, is customarily simplified in the commercial production of the gears by the expedient of proportioning both the gear and pinion according to the dimensions for standard, straight-tooth, bevel gears with the same number of teeth, pitch, and gear ratio. Ordinarily no alteration in the various diameters is made, nor are any modifications in angles necessary. This plan is made possible by the fact that, while the apex points of the engaging gears do not coincide the converging, conical, pitch surfaces of the skew-bevel gears are parallel to the pitch cones of similar straight-tooth bevel gears with a common apex

point (see Fig. 94), the center angles of the gear and pinion members being complements and equal in both instances.

The teeth of both the gear and pinion members are machined with the plane of the cutting-tool offset from the work-carrying spindle, but the amount of this cutter offset is not the same for the respective gear members, unless they are of exactly the same size. In such event, the offset between the work spindle and the plane of the tool is made the same for machining both gears and equal to half the total offset in the drive; the respective gears, of course, rotating in opposite directions during produc-

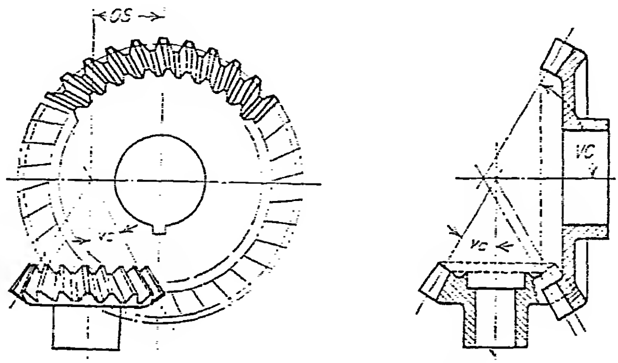


FIG. 94.—Layout for skew-bevel gears.

tion, as they do in service. When the gear ratio is other than 1 to 1, the total offset is divided proportionally to the gear ratio, the smaller value being employed as the cutter offset for machining the pinion teeth and the larger value for the cutter offset for machining the gear teeth.

Skew-bevel gearing with teeth proportioned in this manner runs smoothly and is in some respects superior to standard, straight-tooth, bevel gearing, but, if the offset which governs the angularity of the teeth is relatively large, the strength of the gear teeth may prove inadequate. In such instances, an increase in gear diameter, the amount of increase depending upon the angularity of the skew-bevel gear teeth, will provide the additional tooth thickness required for the development of a tooth strength equal to that of the standard bevel gear.

HYPOID GEARS

The same important feature of efficient gear transmission between offset axes, characteristic of skew-bevel gearing, is

possessed in the class of spiral-bevel gearing of circular tooth curvature, by what are commonly known as *hypoid gears;* gears designed to run on offset axes, approximately conical in form, similar in general appearance to spiral-bevel gears and with teeth generated by methods quite similar to those used for spiral-bevel gears.

While hypoids are machined in practically the same manner as spiral gears, there is usually less spiral angle on the gear member and more on the pinion member, the difference depending upon the degree of pinion offset. The result is that, with the same size mating gear and number of teeth, a hypoid pinion of the usual design will be from 20 to 30 per cent larger than a corresponding spiral-bevel pinion.

In the computations entailed for the hypoid-gear member the pitch angle is governed, naturally, by the number of teeth,

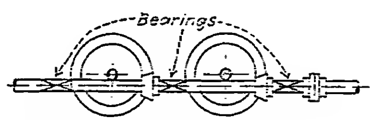


Fig. 95.—Offset pinion shaft permits good bearing support.

and the gear ratio establishes the circular pitch, the latter being influenced by both the spiral angle and the radius of tooth curvature. It is necessary, therefore, to employ a spiral angle and radius of curvature suitable for the hypoid pinion in the offset position in which it operates and in which it is generated. Unless this is done, the curvature of the two sides of the pinion teeth differs too much, and undesirable conditions develop. For this reason, the blank dimensions for hypoid gears have to be worked out to suit the particular spiral angle selected and radius of tooth curvature used.

Correctly proportioned hypoid gears possess all the good features of spiral-bevel gears and have, like skew-bevel gears, other advantages due to the pinion offset. Their operation combines the smooth rolling action of spiral-bevel gears and a limited amount of sliding action of the nature of that of worms. This additional sliding is not sufficient to accelerate tooth wear appreciably and proves of advantage in that the high polish imparted to the gear teeth by the motion tends to build up the efficiency of the gearing with service, *i.e.*, the efficiency of the

gearing tends to improve with running. The sliding also helps to distribute and maintain an oil film between lubricated teeth.

The most important advantage of the offset mountings common to both skew-bevel and hypoid-gear assemblages is that the shafts of the respective gear members can be extended past one another, permitting bearing supports to be located on either side of the gears and affording far greater opportunities for gear applications than have been feasible with ordinary bevel gears operating with a common apex point. An example of how this feature can be applied in machine design is indicated in Fig. 95, where a number of offset pinions can be mounted on the same shaft and each drive its individual gear.

HYPOIDS IN AUTOMOBILE DIFFERENTIALS

One of the applications in which hypoid gearing has demonstrated a high value is in its use in the rear axles of automobiles for differential gearing, a service, incidentally, largely instrumental in the development of the gearing and a service for which spiral-bevel gearing had previously been considered especially suitable. Extended tests between four different spiral-bevel and hypoid-gear differentials of between $4\frac{1}{2}$- and 5-to-1 ratio brought out some interesting statistics. In each case, the number of teeth in gear and pinion and diameter of gear member were the same for spiral bevels and hypoids. The loads measured, furthermore, were based on the same power delivered at the rear axle.

An average of the results secured with hypoid gearing, in terms of like results with spiral-bevel gearing, are:

	Per Cent
Increase in pinion diameter	28
Increase in spiral angle on pinion member	30
Reduction in spiral angle on gear member	40
Reduction in normal tooth load on pinion	10
Reduction in tangential load on pinion	19
Increase in end thrust of pinion on forward drive	8
Reduction in end thrust of pinion on reverse drive	7

SECTION X

INTERNAL GEARING

The internal arrangement of gear teeth possesses so many inherent advantages that it is especially desirable for the gear designer to be, not only thoroughly familiar with internal gearing, but fully cognizant of the limitations of internal gears and to know how accepted modifications in the form of internal involute teeth have greatly extended the fields of application for the gears. The more obvious of internal gearing advantages are, naturally, its compactness, the pinion running inside the gear, and the fact that the rim of the internal gear is itself an excellent gear-tooth guard, providing a safety feature secured with externally meshing gearing only when the gears are enclosed in a protecting casing.

There is greater length of tooth contact in internal gearing, the pitch circle of the internal gear following, instead of receding from, the pitch circle of the pinion, a larger proportion of rolling and less sliding action between engaging gear teeth is secured, more teeth are in engagement at the same time and there is considerably less tooth wear. As a result, there is a wider distribution of the transmitted load than can be secured with externally meshing gearing, less friction is developed, less vibration; higher mechanical efficiency is secured, longer gear life and greater gear strength. Because of the longer tooth contact and reduction in sliding action, furthermore, internal gearing is inherently quiet in operation.

Such a formidable list of important advantages, in view of the fact that with approved methods of gear-tooth generation internal gearing need be little more expensive to produce than externally meshing gearing of plain-spur or helical-tooth type, focuses attention more upon the limitations of the gears and on the methods devised to circumvent the fouling of teeth by modifications of the involute profile of the gear-member teeth. However, these accepted liberties with the involute system of internal gearing can be best taken up individually, attention first being given to the formulas entailed in the design of internal gearing.

The nomenclature and most of the dimensions and proportions of internal gears are similar to those for externally meshing spur gears, with the substitution of the internal diameter ID for the

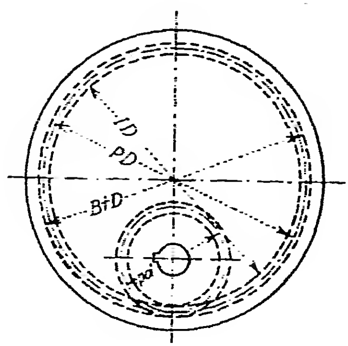


FIG. 96.—Diagram of internal gearing.

outer diameter of plain spur gearing and of the bottom diameter BtD for the root diameter. Also, the center distance in internal gearing is equal to the difference between the pitch radii of the respective gear members, instead of their sum.

Formulas for Internal Gearing

$$CD = \frac{N - n}{2DP} = \frac{(N - n)CP}{6.2832} = \frac{PD - pd}{2} \tag{114}$$

$$ID = PD - 2A = \frac{N}{DP} - 2A \tag{115}$$

$$BtD = 2WD + ID = 2.5A + PD \tag{116}$$

DESIGN OF INTERNAL GEARS

While the design of an involute internal gear differs in only a few minor points from that of an involute external gear, the limitations of the involute system of gearing impose conditions which frequently necessitate quite radical modifications in the addendum portion of the internal gear-tooth profiles. These alterations, though they are made automatically by the pinion-cutter method of internal gear-tooth generation, entail, when necessary, a certain sacrifice in the duration of active tooth contact, *i.e.*, a shortening of the involute line, or path, of action.

This loss is attributable for the most part to the fact that with the pitch circles of the gear and pinion curving in the same direction the involute interference of teeth is much intensified,

and the internal gear teeth foul with the pinion teeth if there is not sufficient difference between the diameters of the respective gear members. The fact that there is a minimum number of teeth in an involute-type pinion, under which excessive undercutting of the pinion teeth occurs, aggravates the situation still further.

When the internal gear is comparatively large and the pinion small, the teeth of the respective gear members will clear without fouling, or with only a limited amount of interference which can be taken care of by a light trimming of the internal gear teeth, with little sacrifice in duration of active tooth contact.

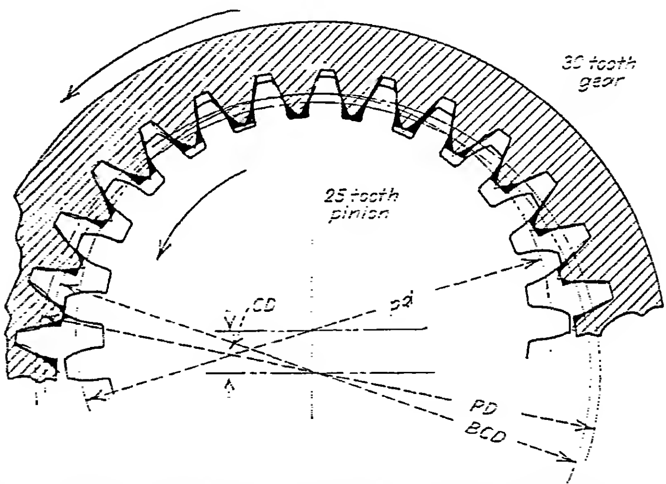


FIG. 97.—Diagram showing how pinion and internal gear teeth foul when center distance is small; $14\frac{1}{2}$-deg., full-depth, involute teeth.

As a general rule, the smallest permissible difference between the number of teeth in the internal gear and the mating pinion, in order to secure proper tooth action without material modification in the shape of the tooth is seven, when employing the 20-deg. stub form of involute tooth, the customary standard for internal gears, or, when using full-depth teeth of the $14\frac{1}{2}$-deg. involute form, 12 teeth.

As the gear-shaper type of cutter employed generally for the generation of involute-type internal gears is essentially a pinion, the cutting generation of internal gears presents the same conditions as when a pinion is mated with the internal gear. It is essential, therefore, that the difference between the number of teeth in the cutter and the number of teeth in the internal

gear it is to generate be such as to permit the cutter to be fed to depth and at the same time rotate with the gear without any excessive trimming or fouling of the internal gear teeth.

In the diagram (Fig. 97), depicting a 25-tooth pinion meshing with a 30-tooth internal gear, the difference between the number of teeth in the respective gear members is only five, with the result that the heavy fouling indicated by the black corners of the internal gear teeth would occur, were these sections not

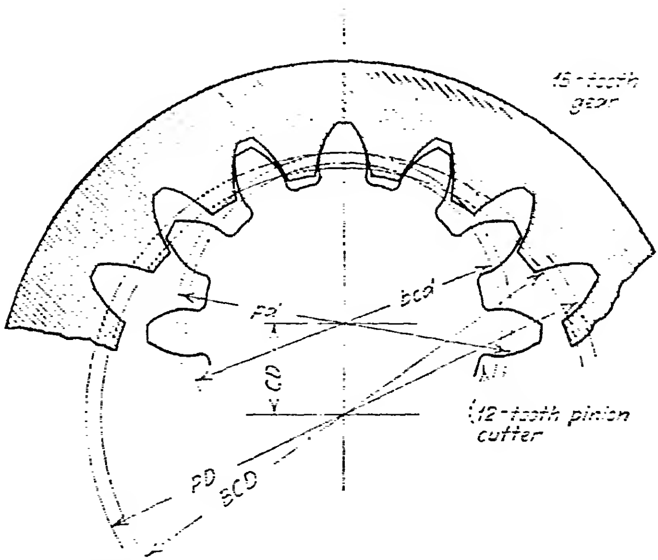


FIG. 98.—Diagram showing how large pinion cutter trims teeth of internal gear member when fed to depth.

removed by the pinion cutter generating the internal gear teeth. These sections, it will be noted, extend well beyond the base circle of the internal gear, sharply limiting the duration of active tooth contact. The much modified addendum sections of the internal gear teeth are inoperative so far as useful work is concerned, for involute tooth contact cannot take place inside the base circle nor does it occur outside the base circle where the involute profile curvature is destroyed, and these sections could be removed by increasing the internal diameter of the gear without affecting the efficiency of the gear assemblage in any way. To improve the situation, either a heavier pressure angle would

have to be employed, thus reducing the degree of fouling, or else a pinion with a smaller number of teeth.

In the layout illustrated in Fig. 98, showing a 12-tooth pinion cutter engaged with an 18-tooth internal gear, trimming of the internal gear teeth, indicated by the black sections of the gear teeth, takes place as the cutter is fed to depth. In this case, the amount of tooth removed is considerably less, a 20-deg. stub tooth being employed and there being a difference of six between the number of teeth in the internal gear and the pinion cutter. The trimming, furthermore, does not extend much beyond the base circle, is not so harmful, and could be avoided entirely by employing, if feasible, a cutter with a smaller number of teeth.

Modification of tooth form by the trimming of the internal gear teeth, or by a reduction in the internal diameter of the gear; the reduction of interference by the employment of a heavier pressure angle; and the use of shortened-depth teeth for both gear members are all means resorted to in the avoidance of fouling between pinion and internal gear teeth in the involute form of internal gearing, but these methods all involve a certain sacrifice in transmission efficiency and generally destroy that questionable advantage of interchangeability distinctive of the involute system. For these reasons, as well as for the merits of the development, another system of internal gearing, the Williams system, is of especial interest, possessing as it does a number of valuable features.

WILLIAMS SYSTEM OF INTERNAL GEARING

In the Williams system of internal gearing, the forms of the gear teeth in both the internal gear and the mating pinion depart radically from those designed according to other systems and secure in so doing certain inherent advantages that give to the gearing an improved operating action, increase the strength of the pinion teeth, reduce wear, and greatly reduce the cost of producing accurate and efficient internal gearing of the spur or helical type. The limitations placed upon internal gearing designed according to other systems are materially reduced, while all the advantages are retained and accentuated.

In this system, the teeth of the internal gear are made with straight-line profiles, the bounding profiles of the tooth spaces being similar to those of an involute rack, while the teeth of the mating pinion are provided with curved profiles of conjugate

form, giving a combination that results in a distinctive path to the point of contact between the gear and pinion teeth through-

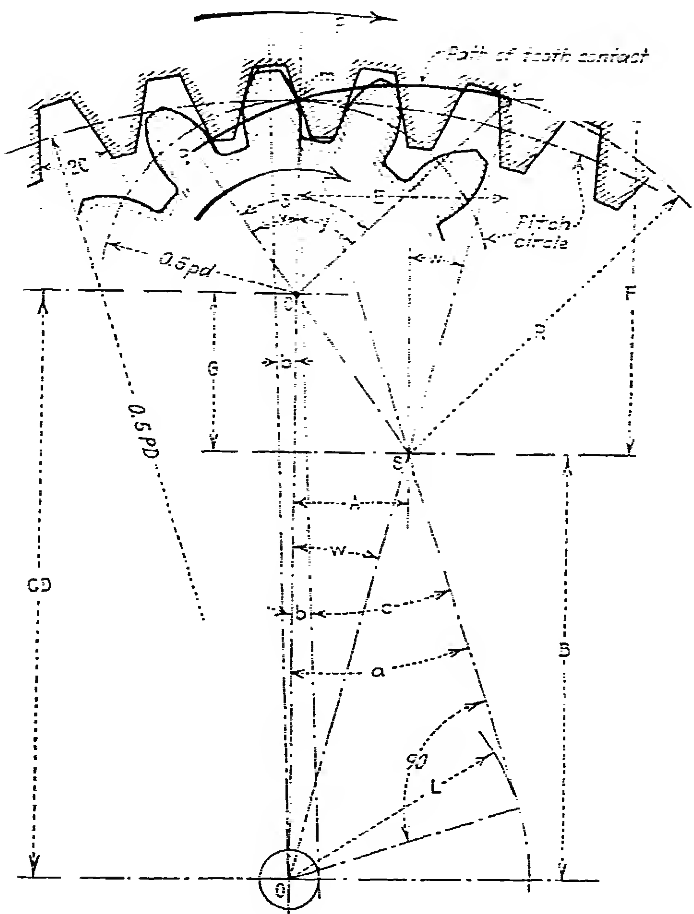


FIG. 99.—Layout of Williams system of internal gearing.
$b = 90/N$. $a = c + b$. $L = 0.5PD \times \sin a$. $R = 0.25PD \times \cos a$. $A = R \times \sin a$. $B = 0.5PD - R \times \cos a$. $\tan w = \frac{A}{B}$. $z = x + y$. $\tan x = \frac{A}{CD - B}$. $E = 2R \times \sin w$. $F = \frac{E - A}{\tan w}$. $\tan y = \frac{E}{F - G}$.

out their arc of contact (see Fig. 99). An extension of each flat gear-tooth profile is tangent to a circle centered on the axis of the internal gear, and the extension which passes through the instant axis m of the mating gears also passes through the

center S of the path of tooth contact, as does also the radial normal from the center of the internal gear to the path of tooth contact.

The distinctive path of tooth contact is, to all intents and purposes, the arc of a circle centered at the intersection point S and passing through the instant axis m of the engaging gears. Its usable section is subtended by the angle between a radial from the center S of the path of tooth contact passing through the center of the pinion member and the radial normal from the center of the internal gear also passing through the center of the path of tooth contact *i.e.*, by the angle qSr. The maximum angle of approach is, consequently, the angle between the pinion radial to the point q of first possible tooth contact and the pinion radial to the instant axis m (angle x), and the maximum angle of recession is the angle between the pinion radial to the instant axis and the pinion radial to the last possible point of tooth contact r (angle y), the total angle of tooth contact being measured by the angle z, the sum of the angles of approach and recession.

In the combination illustrated in Fig. 99, tooth contact does not start quite at the beginning of the usable path of tooth contact, nor is it maintained to the end of the path, active contact beginning at the point at which the inner circle of the internal gear crosses the usable path of point of contact and ends at the point at which the outer circle of the pinion crosses the same path. Despite the failure to utilize the maximum usable section of the path of tooth contact, the actual arc of pinion contact is unusually long for the gearing combination shown. The arc of contact is, in fact, materially longer than that securable with gear and pinion teeth proportioned according to any other of the systems of internal gearing in common usage. To appreciate this important fact in full, a comparison of the paths of tooth contact for internal gearing combinations of similar ratios, but designed according to different systems of tooth form, is necessary.

PATHS OF POINT OF CONTACT

The two diagrams shown in Fig. 100 illustrate in a graphic and illuminating manner the paths of point of contact in the two principal standardized systems of internal gearing. In each instance, the direction of rotation is clockwise and the shaded

area between the pitch circles of the gears and pinions *A* and *B*, respectively, indicates areas within which no part of the path of tooth contact can lie, for in such areas interference between gear and pinion teeth would result.

In the first diagram are depicted the paths of point of contact for $14\frac{1}{2}$-deg. and 20-deg. teeth in the involute system of gears, a–b and a'–b', respectively. The paths are flat planes passing through the instant axis and inclined to the common tangent plane at the respective pressure angles. The commencement of tooth contact on the flank of the pinion, and therefore the limiting factor in the useful depth of the gear teeth face, is fixed

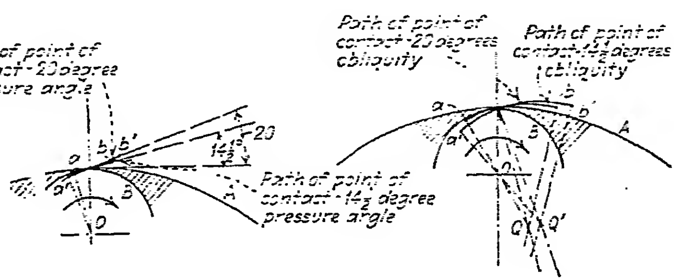


FIG. 100.—Comparison of paths of tooth contact in involute and Williams systems of internal gearing.

by the pinion radius normal to the approach section of the path of point of contact, while the limit to the recession section of the path of point of contact is—theoretically, at least—unlimited. However, there is a practical limit set to the length of the recession section of the path of point of contact by the length of the feasible pinion-tooth addendum, for, if the flank of the internal gear teeth is too long, the tops of the pinion teeth are cut away by the conjugate profiles of the mating gear.

The quite general adoption of 20-deg. involute teeth for internal gears has been brought about by an attempt to secure a longer arc of approach than can be secured with $14\frac{1}{2}$-deg. involute teeth. In spite of such practice, the angle of approach remains quite limited, and an effective angle of contact for involute gears is to be secured only through adopting the stub type of tooth with heavy-pressure angle for the internal gear and decreasing the addendum of the pinion teeth, upon the face of which the wear is more concentrated than it is upon the flank of the internal gear teeth.

The second of the diagrams illustrates the paths of point of contact for $14\frac{1}{2}$-deg. and 20-deg. obliquity teeth at the instant axis formed according to the Williams system, a–b and a'–b', respectively. The much greater approach section to the path of point of contact is very apparent and well typifies the reason for the improved operating action of the Williams gear. A Williams gear tooth of $14\frac{1}{2}$-deg. obliquity has a considerably longer arc of approach than has a 20-deg. involute tooth, and the usable section of the recession path of point of contact for the $14\frac{1}{2}$-deg. Williams tooth is also much longer than can be made use of for a 20-deg. involute tooth. In the case of 20-deg. Williams tooth, the increase in length of arc of action is as pronounced as in the case of involute teeth.

It will also be noted that the wear on the teeth is quite evidently much more uniformly distributed in the Williams system of gearing than it is in the involute and, furthermore, that such concentration of wear, as does occur, is toward the bottom of the flanks of both pinion and gear teeth, where the greatest amount of metal is and where, owing to the distinctive form of the teeth, wear can occur with less sacrifice of tooth strength than in any other form of tooth. Wear at such points will also have the tendency to ease off the shock of initial contact and so produce quieter and smoother operating gears.

STRENGTH OF PINION TEETH

In the matter of strength, the pinions (always the weaker of a pair of mating gears) proportioned according to the Williams system are considerably superior to those of equivalent involute form, and this peculiarity is clearly demonstrated by a comparison of the forms of the respective internal gear teeth. The teeth of the Williams pinion have curved profiles of exact conjugate form for the particular Williams internal gear with which it is to operate, while the accurate curves of the profiles of the involute pinion teeth are generated by the same or similar involute rack tool employed for the generation of the mating internal gear.

The tooth forms illustrated in Fig. 101 are accurate representations of: (*a*) involute internal gear teeth; (*b*) an accurately mating pinion tooth; (*c*) equivalent Williams internal gear teeth; (*d*) a mating Williams pinion tooth; and (*e*) the two forms of pinion teeth superimposed to emphasize the difference in their

respective tooth profiles. The curved profiles of the involute internal gear teeth, both above and below the pitch circle, it will be noted, are concave and thus curve away from the contact planes tangent to the tooth profiles on the pitch line. The straight profiles of the Williams internal gear teeth, on the other hand, lie in the contact planes of the gear teeth. The extra

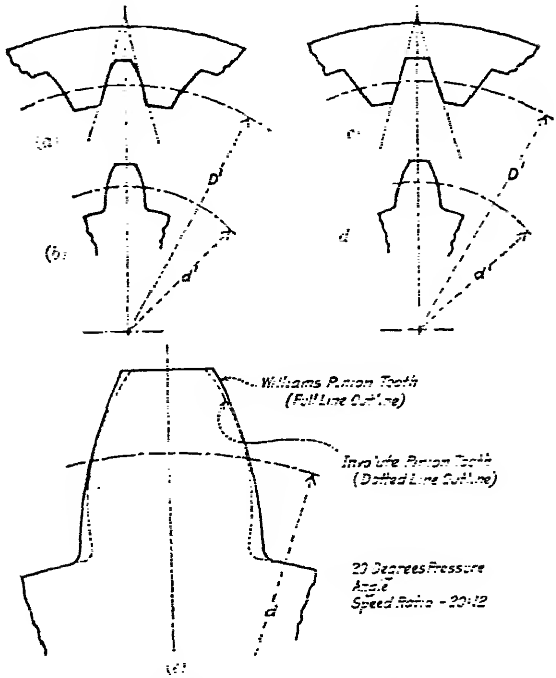


FIG. 101.—Comparison of internal gearing tooth forms.

root thickness of the involute internal gear tooth, though adding to its strength, results in a corresponding decrease in the thickness of the top of the mating pinion tooth, which though perhaps not weakening the tooth does tend to limit the depth of its addendum. The extra top thickness of the gear tooth, however, has a more serious effect, for it tends to weaken the pinion tooth by the necessary undercutting required for clearance. The composite diagram (e) forcibly illustrates the strength superiority of the Williams pinion tooth, for though the pinion teeth selected for comparison have been chosen as only slightly undercut—the involute tooth is a 20-deg. 80 per cent stub tooth of a 12-tooth

pinion—the difference in their root thicknesses is quite sufficient to indicate an appreciable difference in the strength of the two types of pinion teeth. The strength of the respective teeth is proportional to the square of their root thicknesses.

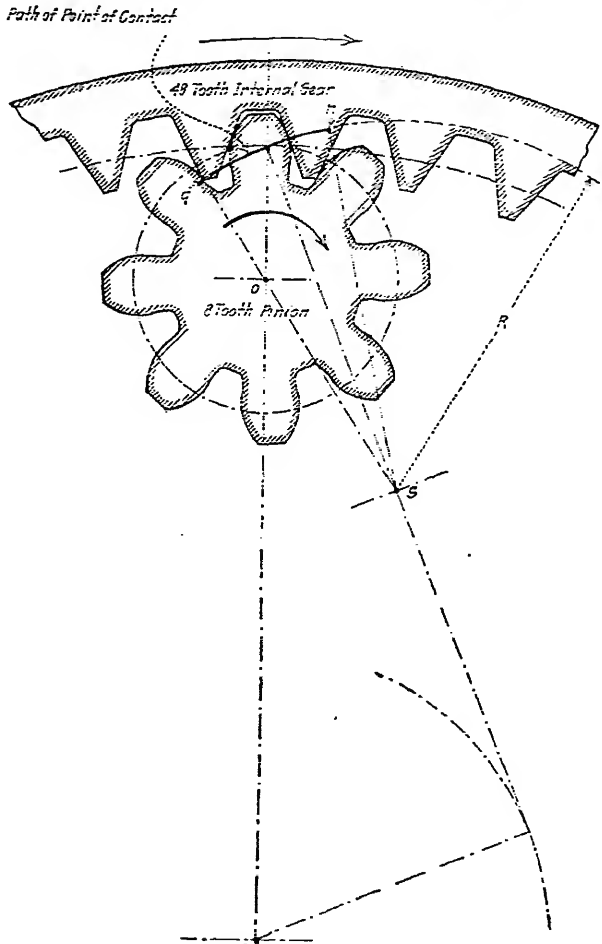


FIG. 102.—Williams internal gearing, 6 to 1 ratio.

The proportions of Williams internal gear teeth, furthermore, are such as to permit a modification in customary design whereby the strength of the gearing—measured by the strength of the pinion teeth—is materially increased by making the pinion

teeth as strong as the gear teeth. This is accomplished, as illustrated by the 8-tooth (6 to 1) combination shown in Fig. 102, by increasing the thickness of the pinion teeth and correspondingly decreasing the thickness of the gear teeth, so that the root thicknesses of the respective teeth are equal. The pitch of the gears is not altered, simply the thickness of the pinion teeth increased and the space between adjacent gear teeth widened by reducing the thickness of the gear teeth.

This method of securing a high-speed ratio with a pinion of few and unusually strong teeth cannot be resorted to as effectively in the case of gears proportioned according to the involute system. In gearing so proportioned, the undercutting of the pinion with few teeth, though they be amply thick on the pitch circle, would be so excessive as to necessitate unduly reducing the thickness of the gear teeth to equalize the strength of the pinion and gear teeth.

WEAR

The superiority of the Williams system of internal gearing in the matter of longer arc of contact, or action, is well demonstrated by a graphic comparison of similar gear combinations of the Williams and involute forms. Figure 103 diagrammatically illustrates the operating relation between the Williams internal gear and its mating pinion shown in Fig. 99, and also the operating relations for involute gears of the same ratio and proportions.

The section of the path of point of contact actually utilized in this combination of Williams gears is from the point *s* to the point *t*, or the section included between the intersections of the path of point of contact and the inner circle of the internal gear and the outer circle of the mating pinion respectively. The approach of the pinion teeth is measured by the angle *e* and their recession by the angle *f*, or the length of the pinion arc of contact, or action, is measured by the angle *g*. These angles, it will be noted, are materially less than the usable angles of approach and recession as limited by the available arc of contact, fixed by the extreme points *q* and *r* of the path of point of contact, yet they are considerably greater than the equivalent angles for an involute gear combination of the same speed ratio and pressure angle, *i.e.*, when the addendum and dedendum dimensions of the teeth are the same. The oblique line tangent to the Williams path of point of contact at the instant axis of the

gears, the line $s't'$, depicts the path of point of contact for such involute gear combination. Contact between opposing involute profiles cannot commence in advance of the point s', the intersection of the path of point of contact with its radial normal from the pinion center, and cannot continue beyond the point t', where the outer circle of the pinion crosses the path of point of contact.

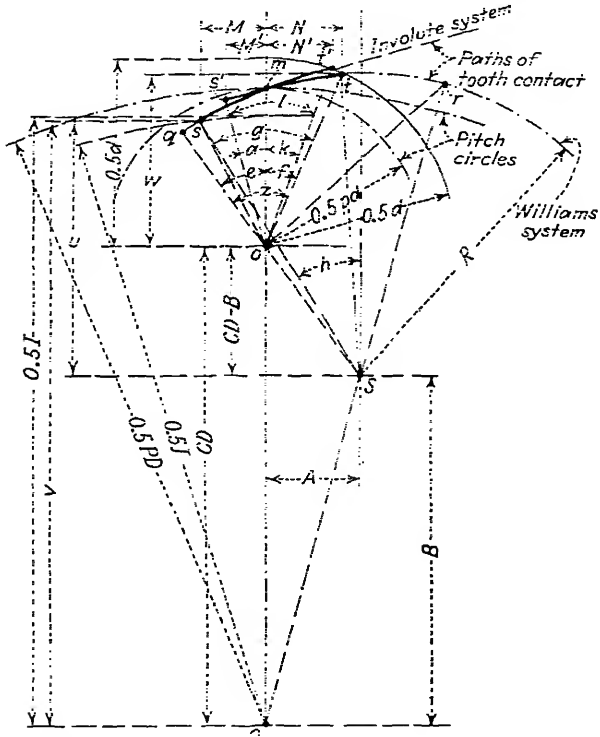


Fig. 163.—Comparison of involute and Williams arcs of contact.

In the involute gear combination, the angle of pinion approach, a, is equal to the pressure angle and is quite noticeably less than the angle of approach, e, for the Williams gear combination. The angle of recession, k, is also less than the corresponding angle for the Williams gear combination, so that the total pinion angle of contact, l, or action, is, in the involute gear combination, only about 85 per cent of its value in the case of the Williams gears. Furthermore, the delayed commencement of contact

in the involute combination limits the tooth-face area over which contact occurs to approximately 75 per cent of that so utilized in the more efficient construction.

With an arc of contact, or action, $17\frac{1}{2}$ per cent greater and 50 per cent more tooth-face area over which contact between the mating teeth takes place, the Williams gear combination has a wear-resisting capacity some 75 per cent greater than the similar involute gear combination, when transmitting the same power at the same speed of rotation—provided, of course, that the respective gears are constructed of materials of similar wear resistance. These comparative values are, naturally, applicable only to the particular gears under consideration and will vary to some extent for other gear combinations, but they are typical.

To secure a utilization of the full depth of the involute tooth profiles for contact, in a combination such as that depicted in Figs. 99 and 103, would necessitate increasing the pressure angle to something over 20 deg. (the obliquity of pressure at the instant axis for the gears as illustrated being $14\frac{1}{2}$ deg.) and though such modification would increase the angle of pinion approach to about that for the $14\frac{1}{2}$-deg. Williams gears, the angle of recession would be quite materially reduced, unless the flank of the gear teeth was considerably increased. Such modifications would be too excessive to be feasible, but if they could be made and were also adopted for the Williams gear combination, the superiority of the latter form of construction in the matter of angle of contact would still be marked, and if the flank of the pinion teeth was also increased (which could be profitably done in the Williams system, but not in the involute) the superiority of the Williams construction in lengthened arc of contact and also increased wear resistance would be substantially the same as in the gear combinations that have been discussed.

REDUCTION IN THE NUMBER OF PINION TEETH

The teeth of the pinion in a Williams combination are proportioned to operate with a particular gear, or, in other words, the pitch and obliquity of the internal gear teeth positively control the profile curvature of the mating pinion teeth. It is obvious, therefore, that pinions with fewer teeth than are feasible in the involute system of gearing can be employed, for if they can be generated at all, they will operate with their "master" gear with-

out interference of teeth. Gear combinations with pinions with a small number of teeth are both feasible and practical in the Williams system of internal gearing.

Figure 104 illustrates a combination of Williams gears—a 5 to 1 ratio—in which the pinion has only six teeth, yet they are

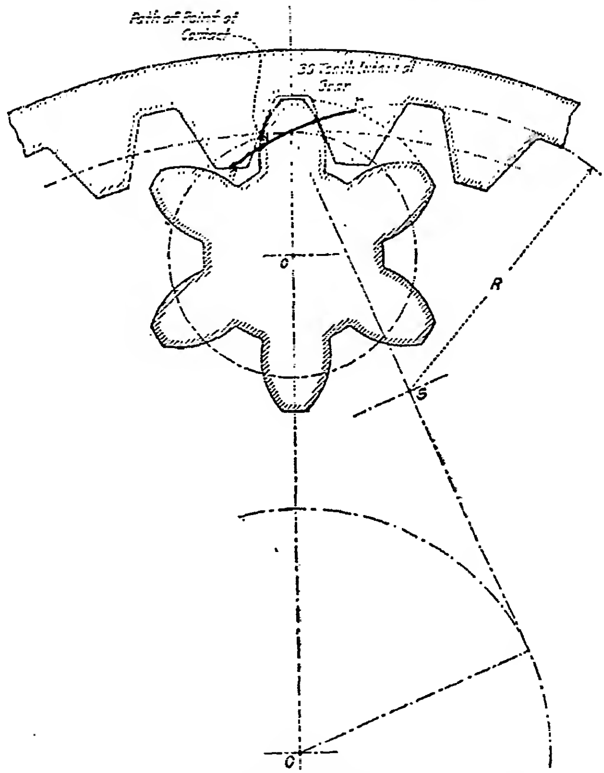


Fig. 104.—Williams internal gearing, 5 to 1 ratio.

well proportioned for strength and free from objectionable undercutting. The obliquity of the gear teeth is marked, it is true, but the reduction in the efficiency of power transmission is far less than it would be in the case of involute gears with a corresponding heavy-pressure angle, for during the recession period of the engaging teeth—the portion of tooth contact during which the transmission of power from the pinion to the gear is considerably more effective than during the approach to the position of so-called "full mesh"—the obliquity of the direction of pressure reduces steadily.

The gear combination shown in Fig. 102 illustrates a method of proportioning the pinion and gear teeth by which a pinion with only eight teeth was employed in a 6 to 1 reduction without sacrifice of strength or adopting a tooth of unusual obliquity.

This distinctive feature of ability to generate and use pinions with a small number of teeth is quite obviously an important advantage of the Williams system of internal gearing. Greater speed ratios are obtainable without unduly increasing the diameter of the internal gear, or a coarser pitch may be employed than is feasible with involute gears, when available space for the accommodation of the gearing is limited.

The ability to employ gears with fewer teeth of heavier pitch than can be used in the involute system (practice always to be recommended) is particularly advantageous in the Williams system on account of its superiority in length of arc of tooth contact and improved operating action.

MACHINING TEETH

The straight-tooth profiles of the Williams internal gear make the accurate cutting of such gear teeth a comparatively simple and inexpensive operation. A plain V-shaped, reciprocating cutting tool will gash out the tooth spaces rapidly and finish accurately both of the bounding tooth profiles, necessitating only the simple adjustment act of indexing the gear blank from tooth space to tooth space. Any ordinary shaper with a suitable indexing mechanism can be employed. If the internal gear is of the ring type, the tooth spaces can be milled with even greater rapidity, the cutter—as in the case of the simple reciprocating tool—being easily ground with the utmost precision as to the accuracy of the cutting edge and its obliquity.

The cutting of the pinion teeth entails the use of a simple generating machine in which the cutting tool, conforming in shape to the straight, flat, profile internal gear tooth, swings about the rotating pinion blank. As the pinion of the ordinary internal-gear combination has comparatively few teeth, incidentally, their generation is a simple task compared with the task of generating the teeth of the mating gear.

The cost of cutting Williams internal gears and gear combinations is hence, quite obviously, much less than for machining gears of other tooth forms and is more and more marked as the difference in the number of teeth in the internal gears and their

mating pinions increases. If the average speed ratio of internal gear combinations is taken as 4 to 1, the relative time consumed in cutting a gear combination proportioned according to the Williams system, compared to that required to generate similar gears proportioned according to the involute system, is substantially 50 to 60 per cent. In addition to this very material saving in the time required to machine the gear teeth, less skilled operators may be employed for the simple operations entailed in cutting Williams gears than can be safely trusted to operate the more intricate machines required for generating the involute type of tooth, so that the total cost of manufacturing Williams internal gears and mating pinions is substantially less than of generating similar gears designed according to the involute system of gearing.

DISTINCTIVE ADVANTAGES

The advantages of the internal type of gearing are now quite generally appreciated and conceded, and in every instance the superiority of the Williams system of internal gearing is conspicuous. Briefly summarized, the more evident of the advantages that are possessed to an accentuated degree by the gear combinations proportioned according to this system are:

1. Greater length of arc of tooth contact, due to the distinctive path of point of contact.
2. Reduction in wear, due to the prolonged contact and the greater proportion of the tooth-profile areas utilized for contact.
3. Increased strength of pinion teeth, on account of the reduction in undercutting.
4. Improved operating action, due to the slight relief in shock of tooth contact occasioned by the concentration of wear on the flanks of the pinion and internal-gear teeth.
5. Reduction in the required number of pinion teeth.
6. Reduction in the diameter of the internal gear for a given speed ratio and load.
7. Greater speed ratios without increase in diameter of internal gear.
8. Possibility of employing a coarser pitch without increasing the diameter of the internal gear.
9. Increased strength of gear combinations.
10. Simplicity of design.
11. Accuracy and ease of gear-tooth reproduction.
12. Greatly reduced cost of manufacture.

SECTION XI

EPICYCLIC-GEAR TRAINS

The inclusion in gear drives of both planetary and axial rotation of gear members, *i.e.*, epicyclic gearing, where one or more of the individual gears are carried on a revolving arm centered on the fixed axis of another of the gear members, broadens the field of gear applications considerably, the arrangement effecting marked changes in the speeds and powers of the driving and driven members of the assemblage. These epicyclic gear trains are used extensively in drives for machine tools, special types of machines, in speed reducers or gear units, and in the transmissions of many other mechanical devices.

Since the computations entailed in the design of this class of gearing are more complex than those for gearing that simply rotates on fixed axes, a clear understanding of the principles of epicyclic gearing is of importance. This knowledge can probably be gained most readily by tracing the development from a simple two-gear epicyclic train to some of the more complicated assemblages of compounded planetary gearing, including simple and compound gears of both external and internal varieties. An effective way of conducting such a study is to present a series of concrete examples featuring some of the more commonly employed arrangements, or types, of epicyclic trains.

As a help in differentiating between the directions of rotation and in establishing gear-ratio values characteristic of basic varieties of epicyclic, or planetary, gearing, rotations in a clockwise direction will be designated as positive (+), rotations to the left as negative (−), and the rotary speed of the driven member expressed in terms of the rotary speed of the driving member.

SIMPLE SPUR-GEAR EPICYCLIC DRIVE

The simplest form of epicyclic gear train consists of two externally meshing spur gears, one stationary, or fixed, and the other free to rotate and carried on a revolving arm which

is centered on the axis of the fixed gear. In such a combination, the arm is the driving member and the free gear the driven, the gear ratio being the number of revolutions made by the driven member during a single revolution of the driving member, which in the case under consideration is the revolving arm.

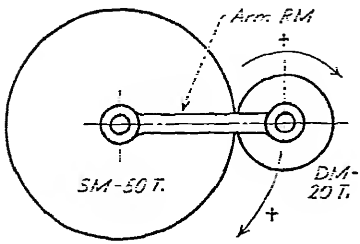


FIG. 105.—Diagram simple spur-gear epicyclic drive.

The driven gear in an assemblage of this kind is quite obviously subjected to both planetary and axial rotation, the former by virtue of being carried around the stationary gear by the driving arm and the latter by being in mesh with the stationary gear about which it rolls. The effect of these combined motions upon the gear ratio of the assemblage is dependent, naturally, upon the number of teeth in, or the diameter of, the respective gear members and can be best demonstrated by citing a typical example, as follows:

Let:

Stationary gear member (*SM*) have 50 teeth.
Driven gear member (*DM*) have 20 teeth.

Then:

Disregarding the fact that the two gears are in mesh, the swinging of the driven gear once around the stationary gear, in a clockwise direction, by the driving arm, has the effect of causing the driven gear to turn over completely once, just as if it had made a revolution on its own axis, also in a clockwise direction. While being carried around by the revolving arm, however, the engaging gear teeth on the stationary gear cause the driven gear actually to turn on its axis, the rotation being the same as would be effected by similar gears, both running on fixed axes, when the driving gear turned in a counterclockwise direction. Since the stationary gear has 50 teeth and the driven gear 20, the driven gear makes $2\frac{1}{2}$ revolutions, on account of its engagement with the stationary gear in the epicyclic train, during each revolution of the driving arm and does so while it is also being rotated once by the action of the revolving driving arm. Consequently, the gear ratio of the epicyclic train is 1 to

$3\frac{1}{2}$, the driven gear acquiring 1 revolution by virtue of its planetary motion and having $2\frac{1}{2}$ revolutions imparted to it, during each revolution of the driving arm, by virtue of its engagement with the stationary gear, in both cases the direction of rotation being positive. A general equation, covering similar assemblages of these simple spur-gear epicyclic drives with any number of teeth in their respective gear members, is

$$DMV = RMV\left(1 + \frac{SG}{DG}\right) \text{ and Gear ratio } (GR) = RMV : DMV$$

where DMV = driven-member velocity.
RMV = driving (revolving)-member velocity.
SG = number of teeth in stationary gear.
DG = number of teeth in driven gear.

SIMPLE SPUR-GEAR EPICYCLIC DRIVE WITH INTERMEDIATE GEAR

Introducing an intermediate idler gear in a simple spur-gear epicyclic drive reduces the gear ratio and often causes the

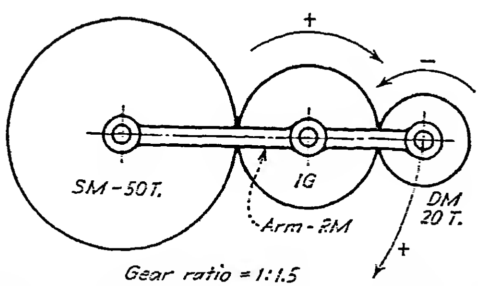


FIG. 106.—Diagram simple spur-gear epicyclic drive with intermediate gear.

directions of rotation of the driving and driven members to be opposed. The extra gear, whatever its size, has the effect of causing the rotation imparted to the driven-gear member by the stationary gear to be in the opposite direction to that of the driving-arm member, but without change in its amount. Consequently, while the turnover of the driven gear by virtue of its being carried around the stationary gear by the revolving arm is in the direction of the arm's rotation, the resultant direction of the driven-gear's combined rotations, planetary and axial, depends upon whether the stationary gear or the driven

gear has the greater number of teeth. The equation for the determination of this point is

$$DMV = RMV\left(1 - \frac{SG}{DG}\right)$$

Where the stationary-gear member has 50 teeth and the driven-gear member 20, as in the foregoing case of a simple spur-gear epicyclic drive, the revolutions of the driven gear would number 1½ for each revolution of the driving-arm member and be in the opposite direction:

$$DMV = RMV(1 - {}^{50}\!/_{20}) = 1.5RMV \quad (-) \quad \text{and} \quad GR = 1{:}1.5$$

On the other hand, if the stationary gear had the 20 teeth and the driven gear 50 teeth, the directions of actual rotation for both the driven gear and the driving arm would be the same and the gear ratio considerably less:

$$DMV = RMV(1 - {}^{20}\!/_{50}) = 0.6RMV \quad (+) \quad \text{and} \quad GR = 1{:}0.6$$

SIMPLE INTERNAL-GEAR EPICYCLIC DRIVE

If, instead of the driven gear being an external spur, it were made an internal gear, the effect would be much the same as inserting an intermediate gear in a simple spur-gear epicyclic drive. The rotation imparted to the driven gear by the stationary spur gear would be opposed to that of the driving arm, though, on account of the internal gear having of necessity more teeth than the stationary spur gear, the internal gear would actually turn in the same direction as the driving arm. For the same reason, the gear ratio would be a reduction.

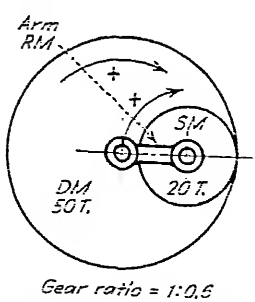


FIG. 107.—Diagram simple internal-gear epicyclic drive.

In this case, the equation for the determination of the driven-member velocity for a simple spur-gear epicyclic drive incorporating an intermediate gear would apply, and the driven-gear velocity and the gear ratio with a 50-tooth, internal-gear and 20-tooth, stationary spur-gear member combination would be

$$DMV = RMV\left(1 - \frac{SG}{DG}\right) = RMV\left(1 - \frac{20}{50}\right) = 0.6RMV \quad (+)$$

$$GR = 1{:}0.6$$

SIMPLE INTERNAL-GEAR EPICYCLIC DRIVE WITH INTERMEDIATE GEAR

The insertion of an intermediate spur gear between the stationary spur gear and the driven internal gear forms an epicyclic drive effecting speed changes similar to those with a simple spur-gear epicyclic drive. The interposition of the additional spur gear has the effect of causing the rotation imparted to the driven gear through its connection with the stationary gear to be in the same direction as the rotation of the driving-arm member, and the resultant velocity of the driven gear is, consequently, the sum of its planetary and axial revolutions. Therefore, the equation for the determination of the driven-member velocity in a simple spur-gear epicyclic drive is applicable and, in the case of a drive of this variety with a 50-tooth, driven internal gear and a 20-tooth, stationary gear, the relative rotary velocities of the driving and driven members would be

$$DMV = RMV\left(1 + \frac{SG}{DG}\right) = RMV\left(1 + \frac{20}{50}\right) = 1.4RMV \quad (+)$$

$$GR = 1:1.4$$

With this variety of epicyclic drive, the internal gear is frequently made the driving member and the revolving arm the driven member, simply reversing the drive, in which case the equation for the determination of the driven-member velocity takes the form:

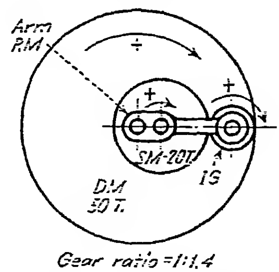


FIG. 108.—Diagram simple internal-gear epicyclic drive with intermediate gear.

$$DMV = RMV\left(\frac{IG}{IG + SG}\right)$$

where IG = number of teeth in driving (internal) gear.

If, with this variety of drive, the same gear proportions are used as in the preceding case, the resulting speed reduction, when the internal gear is the driving member and the revolving arm is the driven member—which entails no change in the direction of the resultant motion—and the gear ratio of the drive are:

$$DMV = RMV\left(\frac{50}{50 + 20}\right) = 0.714RMV \quad (+)$$

$$GR = 1:0.714$$

SUN AND PLANET GEARS

Internal-gear epicyclic drives with one or more intermediate gears are frequently and aptly termed *sun and planet* gears, though this descriptive designation should be reserved for assemblages where the center of the internal gear exactly coincides with the axis of the carrying arm. This more symmetrical

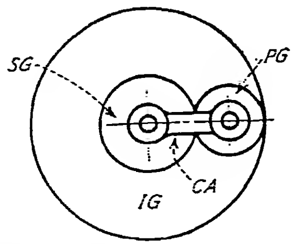


Table 30.—Simple Sun-and-planet Gear Velocities

Symbols for Member Units

Sun gear, *SG*. Planet gear, *PG*. Internal gear, *IG*. Carrying arm, *CA*

(Symbols also denote number of teeth in, or diameters of, gears)

Function Symbols

Stationary member, *SM*. Driving member, *RM*. Driven member, *DM*.

Carrying arm, *CA*

VSG = velocity, sun gear. VIG = velocity, internal gear.

VCA = velocity, carrying arm. VPG = velocity, planet gear.

SM	*RM*	*DM*	Rotary speed per revolution of driving member *VSG*	*VCA*	*VIG*	*VPG**
SG	*CA*	*IG*			$1 + \frac{SG}{IG}$	$\frac{SG}{PG}$
IG	*CA*	*SG*	$1 + \frac{IG}{SG}$	1	0	$\frac{IG}{PG}$
SG	*IG*	*CA*	0	$\frac{IG}{IG + SG}$	1	$\frac{IG}{IG + SG} \times \frac{SG}{PG}$
CA	*IG*	*SG*	$\frac{IG}{SG}$	0	1	$\frac{IG}{PG}$
IG	*SG*	*CA*	1	$\frac{SG}{IG + SG}$	0	$\frac{SG}{IG + SG} \times \frac{IG}{PG}$
CA	*SG*	*IG*		0	$\frac{SG}{IG}$	$\frac{SG}{PG}$
PG	*CA*	*SG* & *IG*				

* Revolutions of planet gear about its own center.

arrangement possesses a number of quite obvious advantages. It is generally a cheaper construction, and usually feasible, since the size of the intermediate gears, usually termed *planet gears*, is a condition that affects only their own individual speed of rotation about their own centers and does not affect in any way the rotary speeds of the other gear members.

With the balanced construction, any one of the gear members, as well as the carrying arm, may serve as either driving, stationary, or driven member, and there are seven different combinations which prove useful. These are listed for simple sun and planet gears in Table 30, where the relative rotary speed of each member is given in terms of a revolution of the driving member for each arrangement of drive. The symbols used in the table, to designate the various member units, are also made use of to denote the number of teeth in, or the diameters of, the respective gear units. That is, in computing the values of the listed ratios, the number of teeth in, or the diameters of, the designated gears should be substituted for the symbols.

COMPOUND SPUR-GEAR EPICYCLIC DRIVE

The employment of compounded gears as members of an epicyclic gear train serves to increase the gear ratio of the

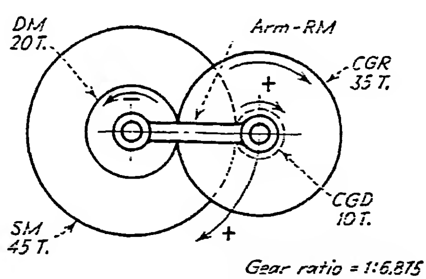


FIG. 109.—Diagram compound spur-gear epicyclic drive.

assemblage, and this construction is shown in its simplest form in Fig. 109 where a compound gear meshes with a stationary gear and with a driven, free-running gear. The stationary and driven gears have a common axis about which the arm carrying the compound gear is also centered.

Disregarding the planetary feature of the assemblage, the speed ratio of such a gear train, all gears being free to revolve (the compound gear as a unit), is such that the velocity of the

last, or driven, gear bears the same relationship to the velocity of the first, or driver, gear as the product of the number of teeth in, or the diameters of, the driver gear and the driver section of the compound gear does to the product of the number of teeth in, or the diameters of, the driven gear and the driven section of the compound gear. Introducing the planetary rotation by fixing the first gear and making the carrying arm the driving member simply adds one more revolution to the rotary velocity of the driven gear.

In such form of planetary drive, the rotation of the compound gear, both sections, is in the same direction as that of the driving arm, and the rotation of the driven gear, consequently, is in the opposite direction. Therefore, the equation for determining the rotary velocity of the driven gear, in terms of the rotary velocity of the driving arm, and the gear ratio of the drive are

$$DMV = RMV\left(1 - \frac{SM \times CGR}{DM \times CGD}\right) \quad \text{and} \quad GR = RMV : DMV$$

where SM = stationary member, number of teeth or pitch diameter.
DM = driven member, number of teeth or pitch diameter.
CGR = compound gear, driver section, number of teeth or pitch diameter.
CGD = compound gear, driven section, number of teeth or pitch diameter.

With 45 teeth in the stationary gear, 20 in the driven gear, and 35 and 10, respectively, in the sections of the compound gear, the rotary speed of the driven gear is $6\frac{7}{8}$ times as great as the speed at which the driving arm revolves and in the opposite direction. The reversal in direction of rotation is due to the fact that the rotary velocity imparted to the driven gear by the compound gear is greater than the rotary speed of the driving arm:

$$DMV = RMV\left(1 \quad \frac{45 \times 35}{20 \times 10}\right) = RMV(1 - 7.875) = 6.875RMV \quad (-)$$

$$GR = 1:6.875$$

COMPOUND SUN AND PLANET GEARS

The substitution of an internal gear for the larger of the spur gears in a compound epicyclic drive causes all the rotating mem-

bers to revolve in the same direction, so that the resultant velocity of the driven internal gear, when the carrying arm is the driver, is the sum of its planetary rotation and the rotary velocity imparted to it by virtue of its engagement with the compound gear. The equation for determining the velocity of the driven member then becomes

$$DMV = RMV\left(1 + \frac{SM \times CGR}{DM \times CGD}\right)$$

A compound epicyclic drive consisting of a 45-tooth internal gear, a 20-tooth stationary, central, spur gear, and a compound gear with 10 and 15 teeth in its respective sections would have a gear ratio of 1 to $1^2\!/_3$. With the arm driving, obviously, the attainable gear ratio in such a drive is limited to the range of between 1 to 1 and 1 to 2, whatever the proportions of the gear members employed.

$$DMV = RMV\ 1 + \frac{20 \times 15}{45 \times 10} = 1.67RMV \quad (+)$$

$$GR = 1{:}1.67$$

This construction, shown in Fig. 110, is of the sun-and-planet gear variety, however, and any one of the gear members can

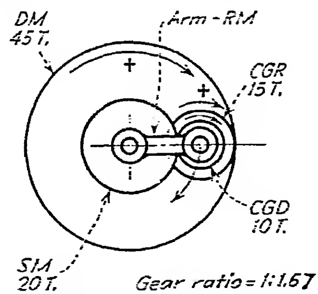


FIG. 110.—Diagram compound sun and planet gear.

be made the driving member and any one the stationary member, so making it possible to secure a much wider range of gear ratios, with the directions of rotation of the driving and driven members always the same. The relationships existing for any of the possible arrangements and the relative rotary speeds for the individual gear members in each arrangement are listed in Table 31, where the symbols employed not only identify the various member units but denote as well the number of teeth in, or the diameters of, the respective gears.

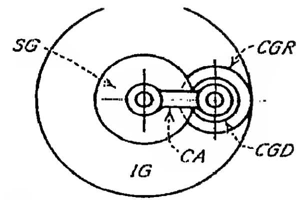


TABLE 31.—COMPOUND SUN-AND-PLANET GEAR VELOCITIES

Symbols for Member Units

Sun gear, *SG*. Internal gear, *IG*. Carrying arm, *CA*.

Compound planet gear $\begin{cases}\text{Driver, } CGR.\\ \text{Driven, } CGD.\end{cases}$

(Symbols also denote number of teeth in, or diameters of, gears)

Function Symbols

Stationary member, *SM*. Driving member, *RM*. Driven member, *DM*. Carrying arm, *CA*.

VSG = velocity, sun gear. *VIG* = velocity, internal gear

VCA = velocity, carrying arm. *VCPG* = velocity, compound planet gear.

SM	*RM*	*DM*	Rotary speed per revolution of driving member			
			VSG	*VCA*	*VIG*	*VCPG**
SG	*CA*	*IG*	0	1	$\frac{CGR \times SG + CGD \times IG}{CGD \times IG}$	$\frac{SG}{CGD}$
IG	*CA*	*SG*	$\frac{CGR \times SG + CGD \times IG}{CGR \times SG}$	1	0	$\frac{IG}{CGR}$
SG	*IG*	*CA*	0	$\frac{CGD \times IG}{CGR \times SG + CGD \times IG}$	1	$\frac{CGD \times IG}{CGR \times SG + CGD \times IG} \times \frac{SG}{CGD}$
CA	*IG*	*SG*	$\frac{IG \times CGD}{SG \times CGR}$	0	1	$\frac{IG}{CGR}$
IG	*SG*	*CA*	1	$\frac{CGR \times SG}{CGR \times SG + CGD \times IG}$	0	$\frac{CGR \times SG}{CGR \times SG + CGD \times IG} \times \frac{IG}{CGR}$
CA	*SG*	*IG*	1	0	$\frac{SG \times CGR}{IG \times CGD}$	$\frac{SG}{CGD}$
CGR & *CGD*	*CA*	*SG* & *IG*	1	1	1	0

COMPOUND EPICYCLIC DRIVES WITH TWO INTERNAL GEARS

A form of compound epicyclic drive that retains the symmetrical arrangement of the sun-and-planet gear arrangement but substitutes for the central sun gear a second internal gear is shown in Fig. 111. This arrangement effects both a reversal in direction of rotation and a reduction in rotary speed, when the carrying arm is the driving member. Under these circumstances, the equation for ascertaining the relative rotary speeds of the driving and driven members are

$$DMV = RMV\left(1 - \frac{SM \times CGR}{DM \times CGD}\right)$$

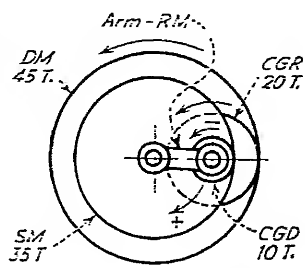


FIG. 111.—Diagram compound epicyclic drive with two internal gears.

With 35- and 45-tooth internal gears, the smaller one being the stationary member of the assemblage and the carrying arm the driving member, and with a compound gear with 10 and 20 teeth in its respective sections, the gear ratio of such an epicyclic drive is a matter of 1 to 0.556. In this construction, should a larger stationary internal gear be employed, the driven internal gear remaining the same size, a somewhat greater reduction in the velocity of the driven gear would be effected:

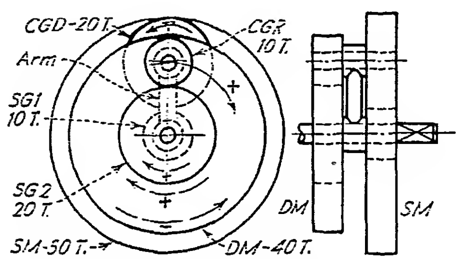
$$DMV = RMV\Big(1 \quad \frac{35 \times 20}{45 \times 10} = 0.556RMV \quad (-)$$

$$GR = 1{:}0.556$$

The addition of two central driving sun gears in an epicyclic drive of this character with its two internal gears, one of which is stationary, exemplifies not only a further modification of interest, *i.e.*, a more complex sun and planet gear, but is well indicative of the type of computations often involved in proportioning this class of gearing. The arrangement is shown in Fig. 112, where the symbols both identify the member gears and denote the number of teeth in, or the diameters of, the respective gear members. While the equation for ascertaining the relative speed of the driven internal gear in terms of the speed of the driving sun gears in this arrangement is not in

itself especially complicated, it might be well to trace its development step by step.

The rotary speeds of the central driving sun gears *SG*1 and *SG*2 are naturally the same, represented by the function symbol *RMV*, and may be taken as being in a positive direction. The rotary speeds of both sections of the compound planet gear, *CGR* and *CGD*, are likewise the same but in a negative direction. Consequently, when the driving gear *SG*1 of the first stage makes one revolution in a clockwise direction, the arm carrying the compound planet gear makes $\frac{SG1}{SG1 + SM}$ revolution in the same direction.



Gear ratio = 1:0.25

FIG. 112.—Diagram sun and planet gear with two internal gears.

In the second stage, if the arm did not move while the driving gear *SG*2, which rotates with *SG*1, makes one revolution in a positive direction, the driven internal gear *DM* would turn in a negative direction by that part of a revolution measured by the ratio *SG*2/*DM*. Also, if the sun gears did not rotate on their centers, the driven internal gear in the second stage would make $1 + \frac{SG2}{DM}$ revolutions in a positive direction for each revolution of the carrying arm. Consequently, during $\frac{SG1}{SG1 + SM}$ revolution of the arm, *i.e.*, the part of a revolution made by the arm while the driving sun gears make one revolution, the driven internal gear would make $\left(1 + \frac{SG2}{DM}\right) \times \left(\frac{SG1}{SG1 + SM}\right)$ revolutions.

The resultant rotary speed of the driven internal gear *DM* is then the sum of this rotation in a positive direction and its

rotation in a negative direction caused by the counterclockwise rotation of the compound planet gear. The final direction of rotation of the driven gear depends consequently upon whether the product of the number of teeth in, or the diameters of, the *SG*1 and *DM* is larger or smaller than the similar product for the gears *SG*2 and *SM*:

$$DMV = RMV\left[\left(1 + \frac{SG2}{DM}\right) \times \frac{SG1}{SG1 + SM} + \left(-\frac{SG2}{DM}\right)\right]$$

$$= RMV\left(\frac{(DM + SG2)SG1}{DM(SG1 + SM)} - \frac{SG2}{DM}\right)$$

$$= RMV\left(\frac{SG2 \times SG1 + SG1 \times DM - SG2 \times SG1 - SG2 \times SM}{DM(SG1 + SM)}\right)$$

That is:

$$DMV = RMV\left(\frac{SG1 \times DM - SG2 \times SM}{DM(SG1 + SM)}\right)$$

With the various gear members proportioned as indicated in Fig. 112, the gear ratio for this somewhat complex epicyclic drive works out as 1 to 0.25, the final direction of the driven internal gear *DM* being counterclockwise, or negative.

$$DMV = RMV\left(\frac{10 \times 40 - 20 \times 50}{40(10 + 50)}\right) = 0.25RMV \quad (-)$$

$$GR = 1{:}0.25$$

SECTION XII

GEAR UNITS

Gear units—speed reducers as they are usually termed in the smaller sizes—constitute what can appropriately be looked upon as the final proving ground of gearing. In these compact, totally enclosed assemblages, the gear trains, running in a bath of oil, operate under the most favorable conditions and develop their highest transmission efficiency. The units conserve power and space, require little attention, run with a minimum of noise and vibration, possess inherently long life, marked durability, and unexcelled safety features; making them unquestionably the most economical of all forms of gearing. In fact, they have made possible the wide use of quality gearing in commercial and industrial applications under all conditions of service and have made feasible the adoption of the modern efficient, high-speed power units which have contributed greatly to the building-up of present-day low-cost manufacturing methods and practices.

Any approved system of toothed gearing, spur, straight, or helical; worms and worm gears; and bevel gears, standard and spiral, can be used for the gear members in these compact gear trains, with driver and driven shafts in line, parallel, in the same or different planes, with the axes of these major shafts intersecting or not. In short, every type of gear and virtually all the many arrangements of gearing are to be found in one or another form of gear unit, the gear members mounted within a confined space under conditions which make the most of the individual merits of the gears and permit provisions for combating and circumventing any inherent drawbacks the gears may possess.

While the materials employed, the design of the gearing, and the skill in workmanship entailed in the production and assembly of the gear members are considerations of prime importance, other essential details in the success of these generally efficient gear units have to do with the method of mounting the gears, equalization of bearing pressures, the symmetrical arrangement

of all rotating parts, and the balance so far as possible of all developed strains and stresses. With these details the gear designer is concerned and, while they differ considerably with the gear-unit arrangement, there is a certain similarity in requirements for specific classes and types of gear units.

There are two general divisions of gear units: those with their driver and driven shafts in line, or, when there is an angular variation in the main-shaft axes, in the same plane; and those with their shafts parallel, or, when there is an angular variation in the main-shaft axes, in different planes. Both these divisions include gear units of radically different types and design, and both are used extensively in machine drives.

GEAR UNITS WITH SHAFTS IN LINE

Gear units with the axes of their driver- and driven-gear members in line may have all their rotating parts arranged

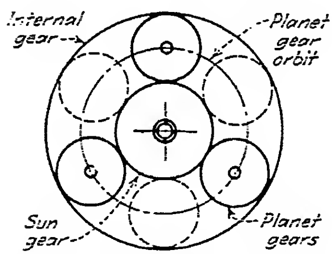


FIG. 113.—Diagram of planetary gear unit.

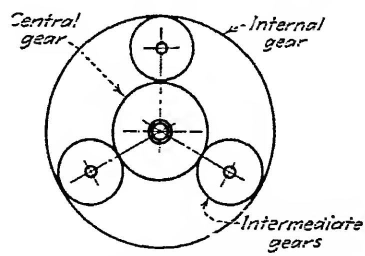


FIG. 114.—Diagram of nonplanetary speed reducer.

concentrically and revolving about the center line of the gear assemblage, or the intermediate gears may be mounted upon individual studs arranged around and equidistant from the major axis of the unit, this division including planetary, nonplanetary, and plain spur varieties of gear units, effecting speed changes in one or several stages. As these varieties of speed reducers are all in common use, and each is well adapted to certain conditions of service, interest centers not so much upon the respective efficiencies of the units as it does upon how to secure the best results with the various constructions, *i.e.*, upon the balancing of bearing pressures and the avoidance, so far as possible, of friction-load losses.

In the planetary construction full advantage is taken of the momentum of the revolving parts, and bearing pressures are

not only effectively equalized but are reduced to a minimum. With accurately proportioned and well-balanced gear members and all gear axes in precise alignment, there is little or no bearing pressure on the individual journals in this variety of speed reducer. This means there is a corresponding reduction in the journal friction and consequently less wear. On the other hand, there is a multiplicity of points of gear-tooth engagement at all of which there is a certain amount of unavoidable sliding action between engaging teeth and consequent friction loss and tooth wear, no matter what system of gear-tooth proportions and curvatures of tooth profile is employed. However, these unavoidable friction losses with well-made gears should not exceed at most 1 per cent of the power transmitted at each point of tooth engagement. Unfortunately, they are accumulative.

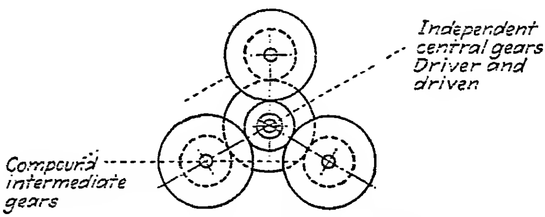


Fig. 115.—Diagram of plain spur-gear unit.

In the nonplanetary units with a symmetrical arrangement of several intermediate gears engaging both a central and an encompassing internal gear member, full advantage cannot be taken of the accumulative momentum of the rotating parts in the equalization of the bearing pressures, either upon the main-driving and driven-gear shaft journals or on the fixed studs upon which the intermediate gears revolve. A good distribution of the bearing pressure on the driving shaft is secured by virtue of the several points of tooth engagement between the central gear and the intermediate gears, however, and the bearing pressure on the internal-gear shaft is likewise well distributed but that on the fixed studs carrying the intermediate gears is concentrated on the sides of the studs, on the circumference of the circle of stud centers. As in the planetary construction, there is also the inescapable tooth-friction loss at each of the several points of tooth engagement between the concentric driving- and driven-gear members and the several intermediate gears, but

the driving torque in the nonplanetary construction is somewhat more effectively applied, and there is less power loss.

A distribution of the bearing pressure on the driver- and driven-gear-member shafts is effected in the plain spur variety of gear unit by the three-point engagement of the central gears with the compound intermediate gears, but, in the case of the pressure on the fixed studs supporting the compound intermediate gears in this construction, the situation is the same as that which pertains when a pair of ordinary spur gears mounted on fixed centers engage. The unit, incidentally, is a compound type of speed reducer effecting two speed changes in a single stage, that between the driver gear and one of the sections on the intermediate compound gears and another between the second section of the compound gears and the driven-gear member.

GEAR UNITS WITH PARALLEL SHAFTS

The equalization of bearing pressures, so essential for the efficient operation of these compact gear assemblages, is more of a question of design in gear units with parallel shafts, for not only do the several functioning shafts have to be accurately aligned, but the localized engagement of the various gear members over confined and limited sections of the meshing teeth does not afford the same opportunities for a distribution of the bearing load as does the multiplicity of points of tooth-engagement characteristic of gear units with their rotating parts revolving about one major axis. The precise alignment of the various gear shafts in the housing of the gear unit is, consequently, of especial importance and this alignment should be such as to assure parallelism of all the functioning shafts, not only when the unit is at rest or running light, but when it is fully loaded and subjected to the most unfavorable operating conditions.

The bearings supporting the shafts cannot, under any circumstances, fit the shafts exactly, a certain amount of running clearance being essential, and this fact must be taken into account when considering the possibilities of misalignment of the sensitively formed gear teeth. The running clearance can only increase and progressively hamper the proper engagement of the gear teeth, should any misalignment develop. Consequently, the unavoidable bearing wear must be kept axially as uniform as possible, and all developed strains and stresses that

are not in the planes of rotation must be balanced with all attainable precision.

Exterior influences, as well as those within the confines of the gear-unit housing, that in any way tend to promote misalignment should be carefully guarded against, for which reason the best gear-unit drives are usually those which are flexibly coupled at both driving and driven ends, thus minimizing the ill effects of any misalignment in the connected shafts. Every effort should be made to make the gear units truly self-contained and well balanced within their housings.

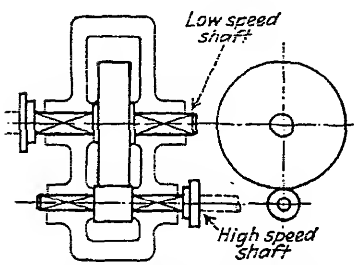


FIG. 116.—Arrangement of gears in simple single-reduction gear unit.

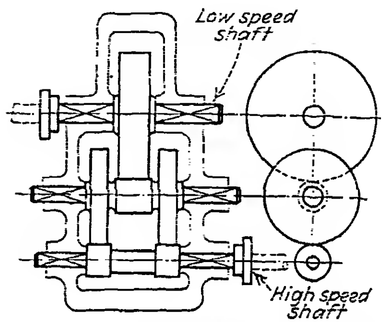


FIG. 117.—Arrangement of gear members in simple double-reduction gear unit.

The problem, aside from the difficulties of mounting even two shafts in exact parallelism, is not especially complicated in gear units of single reduction but becomes progressively more complex as the stages of speed change increase. A symmetry and balance of all functioning parts within the gear-unit housing is the chief requirement.

When two or more changes in speed are involved, necessitating the employment of several pairs of engaging gear members, the individual pairs of mating gears should not be placed side by side, but each alternate train should consist of duplicate pairs of gears, one pair flanking each side of the pair of gears consummating the preceding speed change, as shown in the diagrams depicting two- and three-stage gear units (Figs. 117 and 118). Not only should this straddle arrangement be employed, but the supporting bearings should be symmetrically located in respect to the normal center line of the gear unit, the shafts made adequately stiff for their unsupported lengths, and every

precaution taken to guard against any shaft deflection being caused by the transmitted tooth load.

It is this need of avoiding all preventable shaft deflection that is responsible for and explains the occasional, especially high efficiency of well-proportioned gear units employing standard spur gearing with the ordinary axial arrangement of gear teeth, and of the less pronounced efficiency of similar gear units incorporating gear members with a helical arrangement of gear teeth, even when a fairly symmetrical location of the functioning gearing exists. If the arrangement of the gear members is

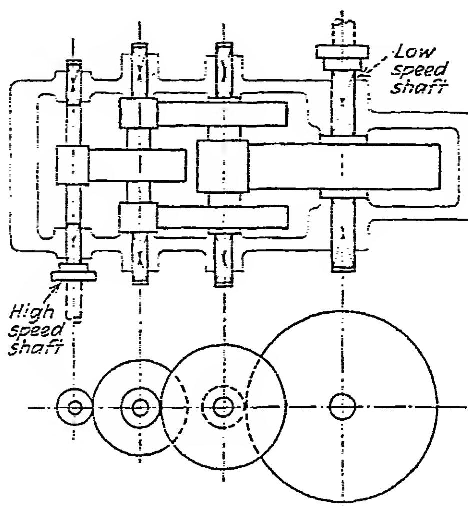


FIG. 118.—Arrangement of balanced gearing in triple-reduction gear unit.

not symmetrical, the situation is naturally more serious, for the resulting misalignment is just so much more harmful. The slight misalignment made possible by the essential running clearance of the functioning shafts in their bearings, trivial though it may be at first, grows progressively more pronounced, on account of the increasing concentration of the transmitted load toward the ends of the helical gear teeth, with the inevitable result that disastrous tooth wear rapidly develops, materially shortening the useful life of the gear unit and sacrificing transmission efficiency.

Considering the sensitiveness of the accurately formed gear-tooth profile to any slight misalignment and the effect of the

misalignment upon the smooth operation of the gears, the important advantages of the progressive engagement of gear teeth secured by their helical arrangement are generally realized best in gear units intended for high-speed service by employing gearing of the herringbone type. With accurately proportioned gears of this variety, the axial thrusts developed by the opposing obliquities of the gear teeth are absorbed by the gears and balance one another. As a result, there are no eccentric stresses tending to localize pressures between meshing teeth.

An exception to this general rule is in the case of large gear units employed for heavy service, *i.e.*, for handling large amounts of power usually at comparatively low speeds of rotation, where the helical gearing successfully employed is of necessity of considerable face width and the helix angle of the gear teeth, consequently, relatively small. Gear units of this character discount the possibilities of any gear misalignment by their very massiveness and proportions; so much so that excellent results are secured with duplicate helical gear members in single-stage units, with teeth of opposing obliquity mounted on the same shaft, preferably with supporting bearings between, as well as flanking the individual pinion members on the high-speed shaft.

WORM-GEAR SPEED REDUCERS

The high transmission efficiency of modern worm-gear-type speed reducers is a result of proper gear design, as well as of the use of suitable gear materials and skillful production. Shaft alignment, usually at right angles, is a consideration of much importance and the helix angle of the worms and gears should be in the neighborhood of 45 deg., to secure a relatively high peripheral worm-gear speed with as limited an amount of sliding action between the engaging worm thread and gear teeth as feasible, as well as for the attainment of high transmission efficiency. The problem, in worm-gear design, is one of frictional heat distribution and control, as well as of the distribution and balance of bearing pressures and of the accurate alignment of worm- and gear-contact surfaces. The qualities and characteristics of the worm and gear materials and the finish of the worm threads and gear-tooth profiles, consequently, command especial consideration.

Chilled cast bronze of a fine dense structure for the rim of the worm gear and a high-grade alloy steel suitably heat treated and hardened, or a low-carbon steel case-hardened, form a combination of gear materials found by experience to be well suited for the arduous and exacting duty placed on the contacting thread and gear-tooth elements. The worms, after hardening, should be carefully ground and polished to insure intimate and uninterrupted tooth contact and be of as small diameter as a heavy, rigid shaft construction permits, the latter to limit the peripheral speed of the worm as much as possible.

Careful consideration should also be given to the bearing constructions of worm and worm-gear members, both requiring efficient thrust as well as radial bearings, the particular variety depending to some extent upon the load and conditions of service. Adequate lubrication is also of the utmost importance in worm-gear-type speed reducers, for the oil supply functions as a cooling as well as a lubricating medium, absorbing and dispersing the frictional heat developed by the sliding engagement of the worm thread and gear teeth.

COMBINATION GEAR UNITS AND ANGLE DRIVES

These several basic varieties of gear units, in single- or multiple-reduction types, provide in themselves any desired gear ratio over quite an extended range and by combinations of the various types the range can be extended to include exceedingly high ratios, as 100,000 to 1 and even higher. The combinations may simply connect several independent units in series or the combination may be effected within the confines of a single housing. The driving unit, usually an electric motor, may also be built into, *i.e.*, incorporated with, the gear unit, but ordinarily the gear unit is an independent, self-contained mechanism interposed between the power unit and the driven mechanisms.

As a protection against eccentric stress being introduced from without the unit, tending to disturb the precise alignment of the gear-unit members that is so essential for the satisfactory operation of the train of gears, the connections to both the high- and low-speed shafts should be effected through suitable forms of flexible couplings. This practice also affords some protection against the jarring shocks of suddenly applied and sharply fluctuating loads.

When there is an angular variation in the main-shaft axes of the unit, as in an angle drive, a condition which also has a tendency to disturb the fine balance and alignment of the important functioning members of the assemblage, the change in axial direction should be made at the low-speed shaft and not at the high-speed shaft or at any intermediate point. By effecting the change in this manner, at the low-speed end, there is a minimum sacrifice in efficiency of transmission and the alteration in rotational speed is consummated under the most favorable conditions.

GEAR-RATIO LIMITATIONS OVERCOME

While any desired, or particular, gear ratio can be secured with any one, or with a combination, of these varieties of gear units, the gear ratio is necessarily fixed, except in certain types of planetary units, where a limited number of different gear ratios can be secured by making different sections of the assemblages the stationary member. Where a definite gear ratio is, or can be made, a permanent requirement, this limitation is no particular drawback, but when some change in the rotary speed of the driven equipment is required from time to time or when conditions are apt to change, making an alteration in gear ratio necessary or advisable, it may be felt that the installation of a quality gear unit, entailing a somewhat higher investment than some other more flexible but less efficient form of gearing, is not justified.

This, ordinarily, is a mistake, for quite often it is possible to interpose a simple train of efficient gears between the driven shaft of the gear unit and the equipment it drives and thus secure a different gear ratio with little loss in transmission efficiency. If several changes in gear ratio are required, provisions for a rapid substitution of the intermediate gears can frequently be made by mounting the gear unit and its driving motor on a suitable form of swinging cradle. In such a case, it is advisable to have the fixed gear ratio of the unit a mean between the required minimum and maximum values.

SECTION XIII

METHODS OF GEAR PRODUCTION

In commercial practice, the production of gears is now almost entirely consummated by a process of generation, in which the machining of the gear teeth is effected by causing the cutting tool and the gear blank to roll together as the tool advances and cuts the tooth spaces, and the gear teeth take form, in a manner closely resembling the engagement of a pair of mating gears. That is, the tool cuts the teeth while it, at the same time, follows the continuous rotation of the gear blank. This method is speedier and produces more accurately proportioned gears than the former usual method of milling the individual tooth spaces with formed cutters, which is now employed only for the production of individual gears that are to be run at relatively low pitch-line velocities. Gear production by generation, being machine controlled, is not only a speedier method but a cheaper one, and the gears that are produced are of better quality and much more uniform. In fact, the quality gearing required for high-speed operation can be produced by no other of the usual machining methods.

The practical application of generation in gear production is effected in a number of different ways, though all are founded on the same basic principle, and these are, in the order of their general employment, by hobbing, planing, grinding, and molding operations. The last, while still in more or less of an experimental stage, is the one that most faithfully applies the principle of generation, but as it entails more of a departure from the involute system of gearing than has as yet been deemed expedient, and much that is foreign to gearing in general, it will be taken up as an independent subject.

HOBBING GEAR TEETH

The process of hobbing gear teeth consists of revolving and advancing a worm-shaped cutter through the revolving gear blank, the respective rotary speeds of the cutter, or hob, and of the gear blank being established by the number of threads

on the hob and number of teeth to be cut in the gear blank For example, if a single-threaded hob is used to cut a 30-tooth gear, the hob rotates 30 times and is fed across the face of the blank once while the gear blank makes a single revolution.

In the performance of this generating operation, the hob is mounted with its arbor set at such an angle in respect to the axis of the gear blank that the helix of the hob is tangent to the sides of the gear teeth it generates as it passes across the face of the gear blank, the walls of the path it cuts as it advances, or, if the gear blank is fed across the hob, as the gear blank advances, being the sides of these teeth. Hence, the normal profile of the generated tooth is that of a conjugate gear of a basic rack having teeth conforming in cross section to the normal projection of the cutting edges of the hob.

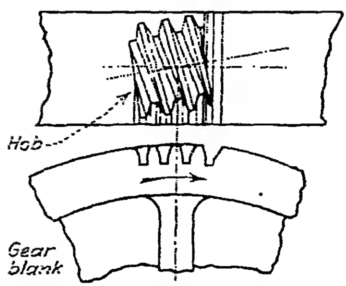


FIG. 119.—Hobbing spur gear.

In the involute system of gearing, the normal projection of the cutting edges of the generating hob is similar in outline to a cross section of the basic involute rack tooth with its inclined, straight-line profiles. Consequently, the teeth of any gear generated by a hob having such straight and inclined cutting edges are conjugate teeth of an involute rack and all gears thus generated will engage and mate with this particular rack.

In this process of hobbing gear teeth, the quality of the generated gear and the accuracy of its tooth profiles depend, naturally, upon the quality and accuracy of the tools employed, as well as upon the skill of the machinist in setting up the work. Both the accuracy of the hobbing machine and the accuracy of the cutting hob play important parts, for any lack of precision in concentricity and alignment of machine parts, errors in timing or the least disturbance to the essentially sensitive synchronism required between rotating speeds and rates of feed, insufficient rigidity in the support of the gear blanks and vibrations of any kind, are detrimental to securing a satisfactory gear-tooth profile.

Hobbing machines with vertical work spindles, with the hob traveling, are favored for the production of large gears, as the machine construction usually affords better opportunities of firmly clamping the gear blank in place and there is somewhat

less tendency of the setting being disturbed under the heavy cutting stresses. For small work, either a vertical or a horizontal work arbor machine is satisfactory and frequently a number of small gears, mounted on the same arbor, can be cut in multiple. The passage across the face of the gear blank can be effected with horizontal arbor machines, either by having the hob travel or the gear blank, both methods being successfully employed.

The accurate setting of the generating hob is also a detail of much concern, as the hob is usually set at an angle to the gear blank that corresponds to the helix angle of the hob at its pitch line. If the axis of the hob were set at right angles to the axis of the gear blank, an axial section of its thread would correspond to the normal outline of the basic rack tooth. Therefore, when the hob is obliquely mounted, the angle of the hob tooth and the lead of the hob have to be altered accordingly. Equations for the determination of the required hob-tooth angle (the normal inclination of the straight-tooth profile of the hob) and of the lead of the hob are

$$\tan VHT = \frac{\tan VP}{\cos VHS} \tag{117}$$

$$LH = \frac{n \times CP}{\cos VHS} \tag{118}$$

where VP = angle of sides of basic rack tooth (pressure angle).
VHS = angle at which hob is set.
VHT = angle of sides of hob tooth.
LH = lead of hob.
CP = circular pitch.

While this angular setting of the hob does not affect the accuracy of the involute-profile curvature of the generated gear teeth, it does have an influence upon the height of the fillet at the root of the hobbed gear teeth, which if too high leads to so termed "edge contact" at the beginning of tooth engagement, interference which is often responsible for noisy and troublesome gearing. Consequently, there is a definite limit to the permissible angular setting of the hob, which varies not only with the number of teeth in a gear, being a minimum in the case of a rack, but also with the tooth proportions. In practice, this limit is ordinarily determined empirically, as the correction of the interference necessitates either the use of a hob of lesser lead or the trimming of the gear teeth.

The advisable cutting speed of the hob is another of the questions that can best be answered by experience. It depends to a large extent upon the physical characteristics of the gear material cut, and those of the hob, and should be somewhat less than the cutting speed of an ordinary milling cutter under similar conditions. A suitable speed once established, the problem then becomes one of selecting the most economical hob diameter for the job, an important question, for the smaller the diameter of the hob the more revolutions the hob has to make to attain the required cutting speed and, as the speed relationship between the rotary speed of the hob and of the gear blank is fixed, the higher is the rate of production. On the other hand, the smaller the diameter of the hob, the fewer, as a rule, are its cutting edges and flutes and the finer has to be its feed for a satisfactory cutting of the metal, requirements which tend to slow up production and impose greater importance upon the accuracy of the hob's proportions.

These and other similar requirements that have to do more with the machine and generating tools, how the production equipment is used and its limitations, than with gear design, apply generally to the hobbing of teeth for all varieties of gears that can be so generated, including the cutting of worm gears. In hobbing this latter variety of gear, however, there is no cross-feeding between the hob and the gear blank, simply the synchronized rotation of the cutter and the work.

PLANING GEAR TEETH

While the planing of gear teeth with a form tool corresponding in form to the tooth space between the teeth, or with a plain tool that is guided by a template conforming in outline to the shape of the tooth space are methods which, like the milling of gear teeth by formed cutters, are now relegated to the production of a few individual gears intended for relatively slow-speed operation or to the production of gears of abnormal proportions, the generation of gears by a planing process is an approved method of production in commercial practice. Two general methods of planing generation are in common use, one in which the gear teeth are cut in a shaper by a rotating pinion-type cutter with cutting edges accurately generated to involute form and the other in which the planing tool takes the form of a section of the

basic involute rack and follows the rotation of the gear blank as the tool planes out the tooth spaces.

In the method employing a pinion-shaped cutter, the gear blank, firmly mounted on the work arbor of a gear shaper, slowly revolves, as the pinion cutter, mounted on the reciprocating ram of the machine and rotating at a synchronized speed conforming to the ratio existing between the number of teeth to be cut in the gear blank and the number of cutting teeth on the pinion tool, planes out the gear teeth. In action, the operation is similar to that which would occur if an involute pinion of the size of the cutter ran with a mating gear of the size of the one generated. The teeth of the pinion-shaped cutting tool being accurately generated to involute form, the teeth cut in the gear blank are of conjugate form to the teeth on the cutter and to those of the basic involute rack to which the pinion cutter is, in effect, a conjugate gear.

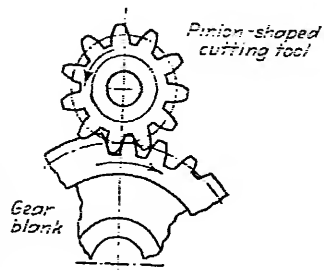


FIG. 120.—Generating gear with pinion-shaped cutting tool.

As in the hobbing process of gear-tooth generation, the accuracy of the product is dependent, to a large extent, upon the quality of the machine and tool, and the care with which the various settings are made and maintained. The pinion cutter, especially, must be accurately proportioned and not only must the profiles of its cutting teeth be accurately generated to involute form, but its teeth must be relieved in a manner that will assure a retention of the proper cutting-edge profile after a reasonable number of dressings.

Naturally, there is a practical limit to the size, or diameter, of the pinion cutter, so that cutters with only a few coarse teeth cannot be produced with full-involute profiles, the flanks of the cutting teeth extending inside the base circle of even the 20-deg. stub-tooth system, which is that generally employed for gearing generated by this pinion-cutter method. When this occurs, the flank of the cutter teeth inside the base circle is usually made with straight-line profiles that are substantially radial in direction. The effect of this necessary modification in profile of the cutter teeth is to increase the height of the fillet at the root of the generated gear teeth and to modify to some extent the curvature

of the tips of the generated teeth, changing them from an involute to a cycloidal form.

An increase in the pressure angle could be resorted to, to decrease the amount of modification in the involute curvature at the tips of the generated teeth, but this would also have the effect of increasing the height of the fillet at the root of the teeth, making it necessary to increase the amount of clearance provided for gear teeth of standard proportions. However, the limitation thus placed on this method of gear-tooth generation by the practical considerations which hold down the diameter of the pinion cutter in commercial practice is not so serious as it might be, for the smallness of the standard pinion cutters confines the use of the method to the production of gears of relatively fine pitch and small diameter, the larger gear units being almost always cut by the other method of planing generation.

PLANING GEAR TEETH WITH RACK-SHAPED CUTTER

This other method of planing, or shaping, gear teeth makes use of a machine with a reciprocating ram carrying a planing tool in the form of a section of an involute rack and a vertical work spindle, on the horizontal bed of which the gear blank is firmly clamped. Between the cutting strokes of the rack tooth, the gear blank is made to revolve and at the same time move sideways a short distance across the path of the reciprocating tool, exactly as a gear of the same pitch diameter would roll over a stationary rack. When this advance, the combination of the rolling and linear motions, of the gear blank amounts to a distance equal to the linear pitch of the rack tooth, the gear blank is automatically returned, without rotary motion, to its starting point and the same cycle of operations continues until all the teeth in the blank have been cut. In this manner, the successive teeth in the gear are all generated by the same few rack-tool cutting teeth.

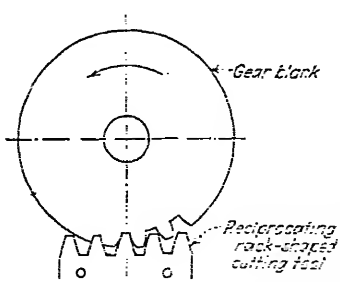


FIG. 121.—Generating gear with rack-shaped cutting tool.

The surface of the gear tooth generated by the reciprocating rack-form cutter consists, obviously, of a series of narrow, flat

surfaces extending across the face of the gear, being in this respect almost identical with the tooth surface generated by the pinion-shaped cutters. However, by making use of a sufficiently fine feed in the rotation of the gear blank, with either method of generating gears by the action of reciprocating tools, the ridges at the intersections of the flat planed surfaces break down and burnish rapidly to a smooth continuous surface when the gears are operated under load, either in service or as a final burnishing operation.

The largest, as well as smaller, gears are produced by this rack-cutter method and the gearing, being generated by full-involute cutter teeth, is free from the limitations imposed by high fillets and by the modifications in tooth-profile curvature common to gearing produced by other methods of gear-tooth generation. From the point of view of productivity, the rack-tooth planer is capable of generating the greatest number of large gears in a given time, but in the generation of small and medium-sized gears more rapid production can be made by the gear hobber.

GRINDING GEAR TEETH

The grinding of gear teeth, heretofore employed simply for the truing up of the profiles of the teeth of hardened gears that have been distorted by the heat treatment to which they were subjected, can be made a valuable process in the production of quality gearing, as an operation for overcoming a large part of the limitations inherent in, and imposed by, other methods of gear-tooth generation. This will entail grinding by a generation process, in preference to form grinding, although both these methods have been applied with considerable success in truing-up service.

Form grinding differs from the form milling of gear teeth only in that a formed grinding wheel is used in place of a milling cutter and is subject to much the same limitations, though to a somewhat lesser degree, since the accuracy of the grinding surfaces of the wheel is customarily maintained by a truing device under the control of carefully proportioned templates of enlarged size. Nevertheless, a particular grinding wheel and guide template should be employed for each particular size and pitch of gear, if true involute tooth profiles are desired.

Grinding by a generation process, on the other hand, makes use of the flat edge of a comparatively large, usually disk-shaped grinding wheel that is made to follow, at a high speed of revolution, the rotation of the gear, much in the same manner that the rack-shaped cutter follows the rotation of the gear blank in the planing generation of gear teeth, to which process, grinding generation may well be likened. The slightly flattened edge of the grinding wheel, the grinding surface used in preference to a flat side surface, though the latter is also used successfully, corresponds to a short section of the flat profile of the rack tool in the planing process and is fed rapidly back and forth across the face

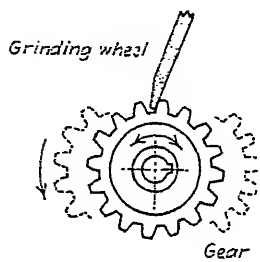


FIG. 122.—Grinding gear teeth by generating process.

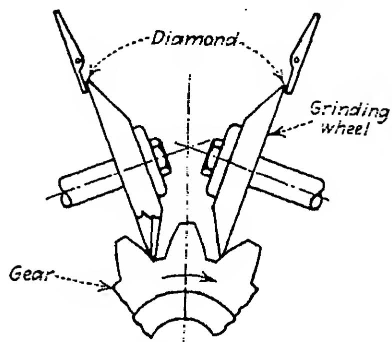


FIG. 123.—Automatic adjustment for wear on grinder wheels.

of the gear teeth as the gear is rotated on its axis. Hence, the resulting ground tooth profile is controlled by and is conjugate to the profile of a tooth of full-involute form.

The ground gear tooth generated by this process is of correct involute profile over the entire section outside the base circle and, consequently, requires no modifications to avoid interference occasioned by high fillets and alterations in the profile curvature at the tips of the gear teeth. Therefore, if this guiding disk process is applied to gear teeth with these blemishes in profile curvature, it will, if the teeth are left somewhat full or oversize, overcome much of the limitation imposed by these defects. The result of this correction is to broaden materially the range of gearing that can be produced by generating methods in which the introduction of these profile errors is inherent.

For the successful application of disk-grinding generation, it is essential that the grinding edge of the abrasive wheel be

maintained continuously in a plane tangent to the true involute curvature of the gear teeth. This is accomplished quite successfully, where a saucer-shaped grinding wheel is employed, by an ingenious truing device, consisting of a flat-faced dressing diamond carried on the short arm of a lever and held in such manner that when the lever is in its normal position the diamond just clears the edge of the grinding wheel (see Fig. 123). Every few seconds, this lever automatically swings forward until the flat surface of the diamond touches the edge of the grinding wheel and, incidentally, dresses it. If any grinding wheel wear has occurred, so that the lever swings beyond its zero position, connection is made with a control pawl that causes a fine feed screw to draw in and reset the wheel with its grinding edge in the required tangent plane. If there has been no recession of the grinding edge, *i.e.*, no wear, the connection is not made with the control pawl and the position of the grinding wheel remains unaltered.

The granular structure of the grinding wheels should be as fine as grinding requirements permit, else the scratches made on the delicate profile surfaces of the teeth will be productive of a distinctive and objectionable noise at high pitch-line velocities and, what is more serious, there will be a reduction in the load-carrying capacity of the gear teeth. With a fine abrasive, the scratches are not deep and will disappear as the profile surfaces of the teeth become highly burnished in operation under load; ground gears that were at first objectionably noisy, frequently becoming comparatively silent and more efficient with relatively short operating service.

LAPPING GEAR TEETH

When hardened gear teeth have been machined with a high degree of precision, they can frequently be finished to advantage by a lapping process. The gears are simply run under load with an abrasive introduced between the teeth with the lubricant, but, as the amount of sliding between well-proportioned gear teeth is quite limited, the process is really only suitable when the profile irregularities are relatively trivial. Even under such conditions and when the teeth are well proportioned, the abrasive to function effectively has to be rubbed over the entire surface to be lapped and this entails a wider distribution of the lapping

material than can be effected by the limited sliding action between well-proportioned gear teeth.

Unless the abrasive material is well distributed, in fact, the lapping process is apt to be restricted largely to a grinding and crushing of the abrasive with little polishing or lapping of the gear-tooth profiles, for not only is the normal sliding action between meshing gear teeth insufficient in amount, but it is not uniform enough for effective lapping action. To secure more sliding and so effect a better distribution of the abrasive, hardened gears are sometimes run with softer gears of considerably wider face, with a combination of rotating and transverse-sliding motions. The gears are rotated together slowly and at the same time the hardened gear is made to slide rapidly back and forth across the face of the wide, softer gear.

Another approved method of lapping is to mount several of the hardened gears on an arbor and mesh them with a wide rack of basic rack-tooth form. The gears are rolled back and forth along the rack, under pressure, while they are made to slide across the broad face of the lap. Still another method is to employ a lap in the form of a wide-face internal gear, reciprocating the hardened gears through the internal gear, indexing the gears at the end of each stroke to distribute the lapping as much as possible. As when grinding gear teeth, the granular structure of the lapping abrasive must be fine, to avoid scratching the delicate profile surfaces of the gear teeth, else the gearing is apt to develop objectionable noise in operation.

The elimination so far as possible of noise at high pitch-line velocities is now recognized as the final test in gaging the quality of gearing, indicating as it does that the gear teeth are correctly formed and that the profile surfaces of the meshing teeth are smooth and highly polished. This combination is essential for modern high-speed gearing and of the two requirements that of accurate and definite tooth form is the more important, since, if the general profile curvatures of the meshing teeth are correct, any minor surface irregularities that are productive of noise can be best ironed out by the cold-working of the gear-tooth profile surfaces that occurs in service when the gearing is run under load.

Efforts to secure silent-running gears by arbitrary modifications of the tooth profiles cut by the generating machine, other than those which may be needed to overcome inherent limitations

in the operation of these machine tools, are, at best, a risky undertaking. The duration of tooth contact is apt to be shortened, and transmission efficiency impaired. Greater care exercised in performing the final, regular production operation on the gear teeth, followed if necessary by a burnishing run with the gears under load, is generally more satisfactory.

SECTION XIV

MATERIALS AND HEAT TREATMENT

Some familiarity with the materials used in the manufacture of gears and of the heat treatments to which the gears and gear blanks may be subjected is required by the gear designer as well as by the machinist. The designer is responsible not only for the proportioning, appearance and general design of the finished gearing, but he is accountable for the suitability of the gear blanks with which the machinist is provided. He can only plan successfully when he has knowledge of the materials in which his creations are to be reproduced, the physical characteristics of these materials and their limitations, whether his designs are to be made in the form of castings or forgings, and the provisions he should make to facilitate production processes. For the machinist, information concerning the materials with which he has to work is required to enable him to use his tools to the best advantage.

The arduous duty gearing performs, frequently under decidedly unfavorable working conditions, and the fact that the gears are expected to withstand strains and shocks that would quickly wreck other forms of mechanisms make the selection of gear materials an exacting responsibility, especially when high pitch-line velocities are entailed, or are to be anticipated, and heavy tooth loads carried. So rapidly have the service demands on gearing increased, moreover, that the quality of the materials used for gears has had to be much improved to avoid undue increase in weight and bulk of the gears and to prevent their cost becoming prohibitive.

HEAT-TREATED STEELS

The quality of the steel used in gears has been raised by the addition of alloying materials and by the heat treatment of the better grades of carbon steel. The alloy steels are, naturally, the more expensive, require heat treatments, and are more difficult to machine, but, when space limitations are exacting and tooth loads and pitch-line velocities are high, their use is

frequently advisable and at times essential, despite the added cost in the manufacture of the gears.

The heat treating of carbon steel is not only less expensive, but gears made from the resulting materials are quite capable of meeting all usually imposed requirements. This heat treating, performed after the main machining processes on the gears have been consummated, consists, broadly speaking, of heating the steel products to a relatively high temperature and then quenching them in a bath of water, oil, brine, or other solution, different results being secured with the various quenches. The grade of steel treated is naturally a matter of concern, and this question has been carefully investigated by the A.G.M.A., the recommendations of which body for rolled and forged carbon steels suitable for heat treatment, as well as for carbon steels well adapted for the manufacture of untreated steel gears, being listed in Table 32.

TABLE 32.—FORGED AND ROLLED CARBON STEEL FOR GEARS

Use	Chemical composition, per cent			
	Carbon	Manganese	Phosphorus	Sulphur
Case-hardened........	0.15–0.25	0.40–0.60	0.045 max.	0.05 max.
Untreated............	.25– .50	.50– .80	.045 max.	.05 max.
	.40– .50	.40– .60	.045 max.	.05 max.
Hardened...........	.40– .50	.40– .60	.045 max.	.05 max.

Case-hardened gears are well suited for installations where the gears must transmit a uniformly high torque without sudden shocks, and for services where the wear-resisting quality of the gear teeth is the outstanding requirement. This process of case-hardening, which gives a hard-surface shell, while leaving the inner metal, or core, comparatively soft, consists of packing the steel gears with some carbonaceous material in a pot, sealing the pot and heating the charge in a furnace to a temperature of from 1600 to 1650°F. As the pot is brought to heat, the carbonaceous material gives off gas which penetrates the steel increasing its carbon content, the depth of penetration, or case, depending upon the length of time the heat is maintained. Following this carburizing step, the gears are quenched and then drawn at a temperature of between 250 and 500°.

For high-grade gears, it is advisable to precede the carburizing process by a normalizing heat at a temperature of between 1650 and 1750°, to improve the structure of the metal by relieving internal stresses and to reduce the liability of distortion as a result of subsequent heat treatment. Also, a treatment of heating the gear blanks of steel recommended for untreated steel gears to a temperature of 1575 or 1675°, quenching them in water and then drawing them to the required hardness before cutting the gear will frequently improve the machining qualities of that grade of carbon steel.

If the case-hardening of a steel of the first grade listed in Table 32 is simply to secure a hard-surfaced gear tooth, one reheat to a temperature slightly below that of the first carburizing heat and a second quench in advance of the draw usually gives satisfactory results, but, if the maximum refinement in both case and core, with a minimum of distortion, is desired, a second reheat and a third quench before the draw is customary. Each of these reheats should be to a temperature 50 or 100° below that of the preceding heat. Case-hardening of this order will increase the surface hardness of the gear teeth about four times, with the core becoming tougher but little harder than before the treatment.

When the load to be transmitted by the gearing is variable, and the gears are liable to be subjected to severe shocks, case-hardened gears, despite their high wear-resisting qualities, are not entirely satisfactory, for the case with its high-carbon content is quite brittle and apt to fail. Under these circumstances, if extreme hardness can be sacrificed for high ductility, an oil treatment, in which the gear is simply heated to slightly above the critical temperature of the steel, quenched in oil, reheated and drawn back to relieve the stresses set up by rapid cooling, gives good results and is an inexpensive process. The steel is made stronger, tougher and about 75 per cent harder than the untreated steel stock.

Where a tough-hard gear is required, however, a steel of higher carbon content should be used, such as that listed last in Table 32, subjected to a heat of 1400 to 1450°, quenched in oil and then drawn to the required hardness. The tensile strength of the treated steel, which is exceedingly tough, will run about 250,000 lb. per square inch at the surface of the gear tooth, where it is only slightly less hard than high-grade case-hardened steel grading down to about 125,000 lb. per square inch at the center

of the tooth. It is not quite "glass hard," for it can be filed, but it is almost impossible to machine the outer surface of the treated steel.

A treatment similar to the one outlined for tough-hard-forged steel is well suited for gears cast from steel made by open hearth, crucible, or electric-furnace processes, the chemical composition of which, as recommended by the A.G.M.A., is listed in Table 33. Suitable steels, whether forged or cast, when heat treated in this manner acquire properties which make the gears well suited for all kinds of severe service. The high surface strength is where the stresses on the gear teeth are the heaviest and the decline in strength where the stresses are balanced about the neutral axis of the teeth.

TABLE 33.—A.G.M.A. STANDARDS FOR STEEL CASTINGS

Use	Chemical composition, per cent				
	Carbon	Manganese	Phosphorus		Sulphur
			Acid	Basic	
Case-hardened....	0.15–0.25	0.40–0.60	0.06 max.	0.05 max.	0.6 max.
Untreated or hardened..........	.30– .40	.40– .60	.06 max.	.05 max.	.6 max.

In the heat treatment of the several varieties and grades of alloy steels used in the manufacture of gears to meet especially exacting working conditions, a preliminary normalizing process at a temperature slightly above that of the heat treatment will in all cases prove desirable, improving the structure of the steels and guarding against distortion caused by the subsequent heating and quenching processes. In the treatment of some of these alloy steels, in which the carbon content is low, such as in grades of nickel, nickel-chromium, chromium, and chromium-vanadium steels suitable for the manufacture of gears, carburizing processes are entailed, while in others in which the hardening carbon content is adequate, such as in suitable grades of molybdenum and silicomanganese steels, simply a hardening, *i.e.*, heat, quench, and draw-to-required-hardness treatment is needed. With these alloy steels, virtually any degree and combination of required

physical characteristics for wear-resisting qualities, tooth strength, etc., can be secured.

HEAT TREATMENTS OF ALLOYED GEAR STEELS

Nickel Steels

S.A.E. Steel 2315.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.10–0.20	Nickel		3.25–3.75
Manganese	0.30–0.60	Silicon	basic open hearth	0.15–0.30
Phosphorus	0.04 max.		electric and acid open hearth	0.15 min.
Sulphur	0.05 max.			

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1600–1650°F.
(3) Quench from box in oil.
(4) Reheat to 1500–1550°F.
(5) Quench in oil.
(6) Reheat to 1350–1400°F.
(7) Quench.
(8) Draw at 250–500°F.

S.A.E. Steel 2320.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.15–0.25	Nickel		3.25–3.75
Manganese	0.30–0.60	Silicon	basic open hearth	0.15–0.30
Phosphorus	0.04 max.		electric and acid open hearth	0.15 min.
Sulphur	0.05 max.			

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1600–1650°F.
(3) Quench from box in oil.
(4) Reheat to 1500–1550°F.
(5) Quench in oil.
(6) Reheat to 1350–1400°F.
(7) Quench.
(8) Draw at 250–500°F.

S.A.E. Steel 2350 (For gears of large section).

CHEMICAL COMPOSITION, PER CENT

Carbon	0.45–0.55	Nickel		3.25–3.75
Manganese	0.50–0.80	Silicon	basic open hearth	0.15–0.30
Phosphorus	0.04 max.		electric and acid open hearth	0.15 min.
Sulphur	0.05 max.			

HEAT TREATMENT

(1) Normalize at 1600–1650°F.
(2) Reheat to 1375–1425°F.
(3) Cool in furnace
(4) Machine.
(5) Reheat to 1400–1450°F.
(6) Quench in oil.
(7) Draw to required hardness.

Nickel-chromium Steels

S.A.E. Steel 3115.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.10–0.20	Nickel		1.00–1.50
Manganese	0.30–0.60	Chromium		0.45–0.75
Phosphorus	0.04 max.	Silicon	basic open hearth	0.15–0.30
Sulphur	0.05 max.		electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1625–1675°F.
(3) Cool in box.
(4) Reheat to 1400–1450°F.
(5) Quench.
(6) Draw at 250–500°F.

S.A.E. Steel 3215.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.10–0.20	Nickel		1.50–2.00
Manganese	0.30–0.60	Chromium		0.90–1.25
Phosphorus	0.04 max.	Silicon	basic open hearth	0.15–0.30
Sulphur	0.045 max.		electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1625–1675°F.
(3) Cool in box.
(4) Reheat to 1375–1425°F
(5) Quench in oil.
(6) Draw at 250–500°F.

S.A.E. Steel 3250.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.45–0.55	Nickel		1.50–2.00
Manganese	0.30–0.60	Chromium		0.90–1.25
Phosphorus	0.04 max.	Silicon	basic open hearth	0.15–0.30
Sulphur	0.045 max.		electric and acid open hearth	0.15 min.

S.A.E. Steel 3335.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.30–0.40	Nickel	3.25–3.75
Manganese	0.30–0.60	Chromium	1.25–1.75
Phosphorus	0.04 max.	Silicon: basic open hearth	0.15–0.30
Sulphur	0.045 max.	Silicon: electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1600–1700°F.
(2) Reheat to 1200–1250°F.
(3) Cool slowly in furnace.
(4) Machine.
(5) Reheat to 1425–1475°F.
(6) Quench in oil.
(7) Draw to required hardness.

S.A.E. Steel 3415.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.10–0.20	Nickel	2.75–3.25
Manganese	0.30–0.60	Chromium	0.60–0.95
Phosphorus	0.04 max.	Silicon: basic open hearth	0.15–0.30
Sulphur	0.045 max.	Silicon: electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1600–1650°F.
(3) Cool in box.
(4) Reheat to 1400–1450°F.
(5) Quench in oil.
(6) Draw at 250–500°F.

S.A.E. Steel 3450.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.45–0.55	Nickel	2 75–3.25
Manganese	0.30–0.60	Chromium	0 60–0.95
Phosphorus	0.04 max.	Silicon: basic open hearth	0.15–0.30
Sulphur	0.045 max.	Silicon: electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1550–1650°F.
(2) Reheat to 1250–1300°F.
(3) Cool slowly in furnace.
(4) Machine.
(5) Reheat to 1400–1450°F.
(6) Quench in oil.
(7) Draw to required hardness.

Molybdenum Steels

S.A.E. Steel 4615.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.10–0.20	Nickel	1.50–2.00
Manganese	0.30–0.60	Molybdenum	0.20–0.30
Phosphorus	0.04 max.	Silicon, basic open hearth	0.15–0.30
Sulphur	0.05 max.	Silicon, electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1625–1675°F.
(3) Cool in box.
(4) Reheat to 1475–1525°F.
(5) Quench.
(6) Draw at 250–500°F.

Chromium Steels

S.A.E. Steel 5120.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.15–0.25	Chromium	0.60–0.90
Manganese	0.30–0.60	Silicon, basic open hearth	0.15–0.30
Phosphorus	0.04 max.	Silicon, electric and acid open hearth	0.15 min.
Sulphur	0.05 max.		

HEAT TREATMENT

(1) Normalize at 1600–1700°F.
(2) Carburize at 1650–1700°F.
(3) Cool in box.
(4) Reheat to 1525–1575°F.
(5) Quench.
(6) Draw at 250–500°F.

Chromium-vanadium Steels

S.A.E. Steel 6120.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.15–0.25	Chromium	0.80–1.10
Manganese	0.30–0.60	Vanadium	0.15 min.–0.18 min. desired
Phosphorus	0.04 max.	Silicon, basic open hearth	0.15–0.30
Sulphur	0.045 max.	Silicon, electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1650–1700°F.
(3) Cool in box.
(4) Reheat to 1525–1575°F.
(5) Quench.
(6) Draw to 250–500°F.

S.A.E. Steel 6150.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.45–0.55	Chromium	0.80–1.10
Manganese	0.50–0.80	Vanadium	0.15 min.–0.18 min. desired
Phosphorus	0.045 max.	Silicon basic open hearth	0.15–0.30
Sulphur	0.05 max.	Silicon electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Reheat to 1250–1350°F.
(3) Cool slowly.
(4) Machine.
(5) Reheat to 1525–1625°F.
(6) Quench in oil.
(7) Draw to required hardness.

Silicomanganese Steels

S.A.E. Steel 9260.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.55–0.65	Silicon	1.80–2.20
Manganese	0.60–0.90	basic open hearth	0.15–0.30
Phosphorus	0.045 max.	Silicon electric and acid open hearth	0.15 min.
Sulphur	0.05 max.		

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Reheat to 1400–1450°F.
(3) Cool slowly.
(4) Machine.
(5) Reheat to 1600–1650°F.
(6) Quench in oil.
(7) Draw to required hardness or tests.

BRONZE AND BRASS CASTINGS FOR GEARS

For spur and bevel gears, the recommendation of the A.G.M.A for hard-cast bronze is a mixture of

	Per Cent
Copper	86–89
Tin	9–11
Zinc	1– 3
Lead (max.)	0.20
Iron (max.)	0.06

Good castings made from this quality bronze have a minimum ultimate tensile strength of 30,000 lb. per square inch, a minimum

yield point of 15,000 lb. per square inch, and an elongation in 2 in. of approximately 14 per cent.

For bronze worm gears, the same authority recommends quality grades of either a phosphor-bronze or a leaded gun metal, both of which alloys are well adapted to chilling for hardness and refinement of grain, especially the phosphor-bronze which is also especially suitable for use with worms of great hardness and accuracy of thread. The leaded gun metal is more suitable for use with unhardened worms. Good castings of these alloys should have minimum ultimate strengths of 35,000 lb. per square inch for the phosphor-bronze and 30,000 lb. for the leaded gun metal; a yield point for the phosphor-bronze of 15,000 lb. per square inch and for the leaded gun metal one of 12,000 lb.; and both should have an elongation in 2 in. of close to 10 per cent.

TABLE 34.—A.G.M.A. STANDARD BRONZES FOR WORM-GEAR RIMS

Variety	Chemcial composition, per cent				
	Copper	Tin	Phosphorus	Lead	Zinc and impurities
Phosphor-bronze........	88–90	10–12	1–3	.....	0.50 max.*
Leaded gun metal.......	86–89	9–11	0.25 max.	1–2.5	.50 max.

* Includes any lead.

The chemical compositions recommended by the A.G.M.A. for the bronze in the bushings for gears and for the brass in the flanges of composition pinions, together with the minimum physical characteristics for good castings made from the respective alloys, are given in Table 35. The grade of brass specified for the flanges of nonmetallic pinions, a good cast red brass, is incidentally of sufficient strength and hardness to carry its share of the tooth load when the active face of the pinion encroaches upon or takes in the flanges, *i.e.*, when the pinion teeth extend through the flanges and the total face width of the pinion engages with the teeth of the meshing gear.

TABLE 35.—A.G.M.A. STANDARDS FOR BUSHINGS AND FLANGES

Alloy and use	Chemical composition, per cent								
	Copper	Tin	Lead	Phosphorus	Zinc	Iron	Antimony	Aluminum	Impurities
Bronze bushings	78.5–81.5	9.0–11.0	9.0–11.0	0.05–0.25	0.75 max.	....	....	......	0.25
Brass flanges	83.0–86.0	4.5–5.5	4.5–5.5	4.5–5.5	....	0.35 max.	0.25 max.	None	

Physical Characteristics

Alloy and use	Ultimate strength, lb. per sq. in.	Yield point, lb. per sq. in.	Elongation in 2 in., per cent
Bronze bushings	25,000	12,000	10
Brass flanges	27,000	12,000	10

SECTION XV

MEASUREMENT OF GEAR TEETH

The automatic nature of the generating processes of gear production makes measuring the gear teeth in effect simply a calibration of the accuracy of the generating machine and of the precision in form of its cutting tools. The precision in form of the generated gear tooth is, in other words, a measure of the accuracy of the tools employed in its production, rather than a measure concerned with the degree of manual skill exercised in the operation. The reason for this is that any correction in tooth form entails some modification in the production tools, in their form or in the operating precision of the component parts of the machine; provided, of course, that the gear blank is suitably proportioned and correctly mounted and secured upon the work arbor, or spindle.

Skill and experience are needed for the efficient operation of a gear-generating machine, as they are for the operation of any machine tool, it is true, and care has to be exercised in making the required machine adjustments and in avoiding abuse of the machine; but, except for these fundamental and general requirements, the gear cutter has little or no control over the accuracy of the gear teeth generated by the machine he operates. If the machine were perfect in all respects and its operation founded upon sound basic principles, it would, when run with due regard to ordinary machine care, generate perfect gears and there would be little need for any measurements whatever.

As the perfect gear-generating machine has yet to be built, some critical inspection of the gears produced is still necessary, but there is no need in commercial gear production of making more measurements than are needed to establish the facts either that the gears are satisfactory or that the existing errors are sufficiently pronounced to require suitable machine adjustment or modification. A few simple tests and the measurement of a few controlling dimensions only are required.

accuracy in the findings of which is dependent upon both dimensions being noted at the one time. The corrected-addendum setting of the instrument locates the plane on which the gear-tooth thickness is measured, and, unless both dimensions are correct, the gear is below standard requirements. In such a case, it really makes no difference whether the error lies in the thickness of the teeth or in the pitch, or outer diameters, of the gear. Suitable corrections have to be made in any event.

The circular-pitch thickness of the gear teeth, or the arc of the angle of tooth thickness, is equal to one-half the quotient of the pitch circumference divided by the number of teeth minus the amount of backlash provided, from which the angular measure of the tooth thickness is readily ascertained. With this measure known, the equations for the determination of the vernier settings for the chordal tooth thickness and corrected addendum, as indicated in Fig. 126, are respectively:

$$CTT = 2PR \times \sin \left(\frac{45 \text{ deg.} \times \text{arc } VTT}{1.5708PR} \right) \qquad (119)$$

$$CA = OR - PR \times \cos \left(\frac{45 \text{ deg.} \times \text{arc } VTT}{1.5708PR} \right) \qquad (120)$$

Since,

$$\text{Arc } VTT = \frac{3.1416 \times 2PR}{2N} - BL \quad \frac{3.1416PR}{N} - BL$$

$$\text{Angle } VTT = \frac{\text{arc } VTT}{6.2832PR} \times 360 \text{ deg.} = \frac{45 \text{ deg.} \times \text{arc } VTT}{0.7854PR}$$

$$\text{Angle } \frac{VTT}{2} = \frac{45 \text{ deg.} \times \text{arc } VTT}{1.5708PR}$$

where PR = pitch radius.
BL = backlash.
OR = outer radius.
N = number of teeth.

PIN MEASUREMENT OF SPUR GEARS

Another method of checking the tooth thickness consists, in the case of spur gears, in placing pins, or rolls, in diametrically opposite tooth spaces, when the gears have an even number of teeth, or as nearly opposite tooth spaces as possible, when the gears have an odd number of teeth, and measuring the over-all distance over these pins. The diameter of the pins is somewhat greater than the chordal width of the tooth spaces on the pitch

circle and, preferably, of such measurement that normal contact between the pins and the gear teeth is effected on the pitch-line elements of the gear teeth. By having the pins of this special size, the pitch circles of the gear are checked at the same time and also the pressure angle at the critical pitch point on the gear teeth. In addition to this multiplicity of checking operations the computations involved are more obvious, and certain of the dimensions established are also entailed in taking other important measurements.

This desirable diameter for the test pins is equal to the chordal tooth space on the pitch circle, including whatever amount of backlash is provided, divided by the cosine of the angle between the tooth angle and the chord of the tooth space (formula 121), necessitating the determination of this full chordal tooth space, as follows. Referring to Fig. 127:

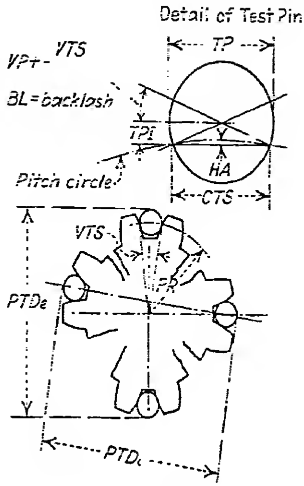


FIG. 127.—Pin measurement of spur gears.

$$\text{Arc } VTS = \frac{3.1416PR}{N} + BL$$

where BL = amount of backlash.

$$\text{Angle } VTS = \frac{\text{arc } VTS}{6.2832PR} \times 360 \text{ deg.} = \frac{45 \text{ deg.} \times \text{arc } VTS}{0.7854PR}$$

$$\text{Angle } \frac{VTS}{2} = \frac{45 \text{ deg.} \times \text{arc } VTS}{1.5708PR}$$

$$CTS = 2PR \times \sin \frac{45 \text{ deg.} \times \text{arc } VTS}{1.5708PR}$$

where CTS = chordal tooth space.

$$TP = \frac{CTS}{\cos\left(VP + \frac{VTS}{2}\right)} \qquad (121)$$

where TP = test-pin diameter.

VP = pressure angle.

With the diameter of the test pin established, the over-all distance over the pins (see Fig. 127) is equal to the sum of the pin and pitch diameters plus the difference between spacing of the pins and the pitch diameter of the gear. Step by step, the derivations of the equations needed for determining this over-all dimension, when the gear has an even number of teeth and when the number of teeth is odd, are

$$HA \text{ (height of chordal arc)} = PR - PR \cos \frac{VTS}{2} = PR\left(1 - \cos \frac{VTS}{2}\right)$$

$$TPI \text{ (distance from pin center to chordal plane)} = \frac{TP}{2} \times \sin VP$$

$$PTDe = PD + TP + 2(TPI - HA)$$

$$PD = PTDe - TP - 2(TPI - HA) \qquad (122a)$$

$$PTDo = PD \times \cos \frac{90}{N} \text{ deg.} + TP + 2(TPI - HA)$$

$$PD = \frac{PTDo - TP - 2(TPI - HA)}{\cos \frac{90 \text{ deg.}}{N}} \qquad (122b)$$

SPACING OF GEAR TEETH

Errors in tooth spacing of generated gear teeth, if not occasioned by inaccuracies in the feeding and indexing mechanisms of the production machine and, hence, calling for prompt machine inspection and adjustment, are usually attributable to inadequate and insecure support of the gear blank, *i.e.*, to lack of necessary rigidity under the heavy stresses developed in cutting the gear teeth. A simple method of testing for these faults, whether occasioned by machine inaccuracies or other failings, is to place pins in adjacent tooth spaces and measure the distance over the pins, repeating the operation until the spacing of all the teeth has been tested. Any variation in the findings indicates a spacing error of about the same amount.

A number of excellent instruments for measuring the tooth spacing of gears with a high degree of accuracy are also available, most of which operate on much the same principle, taking advantage of the fact that the normal pitch of consecutive tooth profiles of similar involute curvature is constant and

equal to the length of the arc of the base circle between consecutive teeth (see Fig. 128).

In one variety of these instruments, an arm, which corresponds in effect to a section of a mating rack or gear, has two finger-

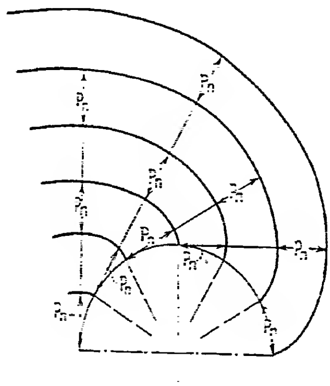


FIG. 128.—Uniformity in normal pitch of involutes.

like projections, one fixed and the other adjustable, separated by a distance conforming to the normal pitch between the profiles of consecutive teeth. In use, the fixed finger is held against the face of one of the gear teeth by means of a type of latch, and the second finger, adjusted to the pitch of the gear, is rocked over into contact with the corresponding profile of the next tooth, in a manner similar to that of tooth engagement in meshing gears. The second finger is mounted on springs and is connected with an indicator on the dial of which any

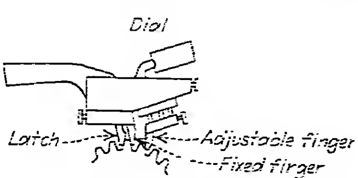


FIG. 129.—Odontometer for measuring gear-tooth spacing.

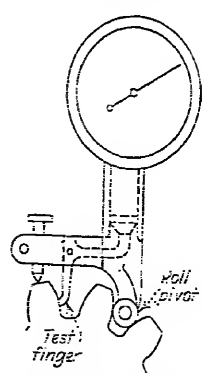


FIG. 130.—Roll-type gear-tooth tester.

variation in the normal pitch is registered.

In another variety of instrument, a roll fits into a tooth space and acts as a pivot about which a test finger centers and is rocked into contact with the opposing profile of the next

tooth. The measuring test finger is also connected with an indicator, and any change in the normal distance between consecutive tooth profiles is registered on the dial of the instrument.

PROFILE TESTING

While uniformity in tooth spacing is essential for high-quality gears, the smooth, uniform transmission of motion and high mechanical efficiency required of modern gearing are also influenced to a marked extent by the accuracy in the curvature of the gear-tooth profiles. Gears with excellent tooth spacing and with teeth well proportioned so far as thickness on the pitch circle is concerned and precise in respect to alignment occasionally prove noisy and irregular in action, the usual explanation offered being that edge contact occurs as the mating teeth engage, *i.e.*, at the beginning of mesh. Unquestionably, something of this nature does occur, and considerable dependence is often placed on what is termed "easing off the profile of involute gear teeth," in order to circumvent this interference.

There are two general ways of doing this, both of which destroy some portion of the involute profile of the gear teeth and reduce to some degree the duration, or length, of contact between meshing teeth. One method is to modify the tooth-profile curvature near the base circle, where tooth contact first occurs, and is effected either by proportioning the generating tools so that they undercut and remove some of the flank of the tooth or by a separate milling operation. The other method is to ease off the tip portion of the gear teeth, so that tooth contact cannot commence quite as close to the base circle.

Still a third method of overcoming the objections to noisy gears, one that avoids rather than destroys the interference responsible for the noisy operation of carefully generated gearing, is to design the gear-tooth form so that the active tooth profile stops at an appreciable distance from the base circle. This practice entails either employing a higher pressure angle or else an increase in the addendum of the pinion member and a corresponding decrease in the addendum of the gear member, but again there is some sacrifice in gear efficiency and in the load-carrying capacity of the gearing.

Both methods of circumventing the interference by destroying sections of the involute profile are subterfuges and their justification is open to question, for if the teeth of noisy gears were of

correct form, there would be no need for any such modifications. Nevertheless, the expediency is occasionally necessary, in view of limitations, not so much of the involute system of gearing as of the generating machines employed for cutting the gear teeth, and the question naturally arises as to which method is the more desirable. Actually, the question should be, which method is the less harmful to the operation of the gears.

This, also, is a disputed point, though the choice in methods should be governed by whether the surplus thickness of the gear teeth responsible for the interference is confined, at least in major part, to the root section of the teeth, or whether it is divided between the root and tip sections of the teeth. In one case, modification on the flank of the teeth is indicated as advisable and, in the other, on both the tip and root sections of the teeth.

In applying either correction, the amount of modification in the form of the teeth should be the minimum needed to avoid edge contact, for which reason it is quite apparent that it would be far more desirable to eliminate, so far as possible, the opportunity of edge contact occurring, thus conserving all possible productive tooth contact, than it is to sacrifice contact and gear efficiency in anticipation of the interference. That gears can be generated that will run smoothly and relatively noiselessly is clearly shown by the fact that only some of the gears generated by standard production machines are inherently noisy, while others, cut under identical conditions and with the same degree of care, are inherently quiet in operation.

This complication simply adds to the importance of making careful investigations of the profile curvature of the teeth of all gears having a tendency to be noisy, with the view of ascertaining the reason for the fault. In this connection, since the curvature of the tooth profile is hard to measure with accuracy near the base circle, it is advisable to study the involute profile of the teeth in its entirety, rather than to center attention on the flanks of the teeth where the interference originates. Incidentally, the flatter section of the tooth profile, that between the base and pitch circles, need not constitute any particular difficulty in the accurate measurement of the curvature of the tooth profiles.

Referring to Fig. 131, the far side of an involute gear tooth is shown in its relationship to the center line of the gear, when a tangent to the point of involute origin on the base circle is at right

angles to the center line of the gear. In this position, the normal distances from the center line to the edge of the gear tooth and to the base-circle end of the involute tooth profile, respectively, are readily computed from other known dimensions, as are also the radial angles subtended by normals from these points to the center line of the gear. The difference in these normal distances constitutes a "check dimension" which covers the entire involute section of the tooth profile, as does also the angular difference

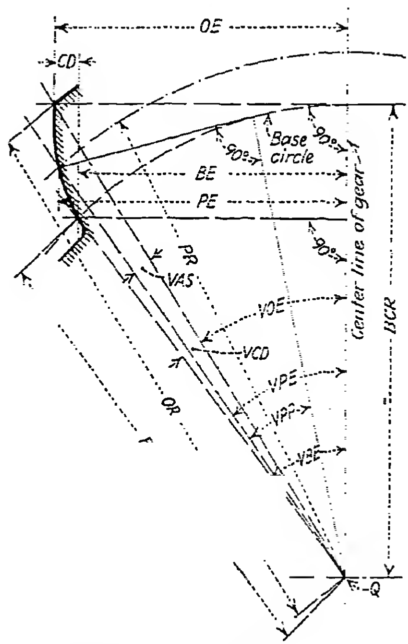


FIG. 131.—Position of gear tooth in profile test measurements.

between the radial angles subtended by the respective normals from these critical points on the tooth profile to the center line of the gear. Expressed algebraically, the simple calculations entailed in definitely establishing these relationships are:

$$BCR = PR \times \cos VP$$

$$\cos VOE = \frac{BCR}{OR}$$

$$OE = OR \times \sin VOE$$

$$VBE = \frac{OE \times 360 \text{ deg.}}{3.1416 \times 2PR} = \frac{OE \times 45 \text{ deg.}}{0.7854 \times PR}$$

$$BE = BCR \times \sin VBE$$
$$CD = OE - BE \qquad (123)$$
$$VCD = VBE - VOE \qquad (123a)$$

where VP = pressure angle.

The check dimension CD can also be readily and accurately measured on the gears under investigation with standard calibrating instruments, and, if the findings differ from the computed

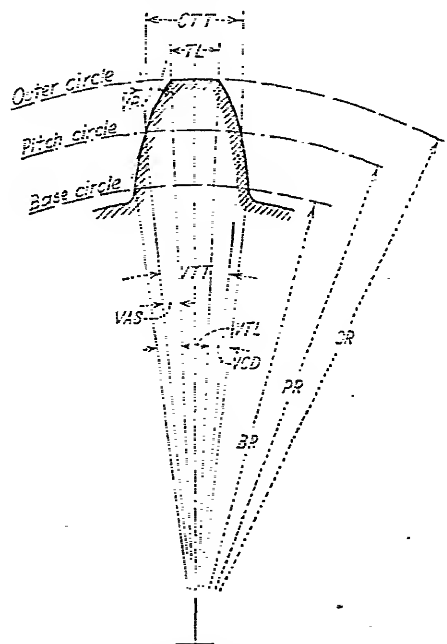


FIG. 132.—Critical angles in top-land measurements

values for the involute form of gear tooth, it is evident that the profiles of the teeth of the inspected gears are not of full-involute form, or curvature. Some modification in the curvature of the generated teeth exists, located—if the spacing of the teeth and the pressure angles are correct—either at the root edge of the tooth face or at the tips of the gear teeth. While some indication of the seriousness of the interference the faults develop is also supplied by the check dimensions, locating the interference on the flank of the tooth or at both the top and root of the teeth necessitates a check of the top land of the teeth.

The radial angle VAS of the addendum section of the tooth profile (see Figs. 131 and 132) provides a logical measure for separating the two portions of the gear-tooth face on which interference due to modifications in the generated gear-tooth profile may occur and, while this angle is not easily measured on a gear under inspection, owing to the extreme sensitiveness of the curvature of the gear-tooth profile, it can be readily computed, and the location of the interference accurately established, by measurements that can be made with preciseness. In fact, the only additional measurement required is the determination of the width of the top land of the teeth. Comparison of the top-land findings with the computed values for the same dimension of a true involute-gear tooth will then definitely establish what proportion of any variation between the measured findings and the true values of the check dimension CD is at the tip of the teeth, and what proportion is at the base-circle extremity of the tooth profile. The equations entailed in these computations are quite simple.

Referring to Fig. 131, the length of the tangent from the pitch point on the gear-tooth profile to the base circle is governed by the known pitch- and base-circle dimensions, while the angle VPE formed by a radial to the pitch point and the center line of the gear in the position shown is included in the base-circle arc VBE. The length of this latter arc, which is governed by the known outer and base-circle dimensions, is the same as that of the tangent OE from the tip of the involute gear tooth to the base circle and thus also measures angle VOE. The difference between the angles VPE and VOE, or the angle VAS, embraces the addendum portion of the involute tooth profile. With this latter angle determined, the angle VTL subtended by the top land of the true involute-gear tooth (see Fig. 132) is easily computed.

Referring to Figs. 131 and 132, the various steps in this determination of the top-land width of the true involute-gear tooth expressed algebraically are:

$$\cos VPP = \frac{BCR}{PR} = \frac{BCD}{PD}$$

$$VPE = VPP + \frac{(OE - PR \times \sin VPP)\ 360 \text{ deg.}}{3.1416PD}$$

$$= VPP + \frac{(OE - PR \times \sin VPP)\ 45 \text{ deg.}}{0.3927PD}$$

$$\cos VOE = \frac{BCR}{OR} = \frac{BCD}{OD}$$

$$VAS = VPE - VOE$$

$$VTL = VTT - 2VAS$$

$$TL\,(\text{chord}) = OD \times \sin \frac{VTL}{2} \tag{124}$$

Test instruments for the calibration of the tooth profiles of gear teeth have also been developed by which a test finger is made to follow the profile curvature of the teeth and register on an indicator dial any variation of the tooth profile from true involute curvature. The principle underlying the operation of these gear testers is that if a straight edge is rolled, without

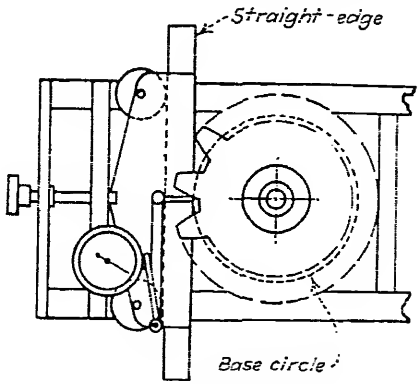


FIG. 133.—Gear-tooth profile tester.

sliding, on the surface of a cylinder any given point on the straight edge will trace an involute curve, the origin of which is on the circumference of the cylinder.

One of these gear testers that well exemplifies the practical application of this principle consists of a framework centered about an arbor on which is mounted a cylindrical guiding disk of the same diameter as the base circle of the gear to be investigated and an exploring mechanism which includes a straight edge on rollers held firmly against the cylindrical guide and on which is mounted a pivoted test finger. In operation, the gear to be tested is placed on the central arbor and adjusted so that the tip of the test finger rests on the profile of one of the gear teeth at the base circle and, when in this position, is clamped securely to the cylindrical guide.

With the gear in place and the test finger adjusted, the frame is rotated about the central arbor, while the guiding disk and gear are kept stationary, causing the straight edge to roll on the guiding cylinder and the test finger to follow the curvature of the gear-tooth profile. As the natural path traced by the test finger is that of the correct tooth profile of involute curvature, any modification in the profile of the gear tooth is detected and registered on the dial of an indicator to which the pivoted test finger is linked. Over the entire active face of the gear tooth, comparison is thus made between the existing curvature of the gear teeth and the profile of a perfect involute tooth of like proportions, locating and measuring the sections of modified curvature.

CORRECTION OF FAULTS

At the completion of these specific error-determining tests, or after a simple "composite test" to note the smoothness in action of a pair of gears (in this latter test the gears are mounted at correct center distance on accurately aligned spindles, run together by hand, the amount of backlash measured by feelers or by center-distance adjustments, and the effects of errors of spacing, etc., observed), methods of correcting the discovered faults have, naturally, to be instigated. It is then that the value of the tests becomes apparent, and the limitations of the gear-cutting equipment employed can be appraised.

The errors found due to careless workmanship, poor machine adjustments, malformed cutting tools, etc., are not only surmountable, but their correction results in material improvement in the quality of the gearing turned out and to lowered production costs. Faults due to machine limitations, on the other hand, entail when they can be overcome or relieved to some extent, either a sacrifice in the transmission efficiency and load-carrying capacity of the gearing or else an additional gear-cutting expense for a corrective finishing operation.

In the outline of the three approved methods of generating involute form gears, the faults due to machine limitations were shown to be confined mainly to those same modifications in the profiles of the generated gear teeth that are chiefly responsible for the noisy operation of gears with well-cut but poorly proportioned teeth, *i.e.*, gears of the involute form that have the profiles of their teeth more or less modified as to profile

curvature. The common subterfuge of easing off the gear teeth has been referred to briefly, and this practice, in view of its connection with the high value of profile testing, can now be discussed a little more fully.

When easing off the profile of involute gear teeth without first making a careful survey of the tooth-face situation, to locate the modifications in the profile curvature of the gear teeth which cause the objectionable interference, there is always grave danger that the operation may do more harm than good. A wise recommendation is, then, not to resort to the operation before having by careful tests definitely placed the location of the troublesome modifications in gear-tooth profile. Then, if the blemishes to the tooth faces as located are found to be attributable to definite machine limitations, and it is not expedient to substitute gears with teeth of proportions which avoid the interference, it is better to leave the gear teeth heavy and finish shaping them on a machine not handicapped by the same limitations.

A generating machine making use of cutting tools of true involute form is needed for the finishing operation and, in view of the delicacy of the process, a gear-tooth grinder employing flat or saucer-shaped abrasive wheels is an excellent, and frequently the best variety of, finishing machine. With the aid of a tool of this type, gears with teeth left a trifle full can be rapidly and accurately finished which would, if completely cut by the machine handicapped by the limitation from which a substantial, well-constructed gear grinder of the generating variety is especially free, have proved unsatisfactory.

When a machine free from the limitations imposed by the enforced use of cutting tools which are not of true involute form is not available, and the easing of the teeth by some other corrective machine is the only way of improving the running qualities of the gearing, the gear teeth should be finished on the first machine up to the point of securing accurate tooth thickness on the pitch circle, in order to make it unnecessary to touch the portion of the tooth faces bounding the critical pitch-line-element zone of the teeth in the final dressing operations. Subsequent easing cuts, if required at the root of the teeth to reduce the height of the fillet should be, not only extremely light, but radial at the base circle, where the involute profile originates and tooth contact commences. At the other extremity, the

easing should not be carried beyond the point where the width of the top land is either less or more than that of a true involute gear tooth of similar proportions.

Easing so skillfully consummated as to leave the teeth of the gear of the correct thickness on the pitch circle, radial at the base circle and of proper top-land proportions will convert a gear with malformed teeth into one with tooth profiles of virtually true involute curvature, transforming the unsatisfactory gear into a smooth- and quiet-running transmission of excellent mechanical efficiency and high-load carrying capacity. The task may tax the skill of the machinist, but it can be done successfully.

SECTION XVI

ROLLED GEARING

The commercial development of rolled gearing, in which the gear teeth are forged on gear blanks heated to a semiplastic condition, is not only an outstanding achievement in methods of gear production but a practical realization of the fundamental principles upon which all methods of cutting gear teeth by generating processes have been founded. The process is one of pure generation and the forged gear teeth are stronger, tougher, harder, and more accurately formed than are the gear teeth produced by any machine-cutting process and the finished gearing is produced at much lower unit cost and with far greater economy in the use of materials. The development in its effects upon methods of gear production is revolutionary in character and appears destined to displace machine-cutting methods in large-scale production of gearing, where low production costs, accuracy of tooth profile, marked wear-resisting qualities, and high transmission efficiency are considerations. For these reasons, a critical analysis of this variety of gearing is highly desirable and preferably should be made after a thorough study of the other methods of producing high-quality gearing.

Forging the gear teeth by synchronized rolling of heat-softened gear blanks under heavy pressure against revolving, hardened die rolls, the one and only operation in building up the gear teeth, insures a smooth, unblemished tooth surface for rolled gears, free from any of the face irregularities that cannot be entirely avoided when two such unrelated operations as the rotation of the gear blank and the advance of the cutting tool in the cutting of gear teeth by generating machines have to be performed at the same time. Furthermore, the heavy pressure under which the softened gear-blank metal is worked into teeth during the rolling process effects the marked increase in the strength of rolled gearing by bringing about a rearrangement and modification of the metal structure of the gear teeth.

The plastic metal during the gradual molding of the teeth is subjected to a thorough and powerful kneading process which

pressure of 10 to 20 tons, into engagement with the semiplastic gear blank, progressively displacing the metal and gradually building up the molded gear teeth. The work and die are kept in this rotational contact until the teeth are fully formed, the die roll meanwhile advancing to full mesh. The heated gear blank has in the meantime cooled to below its critical temperature, so that the formation of forging scale has also ceased. The die roll is then withdrawn and the rolled gear removed to cool.

As the rotary speeds of the die roll and gear blank are positively synchronized at the speeds of their engaging pitch surfaces, through the heavy timing gears, there is no driving action between the die roll and gear blank, and the teeth of the die roll are constrained to enter the blank on radial lines and in the same relative position on each successive revolution. The advancement of the die roll, with the accompanying displacement of gear-blank metal, is slight per revolution, so that teeth on the gear blank are molded gradually and without strain. Throughout the process of rolling, the temperature of the die roll does not rise above that which is bearable to the hand, being kept cool by a stream of water directed against its face at the point farthest from that of its contact with the heated blank.

When gears of extreme accuracy of tooth structure are required, the work and die-roll shafts are made to reverse their direction of rotation frequently during the greater part of the process of building up the teeth. This reversal is performed at a speed of about 150 r.p.m. and it has the effect of balancing the displacement of metal on either side of the teeth, so forming a perfectly symmetrical tooth structure.

During the rolling operation, the stream of cooling water directed against the die roll serves to wash free any forging scale which may tend to cling to the die-roll teeth and is also instrumental in ridding the gear blank of the scale as rapidly as it is formed. The cool, wetted die-roll teeth coming in contact with the hot gear blank accentuates its rate of shrinkage, loosening the forging scale as it forms. The speed of blank rotation then throws the scale free of the machine, leaving the surfaces of the rolled teeth entirely free of clinging scale. The rolling process is continued until the gear teeth are fully formed.

The finishing operations of boring and facing the gear hubs and the dressing of the shrouding formed by the molding of the gear teeth are performed on suitable automatic machines of

standard type, with the rolled gear blanks chucked on their accurately finished pitch surfaces. This method of mounting assures a precise axial alignment and the concentricity of the gears, which are by virtue of the method by which they were produced perfectly balanced.

The shrouding tying the formed teeth and gear body, or web, into an integral unit is due to the use of die teeth somewhat shorter than the face width of the gear blanks and can be entirely cut away, leaving gears of the usual form, or may be retained in whole or in part to add strength to the gear. In the case of a pair of meshing gears, a full shroud or a part shroud can be retained on the member which is customarily the weaker of the two, making it the equal or superior of the other in strength. Or, shrouds on both members can be so proportioned as to develop the maximum or the most effective strength of the gear combination.

PRODUCTION ECONOMIES

As the process of gear rolling is essentially one for quantity production, the resulting economies are probably of even greater importance than such vital considerations as accuracy of tooth formation, resistance to wear, hardness, toughness, and strength of gear teeth, and in this connection the rolling process shows a number of marked advantages. Important savings are realized in material, labor, and equipment costs and also by the much smaller space required for the accommodation of the plant needed for a given production output of gears.

The teeth being formed on the gear blanks by a molding process, rather than by the removal of any metal, as in all gear-cutting processes, the gear blanks can be so proportioned that only a minimum amount of metal is trimmed away in finishing the gear hubs and the ends of the gear teeth. This enables a saving to be made of from 20 to 40 per cent in the weight of the rough blank.

The crew for operating a rolling machine and its supply furnace for heating the gear blanks consists of only two men, an operator and a helper, who under intelligent direction from a competent foreman may be recruited from the class of intelligent laborers. Such a crew can easily attain and maintain an average hourly output of 90 rolled gears per machine. Compared to the labor expense in simply cutting finished teeth by the most

economical process of machining, at a rate of 90 gears per hour, the saving by the rolling method amounts to between 90 and 97 per cent. Finishing up the gears somewhat cuts down the superiority of the rolling method, but the saving is still very marked, being from 75 to 80 per cent, or even more, depending upon the size and type of gearing produced.

The large output of a gear rolling machine, a 10-in. gear with 2-in. face consuming only a trifle over 14 sec. in the rolling operation, results in considerably fewer of these machines being required for a given rate of production than gear-cutting machines for the same gear production. Consequently, a substantial saving is effected in necessary equipment investment and a considerable saving in floor space. As for the investment saving, the case of a shop having a daily production capacity of 700 ring-bevel gears, 700 bevel pinions, 700 miter-bevel gears, and 1,400 spur gears will prove typical.

The gear rolling machines and heating furnaces for such an output would cost 80 to 90 per cent less than the equivalent equipment in generating machines and machine tools. In the finishing-up operations, *i.e.*, the boring and facing of hubs, ., the additional equipment cost favors the machining process some extent, but in the over-all equipment cost, that for all chines entailed, the rolling process is the more economical by ne 60 or 70 per cent.

DESIGN OF ROLLED GEARING

The question of gear design in rolled gearing concerns itself chiefly with the relatively simple matter of suitable die-roll constructions and proportions, in which the straight-line profile teeth of the die rolls make entirely feasible the commercial production of varieties of gears which are beyond the scope of practical gear cutting. This is especially true in the line of bevel gears and centers attention first upon the die rolls for this class of gearing, in which strict adherence to the involute system of ring is rarely attempted.

There are two general varieties of die rolls for forging bevel rs, the flat bevel die roll and the crown die roll. The former nearly invariably employed for rolling bevel gears of so-called ndard pitches. The reason for this is that the radius of the wn die roll is governed by the cone distance of the gear to rolled, when the gear blank is at its critical temperature, and

this distance is rarely an even multiple of the circular pitch of the die roll. Consequently, it becomes necessary to decrease the center angle of the die roll until its effective cone distance equals that of the rolled gear blank at its critical temperature, thus providing for a full complement of whole teeth for the die roll. The circumference of a circle having a radius equal to the cone distance of the gear to be rolled divided by the circular

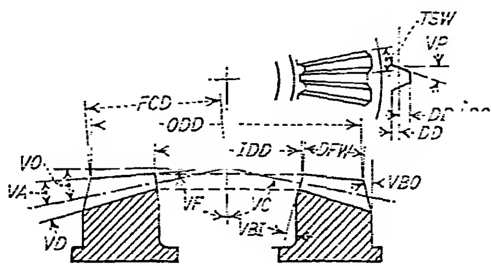


Fig. 135.—Bevel die roll.

pitch of the gear gives the number of teeth for a crown die roll. In the case of a bevel die roll, the quotient is the number of die-roll teeth plus some fraction. To ascertain the pitch diameter of the flat bevel die roll, the fraction is dropped and the product of the whole number multiplied by the coefficient of expansion is divided by the diametral pitch. The center angle of the flat bevel die roll is then readily computed, the necessary offset to the die-roll shaft, etc.

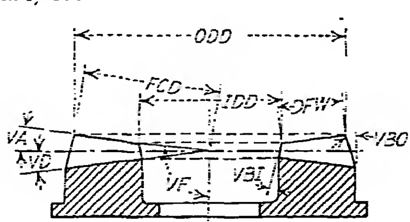


Fig. 136.—Crown die roll.

The use of the flat bevel roll is required when it is necessary to produce a rolled gear of established pitch and, consequently, of fixed diameter, but it is quite conceivable that many occasions might arise where gear proportions slightly smaller or larger might prove much more desirable than proportions established definitely by a standard of pitch. That is, gearing controlled by considerations of available space or by diameter

affords a wider scope than a system in which the diameters of the gears are fixed by the pitch. In any production process, this situation would entail some modification in pitch, but in the gear rolling system this is accomplished with the utmost precision by the use of a crown die roll having an effective diameter equal to double the cone distance of the gearing required, when at its critical temperature. In the case of any gear-cutting process, on the other hand, the use of special and costly cutting tools which could not be ground or reproduced with the same precision would be required. The greater strength of rolled gearing, furthermore, affords somewhat greater leeway in the matter of pitch than may be taken with cut gearing.

REDRESSING DIE ROLLS

The redressing, or grinding, of bevel-gear die rolls, an operation which may be necessary after rolling from 500 to 1,000 gears, is

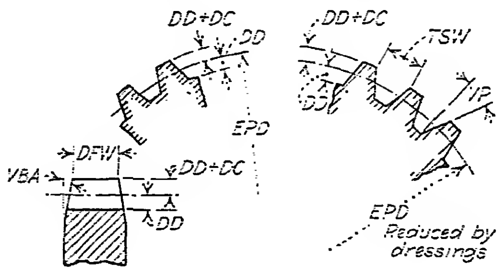


FIG. 137.—Spur-type die roll.

a simple undertaking, as all proportions of die-roll teeth are kept constant, but in the case of die rolls for spur gears the situation is somewhat different. The spur die roll is in the form of a gear with straight-line profile teeth, which if redressed will reduce to some extent the diameter of the die roll and consequently the circular pitch of its teeth. Although this is true, it is possible to redress these die rolls until there is quite a measurable reduction in their pitch diameters without destroying the accuracy and smooth running qualities of the gears they can roll.

This peculiarity is made possible by the positive driving of the functioning shafts through heavy timing gears. As these timing gears control the angular advance of the die rolls and of the gear blanks, the effect of a variation in the respective diameters of die and blank, within reasonable limits, is the introduction of a slight creep between the die and blank as they are brought into

and run in synchronized contact, the width of the tooth spaces of the die roll and not the thickness of the die-roll teeth governing the thickness of the teeth rolled on the plastic gear blanks. Consequently, as the radial spacing of the formed teeth is controlled and kept constant by the timing gears, it is essential that the die-tooth spaces be maintained constant. If this is done, a slight reduction in the thickness of the die-roll teeth, due to the reduction in die-roll diameter through tooth redressing, will not appreciably modify the profile curvature of the rolled teeth and will not affect the accuracy of tooth spacing on the rolled gear. By maintaining constant the tooth spaces of spur-gear die rolls, they can be redressed several times before they should be discarded, making the life of such dies measurable by a production of several thousand gears.

HELICAL AND HERRINGBONE DIE ROLLS

The simple straight-line profile tooth adopted for die rolls, being equally suitable for axial and helical, or spiral, arrangements

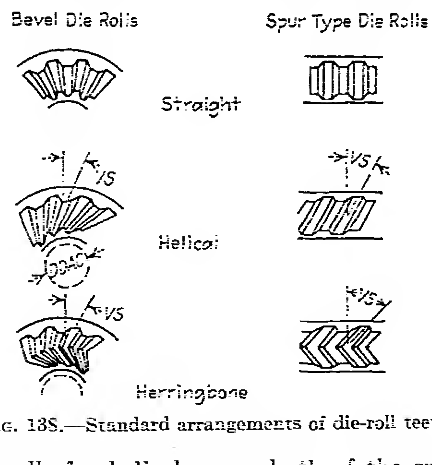


FIG. 138.—Standard arrangements of die-roll teeth.

of teeth, die rolls for helical gears, both of the spur and bevel variety, can be made and kept in condition with almost the same ease as similar die rolls with straight axial teeth. This enables helical gearing to be rolled as cheaply as gearing with ordinary teeth. By making the die rolls in concentric sections, separable for tooth dressing, but otherwise keyed into an integral unit, herringbone varieties of either spur or bevel gearing with rigid binding tooth prows can be rolled just as

cheaply. Herringbone-bevel gears, which exemplify the wide scope of rolled gearing, cannot be produced by any practical process of gear cutting. These gears will doubtless prove of even greater commercial value than the herringbone-gear type of spur

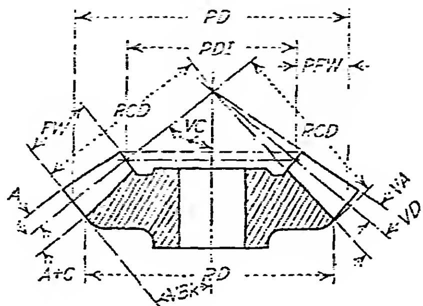


FIG. 139.—Detail rolled bevel gear.

gear, as they will eliminate the objectionable sidethrust in most installations of bevel gearing.

DIE-ROLL FORMULAS AND DESIGN

NOMENCLATURE FOR BEVEL GEARING

(See Figs. 135, 136, 138, and 139)

Die Roll

EPD = effective pitch diameter
N = number of teeth
VC = center angle
VF = face angle
DFW = die-face width
VA = addendum angle
VD = dedendum angle
ODD = outer diameter (die)
IDD = inner diameter (die)
DC = die clearance
DD = die dedendum
VP = pressure angle
VBO = bevel angle (outer)
VBI = bevel angle (inner)
TSW = tooth-space width
NTS = normal tooth space
FCD = face-cone distance
VS = spiral angle
DAC = diameter apex circle, (die)
VO offset angle
PC parting-circle diameter (herringbone gears)
PTS tooth-space width (normal) parting circle
ZE coefficient of expansion = (1 ÷ temperature range) 0.00000672

Rolled Gear

DP = diametral pitch
CP = circular pitch
PD = pitch diameter (outer)
PDI = pitch diameter (inner)
FW = face width
PFW = projected face width
A = addendum
C = clearance
VC = center angle
VBk = back angle
VA = addendum angle
VD = dedendum angle
VP = pressure angle
PCD = pitch-cone distance
RD = root diameter
DAC = diameter apex circle
RCD = root-cone distance
VS = spiral angle
PC = parting-circle diameter

Formulas for Bevel-gear Die Rolls

$$DD + DC = \frac{ODD(A + C)}{PD} \tag{125}$$

$$DD = \frac{ODD \times A}{PD} \tag{126}$$

$$DFW = ZE \times FW + 0.125 \tag{127}$$

$$TSW = 2FCD \times \sin \frac{180 \text{ deg.}}{N} \tag{128}$$

VBO and VBI are arbitrary within reasonable limits.

$$N + \text{fraction} = \frac{6.2832PCD}{CP} \quad \text{(bevel die rolls)} \tag{129}$$

$$N = \frac{6.2832PCD}{CP} \quad \text{(crown die rolls)} \tag{129a}$$

N = largest whole number.

$$EPD = \frac{ZE \times N}{DP} \quad \text{(bevel die rolls)} \tag{130}$$

$$= 6.2832PCD \times ZE \quad \text{(crown die rolls)} \tag{130a}$$

$$\sin VC = \frac{ZE \times N}{PCD \times DP} \quad \text{(bevel die rolls)} \tag{131}$$

$$VC = 90 \text{ deg.} \quad \text{(crown die rolls)}$$

$$VO = 90 \text{ deg.} - VC \tag{132}$$

$$VF = VC + VA \quad \text{(bevel die rolls)} \tag{133}$$

$$= 90 \text{ deg.} + VA \quad \text{(crown die rolls)} \tag{133a}$$

$$RD = PD - 2(A + C) \sin VBk \tag{134}$$

$$RCD = \frac{0.5RD}{\sin (VC - VD)} \tag{135}$$

$$FCD = ZE \times RCD + 0.0625 \quad \text{(bevel die rolls)} \tag{136}$$

$$= \frac{0.5ODD}{\cos VA} \quad \text{(crown die rolls)} \tag{136a}$$

$$ODD = 2ZE \times \cos (90 \text{ deg.} - VA) \tag{137}$$

$$IDD = ODD - 2DFW \times \cos (90 \text{ deg.} - VF) \quad \text{(bevel die rolls)} \tag{138}$$

$$= ODD - 2DFW \times \cos VA \quad \text{(crown die rolls)} \tag{138a}$$

$$DAC = PD \times \sin VS \quad \text{(helical bevels)} \tag{139}$$

$$DDAC = ZE \times DAC \quad \text{(helical and herringbone bevels)} \tag{139a}$$

$$NTS = TSW \times \cos VS \quad \text{(helical and herringbone bevels)} \tag{140}$$

$$PC = ZE\sqrt{(PD)^2 - (PD + PDI)PFW} \tag{141}$$

$$PTS = \frac{NTS \times PC}{ODD} \qquad (142)$$

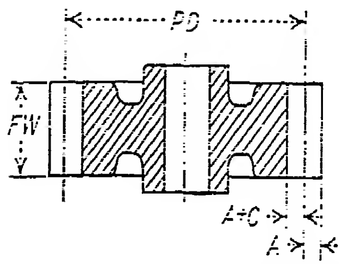


FIG. 140.—Detail rolled spur-type gear.

ADDITIONAL NOMENCLATURE FOR SPUR-TYPE GEARING
(See Figs. 137, 138, and 140)

VBA = bevel angle, edge of spur-type die rolls
GA = gear advance (helical and herringbone spur gearing)
ZE = coefficient of expansion = (1 + temperature range) 0.00000672

Formulas for Spur-type Die Rolls

$$N = 24DP + 1 \qquad (143)$$

$$EPD = \frac{ZE \times N}{DP} \qquad (130)$$

$$DFW = ZE \times FW + 0.125 \qquad (127)$$

$$DD = 0.3183CP \times ZE \qquad (126a)$$

$$DD + DC = 0.3683CP \times ZE \qquad (125a)$$

$$GA = 1.1 \times CP \quad \text{(helical and herringbone gears)} \qquad (144)$$

$$\tan VP = \frac{GA}{FW} \quad \text{(helical gears)} \qquad (145)$$

$$= \frac{2GA}{FW} \quad \text{(herringbone gears)} \qquad (145a)$$

APPENDIX

WORKING FORMULAS AND TABLES IN GEAR DESIGN

APPENDIX

WORKING FORMULAS AND TABLES IN GEAR DESIGN

NOMENCLATURE FOR SYMBOLS[1]

Lineal Dimensions

Symbol

A = Addendum
A' = Special dimension
AD = Apex distance
AF = Active face

B or BL = Backlash
BCD = Base-circle diameter
BCR = Base-circle radius
BD = Bore diameter
BE = Test dimension
BtD = Bottom diameter (internal gears)

CA = Chordal addendum
Corrected addendum
CB = Crown backing (bevel gears)
CD = Center distance
CP = Circular pitch
CTh or CTT = Circular tooth thickness

D = Dedendum
DAC = Diameter apex circle (rolled bevel gears)
DC = Die clearance (rolled gears)
DD = Die-roll diameter (rolled gears)
$DDAC$ = Diameter die-roll apex circle (rolled bevel gears)
DFW = Die-roll face width (rolled gears)
DI = Diameter increment (bevel gears)
DP = Diametral pitch
DPn = Diametral pitch, normal (helical gearing)
DPs = Diametral pitch, inner (bevel gears)

ER = Edge round (worm gears)
EPD = Effective pitch diameter, die roll (rolled bevel gears)

[1] Symbols for pinion members in gear drives are customarily differentiated from corresponding symbols for the larger gear members by the use of small, instead of capital, letters.

Symbol

FCD	Face-cone distance, die roll (rolled bevel gearing)
FL	Face length, worm (worm gearing)
FW	Face width
GA	Gear advance (rolled helical and herringbone gears)
GD	Groove depth (herringbone gears)
GW	Groove width (herringbone gears)
HA	Height chordal tooth-pitch arc
HD	Hub diameter
HE	Hub extension
HL	Hub length
ID	Inner diameter (internal gears)
IDD	Inner diameter, die roll (rolled bevel gearing)
KW	Keyway width
KWD	Keyway depth
L	Lead, normal helix (spiral gears)
LH	Lead of hob
LP	Linear pitch (worm gearing)
LW	Length of worm
MD =	Mounting distance (bevel gears)
NDP =	Normal diametral pitch (spiral gears)
NIP =	Normal involute pitch
NP =	Normal pitch (circular)
NTS =	Normal tooth space, die roll (rolled
NTT =	Normal tooth thickness
OD =	Outer diameter
ODD =	Outer diameter, die roll (rolled gearing)
OR =	Outer radius
OS =	Offset (skew-bevel gears)
PC =	Parting circle, herringbone-bevel gears (rolled gearing)
PCC =	Pitch-circle circumference
PCD =	Pitch-cone distance (rolled bevel gears)
PD =	Pitch diameter
PDe =	Equivalent pitch diameter (skew bevel gears)
PDI =	Pitch diameter, inner (rolled bevel gears)
PFW =	Projected face width (rolled bevel gears)
PR =	Pitch radius
PTDe =	Pin-test diameter (even number of teeth)
PTDo =	Pin-test diameter (odd number of teeth)
PTS =	Tooth-space width, die roll (rolled bevel gearing)

Symbol

RCD = Root-cone distance (rolled bevel gears)
RD = Root diameter
RL = Radial-leverage gear tooth
RR = Root radius
RWF = Radius wheel face (worm gears)
RWR = Radius wheel rim (worm gears)

TD = Throat diameter (worm gears)
TFW = Tooth-flank width
TL = Top land
TP = Test-pin diameter
TPI = Distance, pin center to chordal plane
TSW = Tooth-space width, spur die roll (rolled gearing)

WD = Whole depth, tooth

X = Pitch-line backing (bevel gears)

Angle, Velocity and Miscellaneous Measures

VA = Angle of axes
VAS = Addendum section angle
AA = Arc of action
AP = Arc of approach
AR = Arc of recession
AUS = Allowable unit stress

VB = Bottom (cutting) angle (bevel gears)
VBA = Bevel angle, spur-forging die roll
VBk = Back angle (bevel gears)
VBI = Bevel angle, inner (bevel-forging die roll)
VBO = Bevel angle, outer (bevel-forging die roll)

VC = Center angle (bevel gears)
VCA = Carrying arm, rotary speed (epicyclic gear trains)
C = Character of load factor (Table 25*d*)
CA = Carrying arm (epicyclic gear trains)
CD = Check dimension—testing
CGD = Compound gear, driven member (epicyclic gear trains)
CGR = Compound gear, driving member (epicyclic gear trains

VD = Decrement angle (bevel gears)
DMV = Driven-member velocity (epicyclic gear trains)
DG = Driven gear (epicyclic gear trains)
DM = Driven member (epicyclic gear trains)
DTC = Duration tooth contact

E = Efficiency

Symbol

VF = Face angle (bevel gears)
f = Coefficient of friction

GR = Gear ratio

VH = Helix angle
VHS = Angle of hob setting
VHT = Hob-thread angle
HP = Horsepower

VI = Increment angle (bevel gears)
VIG = Velocity internal gear (epicyclic gear trains)
IG = Intermediate gear, size (epicyclic gear trains)

K = Velocity factor (Table 28)

VL = Lead angle, worm
LC = Load factor (62.5–100)
LC′ = Load factor (175–250)
LC″ = Load factor (Table 12)

VNP = Normal-pressure angle (worm
N = Number of teeth
NTC = Number of teeth in contact

VO = Angle of offset (skew-bevel gears)
Offset angle, die roll (rolled bevel gears)
VOE = Test angle
OE = Test dimension

VP = Pressure angle
VPi = Pitch angle (spiral-bevel gears)
VPn = Pressure angle, normal (helical and herringbone gears)
PLV = Pitch-line velocity
VPG = Planet-gear velocity (epicyclic gear trains)
P = Pitch point
Wear factor (herringbone gears)
PG = Planet gear, size (epicyclic gear trains)

Q = Installation factor (Table 25*c*)

RPM Revolutions per minute
RMV Driving-member velocity (epicyclic gear trains)
RM Driving member (epicyclic gear trains)

VS Spiral angle
VSG Sun-gear velocity (epicyclic gear trains)
S Allowable static stress

Symbol

SWS	Safe working stress
SG, $SG1$, $SG2$	Sun gears (epicyclic gear trains)
SM	Stationary member (epicyclic gear trains)
VTL	Top-land angle
T	Total tooth pressure
T_1	Tangential force
T_2	Axial force
TB	Tooth bearing
TL	Tooth load
W	= Transmitted load
Y	Form factor (Table 10)
	Tooth proportion factor
Z	Section modulus
ZE	Coefficient of expansion (rolled gearing)

$$OR = \sqrt{(BCR)^2 + (CD \times \sin VP)^2} \tag{1}$$

$$or = \sqrt{(bcr)^2 + (CD \times \sin VP)^2} \tag{1'}$$

Limiting proximity of pinion to involute rack:

$$A' = bcr \times \cos VP = pr \times \cos^2 VP \tag{2}$$

Tooth-pressure components:

$$T_1 = T \times \cos VP \tag{3}$$

$$T_2 = T_1 \times \tan VP \tag{3'}$$

TABLE 1.—RANGE OF STANDARD (14½-DEG.) INVOLUTE MILLING CUTTERS

Number	Range
1	135–rack
1½	80–134
2	55– 79
2½	42– 54
3	35– 41
3½	30– 34
4	26– 29
4½	23– 25
5	21– 22
5½	19– 20
6	17– 18
6½	15– 16
7	14
7½	13
8	12

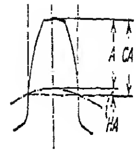


TABLE 2.—CHORDAL THICKNESSES AND (CORRECTED) ADDENDA OF GEAR-TEETH DIAMETRAL PITCH

Number of teeth	1DP		1½DP		2DP		2½DP	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.
8	1.5607	1.0769	1.0405	0.7179	0.7504	0.5385	0.6243	0.4308
9	1.5628	1.0654	1.0419	.7123	.7514	.5342	.6251	.4273
10	1.5643	1.0616	1.0429	.7077	.7521	.5308	.6257	.4246
11	1.5654	1.0559	1.0436	.7039	.7527	.5279	.6261	.4224
12	1.5663	1.0514	1.0442	.7009	.7531	.5257	.6265	.4206
14	1.5675	1.0440	1.0450	.6960	.7537	.5220	.6270	.4176
17	1.5686	1.0362	1.0457	.6908	.7543	.5181	.6274	.4145
21	1.5694	1.0294	1.0463	.6863	.7547	.5147	.6277	.4118
26	1.5698	1.0237	1.0465	.6825	.7549	.5118	.6279	.4095
35	1.5702	1.0176	1.0468	.6784	.7551	.5088	.6281	.4070
55	1.5706	1.0112	1.0471	.6741	.7553	.5056	.6282	.4045
135	1.5707	1.0047	1.0471	.6698	.7853	.5023	.6283	.4019

Number of teeth	3DP		3½DP		4DP		5DP	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.
8	0.5202	0.3589	0.4459	0.3077	0.3902	0.2692	0.3121	0.2154
9	.5209	.3561	.4465	.3052	.3907	.2671	.3126	.2137
10	.5214	.3533	.4469	.3033	.3911	.2654	.3129	.2123
11	.5218	.3519	.4473	.3017	.3913	.2640	.3131	.2112
12	.5221	.3505	.4475	.3004	.3916	.2628	.3133	.2103
14	.5225	.3480	.4479	.2983	.3919	.2610	.3135	.2088
17	.5228	.3454	.4482	.2961	.3921	.2590	.3137	.2072
21	.5231	.3431	.4485	.2941	.3923	.2573	.3139	.2059
26	.5233	.3412	.4485	.2925	.3925	.2559	.3140	.2047
35	.5234	.3392	.4486	.2907	.3926	.2544	.3140	.2035
55	.5235	.3371	.4487	.2889	.3927	.2525	.3141	.2022
135	.5236	.3349	.4488	.2871	.3927	.2512	.3141	.2009

Number of teeth	6DP		7DP		8DP		9DP	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.
8	0.2601	0.1795	0.2230	0.1538	0.1951	0.1346	0.1734	0.1197
9	.2605	.1781	.2233	.1526	.1954	.1336	.1736	.1187
10	.2607	.1769	.2235	.1517	.1955	.1327	.1738	.1180
11	.2609	.1760	.2236	.1508	.1957	.1320	.1739	.1173
12	.2610	.1752	.2238	.1502	.1958	.1314	.1740	.1168
14	.2612	.1740	.2239	.1491	.1959	.1305	.1742	.1160
17	.2614	.1727	.2241	.1480	.1961	.1295	.1743	.1151
21	.2616	.1716	.2242	.1471	.1962	.1287	.1744	.1144
26	.2616	.1706	.2243	.1462	.1962	.1280	.1744	.1137
35	.2617	.1696	.2243	.1454	.1963	.1272	.1745	.1131
55	.2618	.1685	.2244	.1445	.1963	.1264	.1745	.1124
135	.2618	.1675	.2244	.1435	.1963	.1256	.1745	.1116

TOOTH FORMS—SECTION I

Maximum outer circle for mating gears:

TABLE 2.—CHORDAL THICKNESSES AND (CORRECTED) ADDENDA OF GEAR-TEETH DIAMETRAL PITCH.—(*Concluded*)

Number of teeth	10*DP*		11*DP*		12*DP*		13*DP*	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.
8	0.1561	0.1077	0.1419	0.0979	0.1301	0.0897	0.1201	0.0828
9	.1563	.1068	.1421	.0971	.1302	.0890	.1202	.0822
10	.1564	.1061	.1422	.0965	.1304	.0885	.1203	.0816
11	.1565	.1056	.1423	.0960	.1305	.0880	.1204	.0812
12	.1566	.1051	.1424	.0956	.1305	.0876	.1205	.0809
14	.1567	.1044	.1425	.0949	.1306	.0870	.1206	.0803
17	.1569	.1036	.1426	.0942	.1307	.0863	.1207	.0797
21	.1569	.1029	.1427	.0936	.1308	.0858	.1207	.0792
26	.1570	.1024	.1427	.0931	.1308	.0853	.1207	.0787
35	.1570	.1018	.1427	.0925	.1309	.0848	.1208	.0782
55	.1571	.1011	.1428	.0919	.1309	.0843	.1208	.0777
135	.1571	.1005	.1428	.0913	.1309	.0837	.1208	.0772
	14*DP*		15*DP*		16*DP*		17*DP*	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.
8	0.1115	0.0769	0.1040	0.0718	0.0975	0.0673	0.0918	0.0633
9	.1116	.0763	.1042	.0712	.0977	.0669	.0919	.0628
10	.1117	.0758	.1043	.0709	.0978	.0664	.0920	.0624
11	.1118	.0754	.1044	.0704	.0978	.0659	.0921	.0621
12	.1119	.0751	.1044	.0701	.0979	.0657	.0921	.0618
14	.1119	.0746	.1045	.0696	.0980	.0652	.0922	.0614
17	.1120	.0740	.1046	.0691	.0980	.0648	.0923	.0609
21	.1121	.0735	.1046	.0686	.0981	.0643	.0923	.0605
26	.1121	.0731	.1046	.0682	.0981	.0640	.0923	.0602
35	.1122	.0727	.1047	.0678	.0981	.0636	.0924	.0598
55	.1122	.0722	.1047	.0674	.0981	.0632	.0924	.0595
135	.1122	.0718	.1047	.0670	.0981	.0628	.0924	.0591
	18*DP*		19*DP*		20*DP*		24*DP*	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.
8	0.0867	0.0598	0.0821	0.0567	0.0780	0.0538	0.0650	0.0448
9	.0868	.0593	.0822	.0562	.0781	.0534	.0651	.0445
10	.0869	.0589	.0823	.0558	.0782	.0530	.0651	.0443
11	.0869	.0586	.0824	.0555	.0783	.0528	.0652	[illegible]
12	.0870	.0584	.0824	.0553	.0784	.0525	.0653	.0437
14	.0871	.0580	.0825	.0549	.0784	.0522	.0653	.0435
17	.0871	.0575	.0826	.0545	.0784	.0518	.0653	.0432
21	.0872	.0572	.0826	.0542	.0785	.0514	.0654	.0429
26	.0872	.0568	.0826	.0538	.0785	.0511	.0654	.0426
35	.0872	.0565	.0826	.0535	.0785	.0508	.0654	.0424
55	.0873	.0562	.0827	.0532	.0785	.0505	.0654	.0421
135	.0873	.0558	.0827	.0528	.0785	.0502	.0654	.0419

TABLE 3.—CHORDAL THICKNESSES AND CORRECTED ADDENDA OF GEAR-TEETH CIRCULAR PITCH

Number of teeth	5/8-in. *CP*		3/4-in. *CP*		7/8-in. *CP*		1-in. *CP*	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.
8	0.3105	0.2142	0.3725	0.2570	0.4347	0.2997	0.4965	0.3426
9	.3109	.2125	.3730	.2550	.4353	.2976	.4974	.3400
10	.3112	.2112	.3734	.2534	.4357	.2957	.4978	.3378
11	.3114	.2100	.3737	.2520	.4360	.2941	.4982	.3360
12	.3116	.2091	.3739	.2510	.4363	.2938	.4986	.3346
14	.3118	.2077	.3741	.2492	.4366	.2908	.4988	.3322
17	.3120	.2061	.3744	.2473	.4369	.2886	.4992	.3298
21	.3122	.2048	.3746	.2457	.4371	.2868	.4994	.3276
26	.3123	.2036	.3748	.2443	.4372	.2851	.4997	.3258
35	.3124	.2024	.3748	.2429	.4373	.2833	.4999	.3238
55	.3124	.2011	.3748	.2414	.4374	.2816	.4999	.3218
135	.3124	.1999	.3748	.2398	.4374	.2798	.4999	.3198
	1 1/4-in. *CP*		1 1/2-in. *CP*		1 3/4-in. *CP*		2-in. *CP*	
	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.	Thickness, in.	Addendum, in.
8	0.6210	0.4284	0.7450	0.5140	0.8694	0.5994	0.9936	0.6852
9	.6218	.4250	.7460	.5100	.8706	.5952	.9948	.6800
10	.6224	.4224	.7468	.5068	.8714	.5914	.9956	.6756
11	.6228	.4200	.7474	.5040	.8720	.5882	.9964	.6720
12	.6232	.4182	.7478	.5020	.8726	.5876	.9972	.6692
14	.6236	.4154	.7482	.4984	.8732	.5816	.9976	.6644
17	.6240	.4122	.7488	.4946	.8738	.5772	.9984	.6596
21	.6244	.4096	.7492	.4914	.8742	.5736	.9988	.6552
26	.6246	.4072	.7496	.4886	.8744	.5702	.9994	.6516
35	.6248	.4048	.7498	.4858	.8746	.5666	.9998	.6476
55	.6250	.4022	.7499	.4828	.8748	.5632	.9999	.6436
135	.6250	.3998	.7499	.4796	.8748	.5596	.9999	.6396

TABLE 4.—DIAMETRAL PITCH

Relation between diametral and circular pitches, with corresponding tooth dimensions for standard 14½-deg. teeth (composite system)

Diametral pitch	Circular pitch, inches	Thickness of tooth on pitch line, inches	Whole depth, inches	Dedendum, inches	Addendum, inches
½	6.2832	3.1416	4.3142	2.3142	2.0000
¾	4.1888	2.0944	2.8761	1.5728	1.3333
1	3.1416	1.5708	2.1571	1.1571	1.0000
1¼	2.5133	1.2566	1.7257	.9257	.8000
1½	2.0944	1.0472	1.4381	.7714	.6666
1¾	1.7952	.8976	1.2326	.6612	.5714
2	1.5708	.7854	1.0785	.5785	.5000
2¼	1.3963	.6981	.9587	.5143	.4444
2½	1.2566	.6283	.8628	.4628	.4000
2¾	1.1424	.5712	.7844	.4208	.3636
3	1.0472	.5236	.7190	.3857	.3333
3½	.8976	.4488	.6163	.3306	.2857
4	.7854	.3927	.5393	.2893	.2500
5	.6283	.3142	.4314	.2314	.2000
6	.5236	.2618	.3595	.1928	.1666
7	.4488	.2244	.3081	.1653	.1429
8	.3927	.1963	.2696	.1446	.1250
9	.3491	.1745	.2397	.1286	.1111
10	.3142	.1571	.2157	.1157	.1000
11	.2856	.1428	.1961	.1052	.0909
12	.2618	.1309	.1798	.0964	.0833
13	.2417	.1208	.1659	.0890	.0769
14	.2244	.1122	.1541	.0826	.0714
15	.2094	.1047	.1438	.0771	.0666
16	.1963	.0982	.1348	.0723	.0625
17	.1848	.0924	.1269	.0681	.0588
18	.1745	.0873	.1198	.0643	.0555
19	.1653	.0827	.1135	.0609	.0526
20	.1571	.0785	.1079	.0579	.0500
22	.1428	.0714	.0980	.0526	.0455
24	.1309	.0654	.0898	.0482	.0417
26	.1208	.0604	.0829	.0445	.0385
28	.1122	.0561	.0770	.0413	.0357
30	.1047	.0524	.0719	.0386	.0333
32	.0982	.0491	.0674	.0362	.0312
34	.0924	.0462	.0634	.0340	.0294
36	.0873	.0436	.0599	.0321	.0278
38	.0827	.0413	.0568	.0304	.0263
40	.0785	.0393	.0539	.0289	.0250
42	.0748	.0374	.0514	.0275	.0238
44	.0714	.0357	.0490	.0263	.0227
46	.0683	.0341	.0469	.0252	.0217
48	.0654	.0327	.0449	.0241	.0208
50	.0628	.0314	.0431	.0231	.0200
56	.0561	.0280	.0385	.0207	.0178
60	.0524	.0262	.0360	.0193	.0166

TABLE 5.—CIRCULAR PITCH

Relation between circular and diametral pitches, with corresponding tooth dimensions for standard 14½-deg. teeth (composite system)

Circular pitch, inches	Diametral pitch	Thickness of tooth on pitch line, inches	Whole depth, inches	Dedendum, inches	Addendum, inches
6	0.5236	3.0000	4.1196	2.2098	1.9098
5	.6283	2.5000	3.4330	1.8415	1.5915
4	.7854	2.0000	2.7464	1.4732	1.2732
3½	.8976	1.7500	2.4031	1.2890	1.1140
3	1.0472	1.5000	2.0598	1.1049	.9550
2¾	1.1424	1.3750	1.8882	1.0028	.8754
2½	1.2566	1.2500	1.7165	.9207	.7958
2¼	1.3963	1.1250	1.5449	.8287	.7162
2	1.5708	1.0000	1.3732	.7366	.6366
1⅞	1.6755	.9375	1.2874	.6906	.5968
1¾	1.7952	.8750	1.2016	.6445	.5570
1⅝	1.9333	.8125	1.1158	.5985	.5173
1½	2.0944	.7500	1.0299	.5525	.4775
1 7/16	2.1855	.7187	.9870	.5294	.4576
1⅜	2.2848	.6875	.9441	.5064	.4377
1 5/16	2.3936	.6562	.9012	.4837	.4178
1¼	2.5133	.6250	.8583	.4604	.3979
1 3/16	2.6465	.5937	.8156	.4374	.3780
1⅛	2.7925	.5625	.7724	.4143	.3581
1 1/16	2.9568	.5312	.7295	.3913	.3382
1	3.1416	.5000	.6866	.3683	.3183
15/16	3.3510	.4687	.6437	.3453	.2984
⅞	3.5904	.4375	.6007	.3223	.2785
13/16	3.8666	.4062	.5579	.2993	.2586
¾	4.1888	.3750	.5150	.2762	.2387
11/16	4.5696	.3437	.4720	.2532	.2189
⅝	5.0265	.3125	.4291	.2301	.1989
9/16	5.5851	.2812	.3862	.2071	.1790
½	6.2832	.2500	.3433	.1842	.1592
7/16	7.1808	.2187	.3003	.1611	.1393
⅖	7.8540	.2000	.2746	.1473	.1273
⅜	8.3776	.1875	.2575	.1381	.1194
⅓	9.4248	.1666	.2287	.1228	.1061
5/16	10.0531	.1562	.2146	.1151	.0995
2/7	10.9956	.1429	.1962	.1052	.0909
¼	12.5664	.1250	.1716	.0921	.0796
2/9	14.1372	.1111	.1526	.0818	.0707
⅕	15.7080	.1000	.1373	.0737	.0637
3/16	16.7552	.0937	.1287	.0690	.0592
⅙	18.8496	.0833	.1144	.0614	.0531
1/7	21.9911	.0714	.0981	.0526	.0455
⅛	25.1327	.0625	.0858	.0460	.0398
1/9	28.2743	.0555	.0763	.0409	.0354
1/10	31.4159	.0500	.0687	.0368	.0318
1/16	50.2655	.0312	.0429	.0230	.0199

TABLE 6.—PITCH DIAMETERS FOR 1-IN. CIRCULAR PITCH

Number of teeth	Pitch diameter	Number of teeth	Pitch diameter	Number of teeth	Pitch diameter	Number of teeth	Pitch diameter
8	2.550	43	13.687	78	24.828	113	35.968
9	2.870	44	14.006	79	25.146	114	36.286
10	3.183	45	14.324	80	25.465	115	36.605
11	3.501	46	14.642	81	25.783	116	36.923
12	3.820	47	14.961	82	26.101	117	37.241
13	4.138	48	15.279	83	26.420	118	37.560
14	4.456	49	15.597	84	26.738	119	37.878
15	4.775	50	15.915	85	27.056	120	38.196
16	5.093	51	16.234	86	27.375	121	38.514
17	5.411	52	16.552	87	27.693	122	38.833
18	5.730	53	16.870	88	28.011	123	39.151
19	6.048	54	17.189	89	28.330	124	39.469
20	6.366	55	17.507	90	28.648	125	39.788
21	6.684	56	17.825	91	28.966	126	40.106
22	7.003	57	18.144	92	29.284	127	40.424
23	7.321	58	18.462	93	29.603	128	40.743
24	7.639	59	18.780	94	29.921	129	41.061
25	7.958	60	19.099	95	30.239	130	41.379
26	8.276	61	19.417	96	30.558	131	41.697
27	8.594	62	19.735	97	30.876	132	42.016
28	8.913	63	20.053	98	31.194	133	42.334
29	9.231	64	20.372	99	31.513	134	42.652
30	9.549	65	20.690	100	31.831	135	42.971
31	9.868	66	21.008	101	32.148	136	43.289
32	10.186	67	21.327	102	32.468	137	43.607
33	10.504	68	21.645	103	32.785	138	43.926
34	10.822	69	21.963	104	33.103	139	44.243
35	11.141	70	22.282	105	33.421	140	44.562
36	11.459	71	22.600	106	33.740	141	44.881
37	11.777	72	22.918	107	34.058	142	45.199
38	12.096	73	23.237	108	34.376	143	45.517
39	12.414	74	23.555	109	34.695	144	45.835
40	12.732	75	23.873	110	35.013	145	46.154
41	13.051	76	24.192	111	35.331	146	46.472
42	13.369	77	24.510	112	35.650	147	46.790

TABLE 7.—TOOTH HEIGHTS IN STANDARD $14\frac{1}{2}$-DEG. COMPOSITE SYSTEM (A); $14\frac{1}{2}$-DEG. GENERATED GEAR-TOOTH SYSTEM (B); 20-DEG., FULL-DEPTH, GEAR-TOOTH SYSTEM (C); AND 20-DEG. STUB-TOOTH GEAR SYSTEM (D)

Dimension	A	B	C	D
Addendum	1.0M	1.0M	1.0M	0.8M
Dedendum	1.157M	1.157M	1.157M	1.0M
Working depth	2.0M	2.0M	2.0M	1.6M
Whole depth	2.157M	2.157M	2.157M	1.8M
Clearance	0.157M	0.157M	0.157M	0.2M

M = module.

TABLE 8.—DIMENSIONS OF 20-DEG. STUB-GEAR TEETH (Fellows Gear Shaper Company's System)

Diametral pitch	Thickness of tooth, inches	Addendum, inches	Working depth, inches	Depth of space below pitch line, inches	Clearance, inches	Whole depth of tooth, inches
4/5	0.3927	0.2000	0.4000	0.2500	0.0500	0.4500
5/7	.3142	.1429	.2858	.1786	.0357	.3214
6/8	.2618	.1250	.2500	.1562	.0312	.2812
7/9	.2244	.1111	.2222	.1389	.0278	.2500
8/10	.1963	.1000	.2000	.1250	.0250	.2250
9/11	.1745	.0909	.1818	.1136	.0227	.2045
10/12	.1571	.0833	.1667	.1041	.0208	.1875
12/14	.1309	.0714	.1429	.0893	.0179	.1607

SPEEDS AND POWERS—SECTION II

Strength of gear teeth:

$$W = SWS \times CP \times FW \times Y$$

$$W = \frac{600 \times SWS \times CP \times FW \times Y}{600 + PLV} \quad (4a)$$

$$W = \frac{1{,}200 \times SWS \times CP \times FW \times Y}{1{,}200 + PLV} \quad (4b)$$

$$W = \frac{78 \times SWS \times CP \times FW \times Y}{78 + \sqrt{PLV}} \quad (4c)$$

$$W = \left(\frac{150}{200 + PLV} + 0.25\right) SWS \times CP \times FW \times Y \quad (4d)$$

TABLE 9.—SAFE WORKING STRESS *SWS*

Material	Tensile strength, lb. per sq. in.	Safe stress, lb. per sq. in., 0 speed
Metallic:		
Cast iron	24,000	8,000
Mild steel	36,000	12,000
Bronze	36,000	12,000
Cast steel (S.A.E. 1235)	45,000	15,000
Forged steel (S.A.E. 1030)	60,000	20,000
(S.A.E. 1045)	90,000	30,000
(S.A.E. 3245)	120,000	40,000
Nonmetallic:		
Rawhide, etc.		6,000

American Gear Manufacturers' Association Recommendations Nonmetallic Gears

PLV	*SWS*	*PLV*	*SWS*	*PLV*	*SWS*
100	4,500	700	2,500	1,700	1,974
150	4,071	800	2,400	1,800	1,950
200	3,650	900	2,318	1,900	1,929
250	3,500	1,000	2,250	2,000	1,909
300	3,300	1,100	2,192	2,200	1,875
350	3,136	1,200	2,143	2,300	1,860
400	3,000	1,300	2,100	2,400	1,846
450	2,885	1,400	2,063	2,600	1,821
500	2,786	1,500	2,029	2,800	1,800
600	2,625	1,600	2,000	3,000	1,781

[1] For steady loads on single pairs of gears,
suddenly applied loads on single gears, discount.................... 25 per cent
steady loads on gear trains beyond first mesh, discount............. 40 per cent
suddenly applied loads on gear trains beyond first mesh, discount.... 50 per cent

$$W = \frac{1{,}200 \times SWS \times CP \times FW \times Y \times \cos VH}{1{,}200 + PLV} \qquad (4e)$$

$$W = \frac{78 \times SWS \times CP \times FW \times Y \times \cos VH}{78 + \sqrt{PLV}} \qquad (4f)$$

TABLE 10.—VALUES OF FORM FACTOR Y IN LEWIS FORMULAS

Number of teeth	Gear-tooth system		
	$14\frac{1}{2}$-deg. composite and generated	20-deg. full depth	20-deg. stub tooth
10	0.056	0.064	0.083
11	.061	.072	.092
12	.067	.078	.099
13	.071	.083	.103
14	.075	.088	.108
15	.078	.092	.111
16	.081	.094	.115
17	.084	.096	.117
18	.086	.098	.120
19	.088	.100	.123
20	.090	.102	.125
21	.092	.104	.127
22	.093	.105	.128
23	.094	.106	.130
24	.096	.107	.131
25	.097	.108	.133
26	.098	.110	.135
28	.100	.113	.138
30	.101	.114	.139
35	.105	.120	.143
40	.107	.124	.146
50	.110	.130	.151
60	.113	.134	.154
75	.115	.138	.158
100	.117	.142	.161
150	.119	.146	.165
Rack	.124	.154	.175

TABLE 11.—VELOCITY FACTORS

Ft. per min.	$\frac{600}{600 + PLV}$	Ft. per min.	$\frac{1,200}{1,200 + PLV}$	Ft. per min.	$\frac{78}{78 + \sqrt{PLV}}$
100	0.857	1,200	0.500	4,000	0.553
200	.750	1,400	.461	4,200	.545
300	.667	1,600	.429	4,400	.540
400	.600	1,800	.400	4,600	.535
500	.545	2,000	.375	4,800	.530
600	.500	2,200	.353	5,000	.525
700	.461	2,400	.333	5,200	.520
800	.429	2,600	.316	5,400	.515
900	.400	2,800	.300	5,600	.510
1,000	.375	3,000	.286	5,800	.506
1,100	.353	3,200	.273	6,000	.502
1,200	.333	3,400	.261	6,200	.498
1,300	.316	3,600	.250	6,400	.494
1,400	.300	3,800	.240	6,600	.490
1,500	.286	4,000	.231	6,800	.486
1,600	.273	.....	.....	7,000	.482
1,700	.261	.....	.....	7,200	.479
1,800	.250	.....	.....	7,400	.475
1,900	.240	.....	.....	7,600	.472
2,000	.231	.....	.....	7,800	.468
				8,000	.465
				8,200	.462
				8,400	.459
				8,600	.456
				8,800	.454
				9,000	.451
				9,200	.448
				9,400	.446
				9,600	.443
				9,800	.441
				10,000	.438

Horsepower formulas:

$$HP = \frac{W \times PLV}{33,000} \tag{5}$$

$$HP = \frac{SWS \times CP \times FW \times Y \times PLV}{33,000} \tag{5a}$$

$$HP = \frac{0.000095 \times SWS \times FW \times Y \times PLV}{DP} \qquad (5b)$$

Limitations in load:

$$\frac{W}{fw} = LC \times pd \qquad (6a)$$

$$\frac{W}{fw} = LC' \times \sqrt{pd} \qquad (6b)$$

$$\frac{W}{fw} = \frac{88{,}800 \times LC'' \times CP}{PLV + 32.8} \qquad (6c)$$

TABLE 12.—VALUES OF LC'' FOR USE IN FORMULA (6c)
Lubricated gear teeth

Number of pinion teeth	Speed ratio								
	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8	1:10
12	2.80	3.40	3.80	4.20	4.36	4.54	4.68	4.80	5.00
14	3.20	3.80	4.20	4.60	4.88	5.08	5.24	5.40	5.60
16	3.50	4.20	4.64	5.06	5.36	5.58	5.70	5.84	6.10
18	3.80	4.40	5.00	5.40	5.76	5.96	6.10	6.24	6.90
20	4.20	4.90	5.40	5.90	6.20	6.40	6.64	6.88	6.90
22	4.60	5.34	5.86	6.36	6.62	6.86	7.06	7.24	7.36
24	5.00	5.76	6.30	6.80	7.04	7.30	7.46	7.60	7.80
26	5.36	6.08	6.68	7.20	7.46	7.72	7.90	8.06	8.24
28	5.70	6.40	7.04	7.60	7.88	8.14	8.32	8.50	8.64
30	6.06	6.84	7.48	8.00	8.34	8.60	8.74	8.96	
32	6.40	7.28	7.92	8.40	8.80	9.04	9.24	9.40	
34	6.80	7.70	8.34	8.82	9.20	9.46			
36	7.20	8.10	8.76	9.24	9.60	9.88			
38	7.56	8.48	9.16	9.76	10.02				
40	7.90	8.84	9.56	10.28	10.44				

Duration of tooth contact:

$$DTC = \sqrt{(OR)^2 - (BCR)^2} + \sqrt{(or)^2 - (bcr)^2} - CD \times \sin VP \qquad (7)$$

Normal involute pitch:

$$NIP = \frac{3.1416 \times BCD}{N} \qquad (8)$$

Number of teeth in contact:

$$NTC = \frac{DTC}{NIP} \tag{9}$$

GEAR PROPORTIONS AND DESIGN—SECTION III

$$BD = \sqrt[3]{\frac{HP \times 80}{RPM}} \tag{10}$$

$$\text{Minimum } PD \text{ (pinion)} = BD + 2\left(KWD + D + \sqrt{\frac{0.2 \times N}{DP}}\right) \tag{11}$$

$$Z = \frac{CP \times FW \times Y \times PR}{N} \tag{12}$$

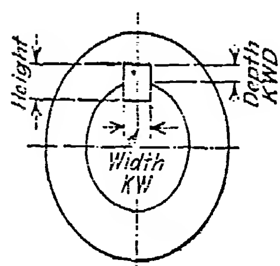


TABLE 13.—A.G.M.A. KEY AND KEYWAY STANDARDS
(Dimensions in inches)

Diameter of holes inclusive	Keyways		Keystock
	Width	Depth	
$\frac{5}{16}$ – $\frac{7}{16}$	$\frac{3}{32}$	$\frac{3}{64}$	$\frac{3}{32} \times \frac{3}{32}$
$\frac{1}{2}$ – $\frac{9}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{8} \times \frac{1}{8}$
$\frac{5}{8}$ – $\frac{7}{8}$	$\frac{3}{16}$	$\frac{3}{32}$	$\frac{3}{16} \times \frac{3}{16}$
$\frac{15}{16}$ – $1\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4} \times \frac{1}{4}$

Table 13.—A.G.M.A. Key and Keyway Standards.—*Continued*

Diameter of holes inclusive	Keyways		Keystock
	Width	Depth	
1 3/16–1 3/8	5/16	5/32	5/16 × 5/16
1 7/16–1 3/4	3/8	3/16	3/8 × 3/8
1 13/16–2 1/4	1/2	1/4	1/2 × 1/2
2 5/16–2 3/4	5/8	5/16	5/8 × 5/8
2 13/16–3 1/4	3/4	3/8	3/4 × 3/4
3 5/16–3 3/4	7/8	7/16	7/8 × 7/8
3 13/16–4 1/2	1	1/2	1 × 1
4 9/16–5 1/2	1 1/4	7/16	1 1/4 × 7/8
5 9/16–6 1/2	1 1/2	1/2	1 1/2 × 1
6 9/16–7 1/2	1 3/4	5/8	1 3/4 × 1 1/4
7 9/16–9	2	11/16	2 × 1 3/8
9 1/16–11	2 1/2	13/16	2 1/2 × 1 5/8
11 1/16–13	3	1	3 × 2

Table 13*a*.—A.G.M.A. Recommendations for Special Keyways
(Dimensions in inches)

Keyways		Keystock
Width	Depth	
1/8	3/64	1/8 × 3/32
3/16	1/16	3/16 × 1/8
1/4	3/32	1/4 × 3/16
5/16	3/32	5/16 × 3/16
3/8	1/8	3/8 × 1/4
1/2	3/16	1/2 × 3/8
5/8	7/32	5/8 × 7/16
3/4	1/4	3/4 × 1/2
7/8	5/16	7/8 × 5/8
1	3/8	1 × 3/4

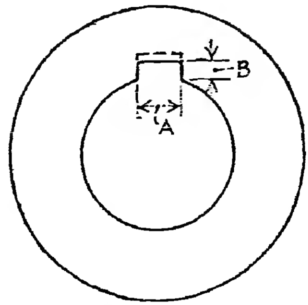


TABLE 14.—TAPER-KEY PROPORTIONS
(Dimensions in inches)

Diameter of holes, incl.	Keyseat taper, width A	$\frac{1}{8}$-in. per foot, height B
1 –1 $\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{32}$
1 $\frac{3}{16}$–1 $\frac{3}{8}$	$\frac{5}{16}$	$\frac{7}{64}$
1 $\frac{7}{16}$–1 $\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
1 $\frac{11}{16}$–1 $\frac{7}{8}$	$\frac{7}{16}$	$\frac{9}{64}$
1 $\frac{15}{16}$–2 $\frac{1}{8}$	$\frac{1}{2}$	$\frac{11}{64}$
2 $\frac{3}{16}$–2 $\frac{3}{8}$	$\frac{9}{16}$	$\frac{3}{16}$
2 $\frac{7}{16}$–2 $\frac{5}{8}$	$\frac{5}{8}$	$\frac{13}{64}$
2 $\frac{11}{16}$–2 $\frac{7}{8}$	$\frac{11}{16}$	$\frac{15}{64}$
2 $\frac{15}{16}$–3 $\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{4}$
3 $\frac{3}{16}$–3 $\frac{3}{8}$	$\frac{13}{16}$	$\frac{17}{64}$
3 $\frac{7}{16}$–3 $\frac{5}{8}$	$\frac{7}{8}$	$\frac{19}{64}$
3 $\frac{11}{16}$–3 $\frac{7}{8}$	$\frac{15}{16}$	$\frac{5}{16}$
3 $\frac{15}{16}$–4 $\frac{1}{8}$	1	$\frac{11}{32}$
4 $\frac{3}{16}$–4 $\frac{3}{8}$	1 $\frac{1}{16}$	$\frac{11}{32}$
4 $\frac{7}{16}$–4 $\frac{3}{4}$	1 $\frac{1}{8}$	$\frac{3}{8}$
4 $\frac{7}{8}$ –5 $\frac{1}{4}$	1 $\frac{1}{4}$	$\frac{27}{64}$
5 $\frac{3}{8}$ –5 $\frac{3}{4}$	1 $\frac{3}{8}$	$\frac{29}{64}$
5 $\frac{7}{8}$ –6 $\frac{1}{4}$	1 $\frac{1}{2}$	$\frac{1}{2}$
6 $\frac{3}{8}$ –6 $\frac{3}{4}$	1 $\frac{5}{8}$	$\frac{17}{32}$
6 $\frac{7}{8}$ –7 $\frac{1}{4}$	1 $\frac{3}{4}$	$\frac{19}{32}$
7 $\frac{3}{8}$ –7 $\frac{3}{4}$	1 $\frac{7}{8}$	$\frac{5}{8}$
7 $\frac{7}{8}$ –8	2	$\frac{21}{32}$

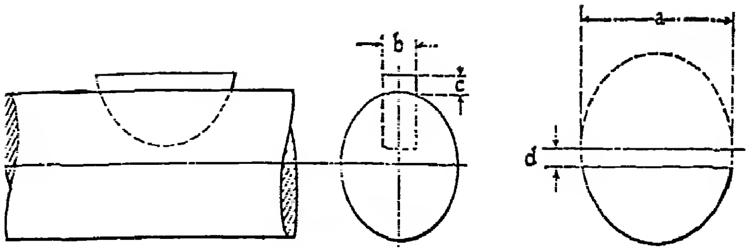


TABLE 15.—STANDARD WOODRUFF KEYS
(Dimensions in inches)

Number of key	Diameter of key	Thickness of key	Depth of key-way	Center of stock, from which key is made, to top of key
	a	b	c	d
1	1/2	1/16	1/32	3/64
2	1/2	3/32	3/64	3/64
3	1/2	1/8	1/16	3/64
4	5/8	3/32	3/64	1/16
5	5/8	1/8	1/16	1/16
6	5/8	5/32	5/64	1/16
7	3/4	1/8	1/16	1/16
8	3/4	5/32	5/64	1/16
9	3/4	3/16	3/32	1/16
10	7/8	5/32	5/64	1/16
11	7/8	3/16	3/32	1/16
12	7/8	7/32	7/64	1/16
A	7/8	1/4	1/8	1/16
13	1	3/16	3/32	1/16
14	1	7/32	7/64	1/16
15	1	1/4	1/8	1/16

Number of key	Diameter of key	Thickness of key	Depth of key-way	Center of stock, from which key is made, to top of key
	a	b	c	d
B	1	5/16	5/32	1/16
16	1 1/8	3/16	3/32	5/64
17	1 1/8	7/32	7/64	5/64
18	1 1/8	1/4	1/8	5/64
C	1 1/8	5/16	5/32	5/64
19	1 1/4	3/16	3/32	5/64
20	1 1/4	7/32	7/64	5/64
21	1 1/4	1/4	1/8	5/64
D	1 1/4	5/16	5/32	5/64
E	1 1/4	3/8	3/16	5/64
22	1 3/8	1/4	1/8	3/32
23	1 3/8	5/16	5/32	3/32
F	1 3/8	3/8	3/16	3/32
24	1 1/2	1/4	1/8	7/64
25	1 1/2	5/16	5/32	7/64
G	1 1/2	3/8	3/16	7[illegible]

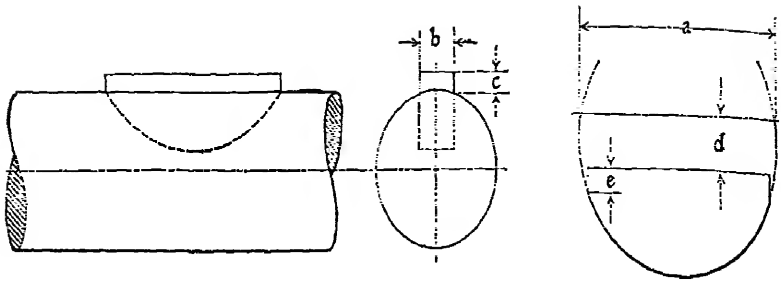


TABLE 15a.—SPECIAL WOODRUFF KEYS
(Dimensions in inches)

Number of key	Diameter of key	Thickness of key	Depth of key-way	Center of stock, from which key is made, to top of key	Width of flat	Number of key	Diameter of key	Thickness of key	Depth of key-way	Center of stock, from which key is made, to top of key	Width of flat
	a	b	c	d	e		c	b	c	d	e
26	2⅛	3/16	3/32	17/32	3/32	31	3½	7/16	7/32	13/16	5/16
27	2⅛	1/4	1/8	17/32	3/32	32	3½	1/2	1/4	13/16	3/16
28	2⅛	5/16	5/32	17/32	3/32	33	3½	9/16	9/32	13/16	5/16
29	2⅛	3/8	3/16	17/32	3/32	34	3½	5/8	5/16	13/16	5/16
30	3½	3/8	3/16	13/16	3/16						

TABLE 15b.—WOODRUFF KEYS FOR SHAFT DIAMETERS
(Dimensions in inches)

Diameter of shaft	Number of keys	Diameter of shaft	Number of keys	Diameter of shaft	Number of keys
5/16–3/8	1	7/8 – 5/16	6, 8, 10	1 3/8 –1 7/16	14, 17, 20
7/16–1/2	2, 4	1	9, 11, 13	1 1/2 –1 5/8	15, 18, 21, 24
9/16–5/8	3, 5	1 1/16–1 1/8	9, 11, 13, 16	1 11/16–1 3/4	18, 21, 24
11/16–3/4	3, 5, 7	1 3/16	11, 13, 16	1 13/16–2	23, 25
13/16	6, 8	1 1/4 –1 5/16	12, 14, 17, 20	2 1/16–2 1/2	25

TABLE 16.—A.G.M.A. STANDARD GEAR-HOLE TOLERANCES

	Nominal size	To ½ in., incl.	To 1 in., incl.	To 2 in., incl.	To 3 in., incl.	To 4 in., incl.	To 5 in., incl.	To 6 in., incl.	To 7 in., incl.	To 8 in., incl.	To 9 in., incl.	To 10 in., incl.	To 11 in., incl.	To 12 in., incl.
						Class 1 Precision Gears for Aircraft, Printing Machinery, Etc.								
Not go	+Limit	0.000	0.000	0.000	0.00025	0.0005	0.0005	0.0005	......	......	0.00075	......	......	0.001
Go	−Limit	.00025	.0005	.00075	.00075	.00075	.001	.001	......	......	.001	......	......	.001
	Tolerance	.00025	.0005	.00075	.001	.000125	.0015	.0015	......	......	.00175	......	......	.002
						Class 2 Automobile Transmission, Machine Tool, Etc., Gears								
Not go	+Limit	0.00025	0.0005	0.00075	0.001	0.00125	0.0013	0.00175	......	......	0.002	......	......	0.0025
Go	−Limit	.00025	.0005	.00075	.001	.001	.0012	.00125	......	......	.002	......	......	.0025
	Tolerance	.0005	.001	.0015	.002	.00225	.0025	.003	......	......	.004	......	......	.005
						Class 3 Standard-jobbing Gears								
Not go	+Limit	0.0005	0.00075	0.001	0.00125	0.0015	0.00175	0.002	......	......	0.003	......	......	0.004
Go	−Limit	.0005	.00075	.001	.00125	.0015	.00175	.002	......	......	.003	......	......	.004
	Tolerance	.001	.0015	.002	.0025	.003	.0035	.004	......	......	.006	......	......	.008

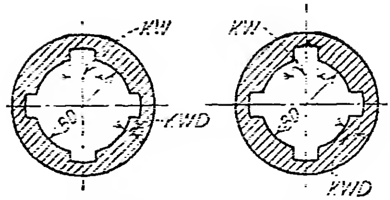


TABLE 17.—S.A.E. STANDARD FOUR-SPLINE FITTINGS
(Dimensions in inches)

Nominal diameter	D Max.	D Min.	d Max.	d Min.	W Max.	W Min.	h Max.	h Min.	
Permanent Fit									
3/4	0.750	0.749	0.637	0.636	0.181	0.179	0.056	0.055	78
7/8	.875	.874	.744	.743	.211	.209	.066	.065	107
1	1.000	.999	.850	.849	.241	.239	.075	.074	139
1 1/8	1.125	1.124	.956	.955	.271	.269	.084	.083	175
1 1/4	1.250	1.249	1.062	1.061	.301	.299	.094	.093	217
1 3/8	1.375	1.374	1.169	1.168	.331	.329	.103	.102	262
1 1/2	1.500	1.499	1.275	1.274	.361	.359	.112	.111	311
1 5/8	1.625	1.624	1.381	1.380	.391	.389	.122	.121	367
1 3/4	1.750	1.749	1.487	1.486	.422	.420	.131	.130	424
2	2.000	1.998	1.700	1.698	.482	.479	.150	.148	555
2 1/4	2.250	2.248	1.912	1.910	.542	.539	.169	.167	703
2 1/2	2.500	2.498	2.125	2.123	.602	.599	.187	.185	865
3	3.000	2.998	2.550	2.548	.723	.720	.225	.223	1,249
To Slide When Not under Load									
3/4	0.750	0.749	0.562	0.561	0.181	0.179	0.094	0.093	123
7/8	.875	.874	.656	.655	.211	.209	.109	.108	167
1	1.000	.999	.750	.749	.241	.239	.125	.124	219
1 1/8	1.125	1.124	.844	.843	.271	.269	.141	.140	277
1 1/4	1.250	1.249	.937	.936	.301	.299	.156	.155	341
1 3/8	1.375	1.374	1.031	1.030	.331	.329	.172	.171	414
1 1/2	1.500	1.499	1.125	1.124	.361	.359	.187	.186	491
1 5/8	1.625	1.624	1.219	1.218	.391	.389	.203	.202	577
1 3/4	1.750	1.749	1.312	1.311	.422	.420	.219	.218	670
2	2.000	1.998	1.500	1.498	.482	.479	.250	.248	875
2 1/4	2.250	2.248	1.687	1.685	.542	.539	.281	.279	1,106
2 1/2	2.500	2.498	1.875	1.873	.602	.599	.312	.310	1,365
3	3.000	2.998	2.250	2.248	.723	.720	.375	.373	1,969

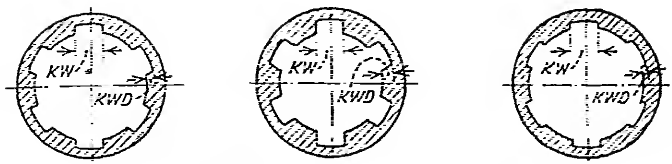


TABLE 18.—S.A.E. STANDARD SIX-SPLINE FITTINGS
(Dimensions in inches)

Nominal diameter	D Max.	D Min.	d Max.	d Min.	W Max.	W Min.	T
			Permanent Fit				
3/4	0.750	0.749	0.675	0.674	0.188	0.186	80
7/8	.875	.874	.788	.787	.219	.217	109
1	1.000	.999	.900	.899	.250	.248	143
1 1/8	1.125	1.124	1.013	1.012	.281	.279	180
1 1/4	1.250	1.249	1.125	1.124	.313	.311	223
1 3/8	1.375	1.374	1.238	1.237	.344	.342	269
1 1/2	1.500	1.499	1.350	1.349	.375	.373	321
1 5/8	1.625	1.624	1.463	1.462	.406	.404	376
1 3/4	1.750	1.749	1.575	1.574	.438	.436	436
2	2.000	1.998	1.800	1.798	.500	.497	570
2 1/4	2.250	2.248	2.025	2.023	.563	.560	721
2 1/2	2.500	2.498	2.250	2.248	.625	.622	891
3	3.000	2.998	2.700	2.698	.750	.747	1,283
			To Slide When Not under Load				
3/4	0.750	0.749	0.638	0.637	0.188	0.186	117
7/8	.875	.874	.744	.743	.219	.217	159
1	1.000	.999	.850	.849	.250	.248	208
1 1/8	1.125	1.124	.956	.955	.281	.279	263
1 1/4	1.250	1.249	1.063	1.062	.313	.311	325
1 3/8	1.375	1.374	1.169	1.168	.344	.342	393
1 1/2	1.500	1.499	1.275	1.274	.375	.373	468
1 5/8	1.625	1.624	1.381	1.380	.406	.404	550
1 3/4	1.750	1.749	1.488	1.487	.438	.436	637
2	2.000	1.998	1.700	1.698	.500	.497	833
2 1/4	2.250	2.248	1.913	1.911	.563	.560	1,052
2 1/2	2.500	2.498	2.125	2.123	.625	.622	1,300
3	3.000	2.998	2.550	2.548	.750	.747	1,873
			To Slide When under Load				
3/4	0.750	0.749	0.600	0.599	0.188	0.186	152
7/8	.875	.874	.700	.699	.219	.217	207
1	1.000	.999	.800	.799	.250	.248	270
1 1/8	1.125	1.124	.900	.899	.281	.279	342
1 1/4	1.250	1.249	1.000	.999	.313	.311	421
1 3/8	1.375	1.374	1.100	1.099	.344	.342	510
1 1/2	1.500	1.499	1.200	1.199	.375	.373	608
1 5/8	1.625	1.624	1.300	1.299	.406	.404	713
1 3/4	1.750	1.749	1.400	1.399	.438	.436	827
2	2.000	1.998	1.600	1.598	.500	.497	1,080
2 1/4	2.250	2.248	1.800	1.798	.563	.560	1,367
2 1/2	2.500	2.498	2.000	1.998	.625	.622	1,688
3	3.000	2.998	2.400	2.398	.750	.747	2,430

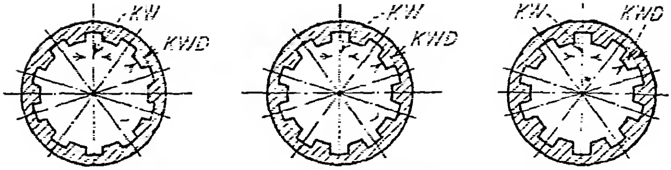


TABLE 19.—S.A.E. STANDARD TEN-SPLINE FITTINGS
(Dimensions in inches)

Nominal diameter	D Max.	D Min.	d Max.	d Min.	W Max.	W Min.	T
			Permanent Fit				
	0.750	0.749	0.683	0.682	0.117	0.115	120
	.875	.874	.796	.795	.137	.135	165
1	1.000	.999	.910	.909	.156	.154	215
1⅛	1.125	1.124	1.024	1.023	.176	.174	271
1¼	1.250	1.249	1.138	1.137	.195	.193	336
1⅜	1.375	1.374	1.251	1.250	.215	.213	406
1½	1.500	1.499	1.365	1.364	.234	.232	483
1⅝	1.625	1.624	1.479	1.478	.254	.252	566
1¾	1.750	1.749	1.593	1.592	.273	.271	658
2	2.000	1.998	1.820	1.818	.312	.309	860
2¼	2.250	2.248	2.048	2.046	.351	.348	1,038
2½	2.500	2.498	2.275	2.273	.390	.387	1,343
3	3.000	2.998	2.730	2.728	.468	.465	1,934
3½	3.500	3.497	3.185	3.182	.546	.543	2,632
4	4.000	3.997	3.640	3.637	.624	.621	3,438
4½	4.500	4.497	4.095	4.092	.702	.699	4,351
5	5.000	4.997	4.550	4.547	.780	.777	5,371
5½	5.500	5.497	5.005	5.002	.858	.855	6,500
6	6.000	5.997	5.460	5.457	.936	.933	7,735
			To Slide When Not under Load				
¾	0.750	0.749	0.645	0.644	0.117	0.115	183
⅞	.875	.874	.753	.752	.137	.135	248
1	1.000	.999	.860	.859	.156	.154	326
1⅛	1.125	1.124	.968	.967	.176	.174	412
1¼	1.250	1.249	1.075	1.074	.195	.193	508
1⅜	1.375	1.374	1.183	1.182	.215	.213	614
1½	1.500	1.499	1.290	1.289	.234	.232	732
1⅝	1.625	1.624	1.398	1.397	.254	.252	860
1¾	1.750	1.749	1.505	1.504	.273	.271	997
2	2.000	1.998	1.720	1.718	.312	.309	1,302
2¼	2.250	2.248	1.935	1.933	.351	.348	1,647
2½	2.500	2.498	2.150	2.148	.390	.387	2,034
3	3.000	2.998	2.580	2.578	.468	.465	2,929
3½	3.500	3.497	3.010	3.007	.546	.543	3,987
4	4.000	3.997	3.440	3.437	.624	.621	5,208
4½	4.500	4.497	3.870	3.867	.702	.699	6,591
5	5.000	4.997	4.300	4.297	.780	.777	8,137
5½	5.500	5.497	4.730	4.727	.858	.855	9,846
6	6.000	5.997	5.160	5.157	.936	.933	11,718
			To Slide When under Load				
¾	0.750	0.749	0.608	0.607	0.117	0.115	241
⅞	.875	.874	.709	.708	.137	.135	329
1	1.000	.999	.810	.809	.156	.154	430
1⅛	1.125	1.124	.911	.910	.176	.174	545
1¼	1.250	1.249	1.013	1.012	.195	.193	672
1⅜	1.375	1.374	1.114	1.113	.215	.213	813
1½	1.500	1.499	1.215	1.214	.234	.232	967
1⅝	1.625	1.624	1.316	1.315	.254	.252	1,135
1¾	1.750	1.749	1.418	1.417	.273	.271	1,316
2	2.000	1.998	1.620	1.618	.312	.309	1,720
2¼	2.250	2.248	1.823	1.821	.351	.348	2,176
2½	2.500	2.498	2.025	2.023	.390	.387	2,688
3	3.000	2.998	2.430	2.428	.468	.465	3,869
3½	3.500	3.497	2.835	2.832	.546	.543	5,266
4	4.000	3.997	3.240	3.237	.624	.621	6,878
4½	4.500	4.497	3.645	3.642	.702	.699	8,705
5	5.000	4.997	4.050	4.047	.780	.777	10,746
5½	5.500	5.497	4.455	4.452	.858	.855	13,003
6	6.000	5.997	4.860	4.857	.936	.933	15,475

SPUR-GEAR CALCULATIONS—SECTION IV

$$CP = \frac{3.1416}{DP} \qquad \frac{3.1416PD}{N} \tag{13}$$

$$DP = \frac{3.1416}{CP} \qquad \frac{N}{PD} \tag{14}$$

$$PD = \frac{N}{DP} = 0.3183N \times CP \tag{15}$$

$$CD = \frac{PD + pd}{2} = \frac{N + n}{2DP} \tag{16}$$

Formulas for Standard-depth Spur Gears

(A.G.M.A. 14½-deg. composite, 14½- and 20-deg. involute full-depth systems)

$$A = \frac{1}{DP} = \frac{CP}{3.1416} = \frac{PD}{N} \qquad \frac{OD}{N+2} \tag{17}$$

$$D = \frac{1.157}{DP} = 0.3683CP \tag{18}$$

$$WD = A + D = \frac{2.157}{DP} = 0.6866CP \tag{19}$$

$$C = \frac{0.157}{PD} = 0.05CP \tag{20}$$

$$OD = \frac{N+2}{DP} = PD + \frac{2}{DP} = \frac{(N+2)PD}{N} = (N+2)A$$

$$= 0.3183(N+2)CP = PD + 0.6366CP \tag{21}$$

$$RD = OD - 2WD \tag{22}$$

$$N = PD \times DP = \frac{3.1416PD}{CP} \tag{23}$$

Formulas for A.G.M.A. 20-deg. stub-tooth gears

$$A = \frac{0.8}{DP} = 0.2546CP \tag{24}$$

$$D = \frac{1}{DP} = 0.3183CP \tag{25}$$

$$WD = \frac{1.8}{DP} = 0.5729CP \tag{26}$$

$$C = \frac{0.2}{DP} = 0.0636CP \tag{27}$$

$$OD = \frac{N + 1.6}{DP} = 0.5092PD \times CP \tag{28}$$

STRAIGHT-TOOTH BEVEL GEARS—SECTION V

Formulas for Standard-depth Bevel Gears—Axes at Right Angles

$$\tan VC = \frac{N}{n} \text{ or } \tan vc = \frac{n}{N} \tag{29}$$

$$\tan VI = \frac{2 \sin VC}{N} = \frac{A}{AP} \tag{30}$$

$$\tan VD = \frac{2.314 \sin VC}{N} = \frac{D}{AP} \tag{31}$$

$$VF = VC + VI \tag{32}$$

$$VB = VC - VD \tag{33}$$

$$AP = \frac{PD}{2 \sin VC} = \frac{N}{2DP \sin VC} \tag{34}$$

$$CP = \frac{3.1416}{DP} = \frac{3.1416PD}{N} \tag{13}$$

$$DP = \frac{N}{PD} = \frac{3.1416}{CP} \tag{14}$$

$$PD = \frac{N}{DP} = 0.3183N \times CP \tag{15}$$

$$DI = 2A \cos VC \tag{35}$$

$$OD = PD + DI \tag{36}$$

$$FW = \frac{AP}{3} \text{ or } \frac{5CP}{2} \text{ (whichever is smaller)} \tag{37}$$

Formulas for Standard-depth Bevel Gears—Axes at Odd Angles

$$\tan VC = \frac{\sin VA}{\frac{n}{N} + \cos VA} \tag{38}$$

$$vc = VA - VC$$

$$\tan VC = \frac{\sin (180 - VA)}{\frac{n}{N} - \cos (180 - VA)} \tag{39}$$

$$vc = VA - VC$$

$$VC = 90 \text{ deg.} \qquad vc = VA - 90 \text{ deg.}$$

$$\tan VC = \frac{\sin VA}{\sin VA - \frac{n}{N}} \tag{40}$$

$$vc = VC - VA$$

Formulas for Parallel-depth Bevel Gears[1]

$$DPs = \frac{DP \times AD}{AD - FW} \tag{41}$$

$$A = \frac{1}{DPs} \tag{42}$$

$$D = \frac{1.157}{DPs} \tag{43}$$

$$WD = A + D = \frac{2.157}{DPs} \tag{44}$$

A.G.M.A. Proportions for Generated Straight-tooth Bevel Gears Operating at Right Angles, Where the Pinion Is the Driver and Has 10 or More Teeth

(Gleason Works System)

Pressure angles:

	PP, Degrees
Ratios having 14 or more teeth in pinion	$14\frac{1}{2}$
13–13 to 13–24	$17\frac{1}{2}$
13–25 and higher	$14\frac{1}{2}$
12–12 and higher	$17\frac{1}{2}$
11–11 to 11–14	20
11–15 and higher	$17\frac{1}{2}$
10–10 and higher	20

Addendum:

(Gear)

$$A = \frac{\text{value, Table 20}}{DP} \tag{45a}$$

(Pinion)

$$a = \frac{2.000}{DP} - A \tag{45b}$$

Dedendum:

(Gear)

$$D = \frac{2.188}{DP} - A \tag{46a}$$

(Pinion)

$$d = \frac{2.188}{DP} - a \tag{46b}$$

[1] For dimensions of parallel-depth bevel gears other than those of tooth-depth proportions, use formulas for standard-depth bevel gears.

Whole Depth:

$$WD = \frac{2.188}{DP} \tag{47}$$

Circular Thickness of Teeth:

(Gear—14½ deg. *VP*)

$$CTh = \frac{1.071}{DP} + 0.5A - \frac{K \text{ (Table 21)}}{DP} \tag{48a}$$

(Gear—17½ deg. *VP*)

$$CTh = \frac{0.971}{DP} + 0.6A \quad \frac{K}{DP} \tag{48b}$$

(Gear—20 deg. *VP*)

$$CTh = \frac{0.871}{DP} + 0.7A - \frac{K}{DP} \tag{48c}$$

(Pinions—14½ deg., 17½ deg., or 20 deg. *VP*)

$$cth = \frac{3.142}{DP} \quad CTh \tag{48d}$$

TABLE 20.—ADDENDUM VALUES FOR ONE DIAMETRAL PITCH FOR DIFFERENT RATIOS
(Gleason Works System)

$$\text{Gear ratio} = \frac{N}{n} = \frac{\text{number of teeth in gear}}{\text{number of teeth in pinion}}$$

Ratios		A	Ratios		A	Ratios		A	Ratios		A
From	To		From	To		From	To		From	To	
1.00	1.01	1.000	1.15	1.17	0.880	1.42	1.45	0.760	2.06	2.16	0.640
1.01	1.02	.990	1.17	1.19	.870	1.45	1.48	.750	2.16	2.27	.630
1.02	1.03	.980	1.19	1.21	.860	1.48	1.52	.740	2.27	2.41	.620
1.03	1.04	.970	1.21	1.23	.850	1.52	1.56	.730	2.41	2.58	.610
1.04	1.05	.960	1.23	1.25	.840	1.56	1.60	.720	2.58	2.78	.600
1.05	1.06	.950	1.25	1.27	.830	1.60	1.65	.710	2.78	3.05	.590
1.06	1.08	.940	1.27	1.29	.820	1.65	1.70	.700	3.05	3.41	.580
1.08	1.09	.930	1.29	1.31	.810	1.70	1.76	.690	3.41	3.94	.570
1.09	1.11	.920	1.31	1.33	.800	1.76	1.82	.680	3.94	4.82	.560
1.11	1.12	.910	1.33	1.36	.790	1.82	1.89	.670	4.82	6.81	.550
1.12	1.14	.900	1.36	1.39	.780	1.89	1.97	.660	6.81	∞	.540
1.14	1.15	.890	1.39	1.42	.770	1.97	2.06	.650			

Table 21.—Values of K for Circular Thickness of Tooth Formulas
(Gleason Works System)

Number of teeth in pinion (n)	Ratios														
	1.00 to 1.25	1.25 to 1.50	1.50 to 1.75	1.75 to 2.00	2.00 to 2.25	2.25 to 2.50	2.50 to 2.75	2.75 to 3.00	3.00 to 3.25	3.25 to 3.50	3.50 to 3.75	3.75 to 4.00	4.00 to 4.50	4.50 to 5.00	5.00 to ∞
10	0.025	0.070	0.100	0.120	0.140	0.160	0.175	0.190	0.205	0.215	0.225	0.230	0.240	0.250	0.255
11	.010	.015	.050	.080	.105	.125	.145	.160	.170	.180	.190	.195	.200	.210	.220
12	.000	.040	.070	.100	.120	.140	.155	.170	.180	.185	.190	.195	.205	.210	.215
13	.000	.015	.040	.045	.050	.060	.070	.080	.090	.100	.110	.120	.135	.150	.165
14	.000	.015	.030	.050	.065	.080	.090	.100	.110	.120	.125	.130	.140	.150	.160
15–17	.000	.000	.010	.020	.030	.045	.060	.070	.080	.090	.095	.100	.110	.115	.120
18–21	.000	.000	.000	.000	.010	.030	.045	.060	.070	.080	.085	.090	.095	.100	.100
22–29	.000	.000	.000	.000	.010	.030	.040	.050	.060	.065	.070	.070	.080	.085	.085
30 up	.000	.000	.000	.000	.010	.025	.035	.040	.045	.050	.055	.060	.065	.070	.070

TABLE 22.—FORM FACTOR Y (LEWIS FORMULAS) FOR GENERATED BEVEL-GEAR TEETH
(Gleason Works System)

Number of teeth in pinion (n)	Ratios 1.00 to 1.25	1.25 to 1.50	1.50 to 1.75	1.75 to 2.00	2.00 to 2.25	2.25 to 2.50	2.50 to 2.75	2.75 to 3.00	3.00 to 3.25	3.25 to 3.50	3.50 to 3.75	3.75 to 4.00	4.00 to 4.50	4.50 to 5.00	5.00 to ∞
10	0.231	0.260	0.280	0.294	0.305	0.315	0.324	0.332	0.340	0.347	0.353	0.358	0.365	0.371	0.377
11	.268	.264	.273	.286	.296	.303	.309	.315	.320	.324	.328	.332	.336	.340	.342
12	.248	.265	.281	.295	.308	.318	.328	.335	.341	.345	.348	.351	.353	.355	.356
13	.264	.278	.291	.280	.278	.286	.291	.295	.298	.299	.301	.303	.305	.307	.310
14	.242	.254	.263	.272	.281	.288	.294	.299	.304	.307	.310	.313	.316	.318	.319
15	.248	.258	.266	.274	.283	.290	.296	.301	.305	.308	.312	.315	.318	.319	.320
16	.252	.261	.269	.277	.285	.292	.298	.304	.308	.312	.314	.317	.319	.321	.323
17–18	.257	.265	.273	.281	.288	.295	.302	.307	.311	.315	.318	.320	.322	.325	.326
19–21	.265	.272	.279	.286	.294	.300	.307	.312	.317	.320	.324	.326	.328	.330	.332
22–25	.274	.281	.288	.295	.301	.307	.314	.319	.324	.327	.331	.332	.335	.337	.338
26–30	.284	.291	.297	.304	.310	.317	.322	.327	.332	.336	.339	.342	.344	.346	.347

HELICAL- AND HERRINGBONE-SPUR GEARS—SECTION VI

Modifications of gear-tooth proportions:

$$od \text{ enlarged} = od + \left(2D - \frac{(\sin VP)^2 \times n}{2DP}\right) \tag{49a}$$

$$od \text{ enlarged} = od + \left(2A - \frac{(\sin VP)^2 \times n}{2DP}\right) \tag{49b}$$

Formulas for Helical- and Herringbone-spur Gears:

$$\tan VPn \text{ (in normal profile plane)} = \tan VP \times \cos VH \tag{50a}$$

$$\tan VP = \frac{\tan VPn}{\cos VH} \tag{50b}$$

$$A = \frac{1}{DP} \text{ (max.)}$$

$$= \frac{0.7}{DP} \text{ (min.)}$$

$$C = \frac{0.3}{DP} \text{ (max.)}$$

$$= \frac{0.157}{DP} \text{ (min.)}$$

$$D = A + C$$

$$WD = 2A + C$$

$$DP = DPn \times \cos VH \tag{51a}$$

$$DPn = \frac{DP}{\cos VH} \tag{51b}$$

$$OD = \frac{N}{DP} + 2A$$

$$AF \text{ (min.)} = \frac{7.22568}{DP \times \tan VH} \tag{52}$$

$$FW \text{ (parted tooth)} = AF + GW \tag{53}$$

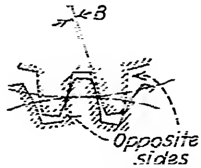


TABLE 23.—BACKLASH LIMITATIONS
(A.G.M.A. Recommendations)

Minimum industrial gears B, inch	DP	Minimum high-speed gears B, inch
0.002	24	0.003
.002	16	.003
.003	12	.004
.003	10	.004
.004	8	.005
.005	6	.007
.006	5	.008
.008	4	.010
.010	3	.013
.012	2½	
.015	2	
.020	1½	
.030	1	

Maximum herringbone-tooth load:

$$TL = \frac{Y \times S \times K}{DP \times P} \tag{54}$$

$$Y = \frac{TFW^2}{6RL} \tag{54a}$$

$$K = \frac{78}{78 + \sqrt{PLV}} \tag{54b}$$

TABLE 24.—ALLOWABLE STATIC STRESS OF MATERIAL

Material	*S*
High carbon or alloy steels heat treated to an elastic limit of approximately 60,000 lb. per square inch	15,000
0.40 to 0.50 carbon steel heat treated to an elastic limit of approximately 50,000 lb. per square inch	12,500

TABLE 24.—ALLOWABLE STATIC STRESS OF MATERIAL.—*Concluded*

0.40 to 0.50 carbon steel untreated with an elastic limit of approximately 40.000 lb. per square inch	10.000
Cast steel A.S.T.M. Class B. Elastic limit approximately 36,000 lb. per square inch	7.500
Cast iron. Tensile strength approximately 24.000 lb. per square inch	4,000
Bronze 88-10-2 Tensile strength approximately 27.000 lb. per square inch	4,000

Horsepower formula (Farrel-Birmingham Company):

$$HP = \frac{S \times FW \times PD^2 \times Q \times C \times RPM}{1,260} \qquad (55)$$

TABLE 25*a*.—GEAR-MATERIAL SPECIFICATIONS

Number	Material	Hardness, Brinell
	Pinion Steel	
1*P*	0.40–0.50 per cent carbon	175–200
2*P*	.50– .60 per cent carbon	175–200
3*P*	.50– .60 per cent carbon	200–225
4*P*	.50– .60 per cent carbon	225–250
5*P*	S.A.E. 3240 or equivalent	225–250
6*P*	S.A.E. 3240 or equivalent	250–275
7*P*	S.A.E. 3240 or equivalent	275–300
8*P*	S.A.E. 2320 case-hardened	450–500
	Gear Steel	
1*G*	Approximately 0.30 per cent carbon	
2*G*	Approximately 0.40 per cent carbon	
3*G*	Special	

TABLE 25*b*.—MATERIAL FACTOR *S*

Material factor *S*	Gear-material specifications	
	Pinion member	Gear member
1.0	1*P*	1*G*
1.1	2*P*	1*G*
1.2	3*P*	1*G*
1.3	4*P*	1*G*
1.4	5*P*	2*G*
1.5	6*P*	2*G*
1.6	7*P*	2*G*
2.0	8*P*	3*G*

TABLE 25c.—INSTALLATION FACTOR Q

Type of installation	Tooth velocity, feet per minute		
	1,000 to 2,000	500 to 1,000	Under 500
Enclosed gears..........	1.0	1.2	1.4
Open gearing...........	.8	1.0	1.2

TABLE 25d.—CHARACTER OF LOADS C

Character of loads	Conditions of daily service			
	Continuous 24 hr.	Continuous 10 hr.	Intermittent over 5 hr.	Intermittent under 5 hr.
Full-load rating with shutdowns only for repairs...	0.45	0.60	0.80	1.10
Friction loads to full motor overloads with frequent power fluctuations.......	0.60	0.80	1.10	1.50
Friction loads to part full-load rating, average running 75% full-load rating	0.80	1.10	1.50	2.00
Friction loads to part full-load rating with majority under 50% full-load rating..................	1.10	1.50	2.00	2.70

$$pd = \sqrt[3]{\frac{HP \times 840}{S \times Q \times C \times RPM}} \qquad (55a)$$

SPIRAL GEARING—SECTION VII

Formulas for Helix Angles of Right-angle Gearing:

$$\tan vh = \frac{pd \times N}{PD \times n} \quad \text{and} \quad \tan VH = \frac{PD \times n}{pd \times N} \tag{56}$$

General Formulas for Spiral Gearing

$$VA = vh + VH \quad \text{(tooth helices of same hand)} \tag{57a}$$

$$VA = vh - VH \quad \text{(tooth helices of opposite hand)} \tag{57b}$$

$$PD = \frac{CP \times N}{3.1416} = \frac{NP \times N}{3.1416 \cos VH} = \frac{N}{NDP \cos VH} \tag{58}$$

$$OD = PD + 2A = PD + \frac{2}{NDP} \tag{59}$$

$$CP = \frac{3.1416PD}{N} = \frac{NP}{\cos VH} \tag{60}$$

$$NP = CP \cos VH \tag{61}$$

$$NDP = \frac{3.1416}{NP} \tag{62}$$

$$A = \frac{NP}{3.1416} = \frac{1}{NDP} = 0.5(OD - PD) \tag{63}$$

$$D = A + 0.1CTh \tag{64}$$

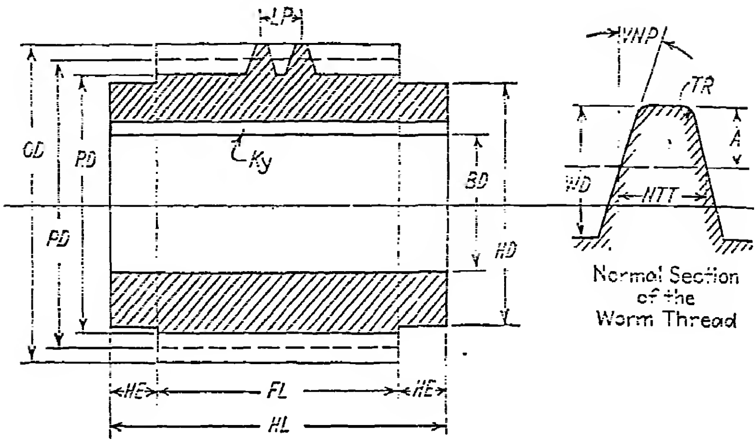
$$C = 0.1CTh \tag{65}$$

$$CTh = 0.5NP \tag{66}$$

$$WD = 2A + 0.1CTh \tag{67}$$

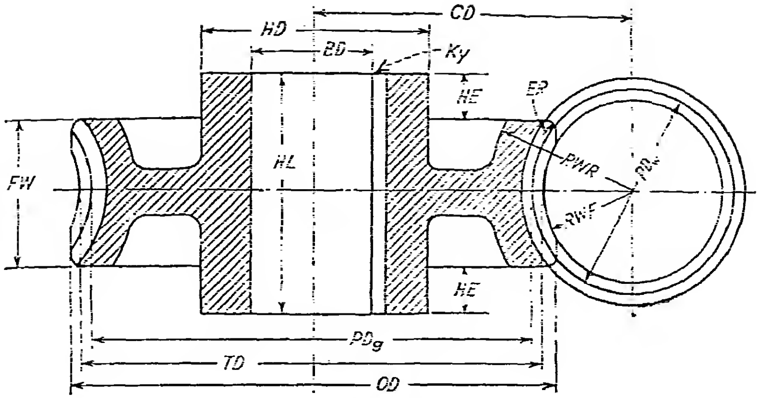
$$L = \frac{N \times CP}{\tan VH} = \frac{3.1416}{DP \tan VH} \tag{68}$$

WORM GEARING—SECTION VIII



Formulas for Standard Worms

Symbol	Single and double thread	Triple and quadruple thread	Formula
PD	2.4 × *LP* + 1.1	2.4 × *LP* + 1.1	(69)
OD	3.036 × *LP* + 1.1	2.972 × *LP* + 1.1	(70)
RD	1.664 × *LP* + 1.1	1.726 × *LP* + 1.1	(71)
HD	1.664 × *LP* + 1	1.726 × *LP* + 1	(72)
BD	*LP* + 0.625	*LP* + 0.625	(73)
FL	*LP* × (4.5 + 0.02*N*)	*LP* × (4.5 + 0.02*N*)	(74)
HE	*LP*	*LP*	(75)
HL	*FL* + 2 × *LP*	*FL* + 2 × *LP*	(76)
Ky	A.G.M.A. Standard	A.G.M.A. Standard	
A	0.318 × *LP*	0.286 × *LP*	(77)
WD	0.686 × *LP*	0.623 × *LP*	(78)
VL	$\cot VL = \dfrac{PD \times 3.1416}{L}$	$\cot VL = \dfrac{PD \times 3.1416}{L}$	(79)
NTT	0.5 × *LP* × cos *VL*	0.5 × *LP* × cos *VL*	(80)
TR	0.05 × *LP*	0.05 × *LP*	(81)
VNP	14½ deg.	20 deg.	



FORMULAS FOR STANDARD WORM GEARS

Symbol	Single and double thread	Triple and quadruple thread	Formula
PD	$N \times 0.3183 \times LP$	$N \times 0.3183 \times LP$	(82)
TD	$PD + 0.636LP$	$PD + 0.572LP$	(83)
OD	$TD + 0.4775LP$	$TD + 0.3183LP$	(84)
HD	$1.875BD$	$1.875BD$	(85)
K_y	A.G.M.A. Standard	A.G.M.A. Standard	
FW	$2.38LP + 0.25$	$2.15LP + 0.2$	(86)
HE	$0.25BD$	$0.25BD$	(87)
HL	$FW + 0.5BD$	$FW + 0.5BD$	(88)
RWF	$0.882LP + 0.55$	$0.914LP + 0.55$	(89)
RWR	$2.2LP + 0.55$	$2.1LP + 0.55$	(90)
ER	$0.25LP$	$0.25LP$	(91)
CD	$(PD_g + PD_w) \times 0.5$	$(PD_g + PD_w) \times 0.5$	(92)
VNP	14½ deg.	20 deg.	

Special formulas:

Single- and double-thread worms—

$$FW = 2\sqrt{(PD_w + A) \times A} + 0.50LP \qquad (93)$$
$$OD = PD_g + 3.5A \qquad (94)$$

Triple- and quadruple-thread worms—

$$FW = 2\sqrt{(PD_w + A) \times A} + 0.25LP \qquad (93a)$$
$$OD(\text{20-deg. } NVP \text{ or greater}) = PD_g + 2.75A \qquad (94a)$$
$$OD(NVP \text{ less than 20 deg.}) = PD_g + 3A \qquad (94b)$$

Efficiency of worm gearing:

$$E = \frac{\tan VL(1 - f \tan VL)}{f + \tan VL} \tag{95}$$

Strength of worm gearing:

$$W = \frac{600AUS(FW \times Y)}{DP(600 + PLV)} \tag{96}$$

TABLE 26.—VALUES OF FORM FACTOR Y FOR WORM GEARS*

Pitch-line velocity (PLV), ft. per min.	Form factor Y	Allowable unit stress (AUS), lb. per sq. in.	
		Cast iron	Phosphor bronze
0	1.000	5,300	8,000
100	.857	4,550	6,800
200	.750	4,000	6,000
300	.667	3,550	5,350
450	.571	3,000	4,500
600	.500	2,650	4,000

* Compiled by Foote Bros. Gear and Machine Co.

Avoidance of worm-gear tooth undercutting:

$$TD \text{ enlarged } (14\tfrac{1}{2} \text{ deg. } VNP) = 0.937PD + 4A \tag{97}$$

$$\cos VNP = \sqrt{1 - \frac{2}{N}} \tag{98}$$

Special worm formulas:

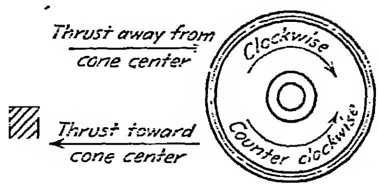
$$PD = 2.35 \times LP + 0.4 \tag{99}$$

$$LW = LP\left(4.5 + \frac{N}{50}\right) \tag{100}$$

[illegible] $LW + LP$ (101)

SPIRAL-BEVEL, SKEW-BEVEL, AND HYPOID GEARS—SECTION IX

Directions of thrust, spiral-bevel gearing:



Driving member		Value of axial thrust Arrow indicates thrust direction of each factor Resultant direction is that of larger factor	Formula
Hand of spiral	Rotation		
Right or left	Clockwise	$ATL = TTL\left(\tan VS \cos VPi - \dfrac{\tan VP \sin VPi}{\cos VS}\right)$	(102a)
Right or left	Counterclockwise	$ATL = TTL\left(\tan VS \cos VPi + \dfrac{\tan VP \sin VPi}{\cos VS}\right)$	(102b)

VP = pressure angle. VPi = pitch angle. VS = spiral angle.

A.G.M.A. Proportions for Spiral-Bevel Gears Operating at Right Angles, Where the Pinion Is the Driver
(Gleason Works System)

Pressure angle:

	VP, degrees
5, 6, or 7 teeth in pinion	20
8 or 9 teeth in pinion	17½
Ratios having 12 or more teeth in pinion	14½
11–11 to 11–19	17½
12–20 and higher	14½
10–10 to 10–24	17½
10–25 and higher	14½

Addendum, gear:

For 5 teeth in pinion,

$$A = \frac{\text{value, Table 27}}{DP} \times \frac{14}{17} \tag{103a}$$

For 6 teeth in pinion,

$$A = \frac{\text{value, Table 27}}{DP} \times \frac{15}{17} \tag{103b}$$

For 7 or 8 teeth in pinion,

$$A = \frac{\text{value, Table 27}}{DP} \times \frac{16}{17} \tag{103c}$$

For 9 or more teeth in pinion,

$$A = \frac{\text{value, Table 27}}{DP} \tag{103d}$$

Addendum, pinion:

5-tooth pinion,

$$a = \frac{1.4000}{DP} - A \tag{103e}$$

6-tooth pinion,

$$a = \frac{1.5000}{DP} - A \tag{103f}$$

7- or 8-tooth pinion,

$$a = \frac{1.6000}{DP} - A \tag{103g}$$

9 or more teeth in pinion,

$$a = \frac{1.7000}{DP} - A \tag{103h}$$

Dedendum, gear:

For 5 teeth in pinion,

$$D = \frac{1.557}{DP} - A \tag{104a}$$

For 6 teeth in pinion,

$$D = \frac{1.657}{DP} - A \tag{104b}$$

For 7 teeth in pinion,

$$D = \frac{1.757}{DP} - A \tag{104c}$$

For 8 teeth in pinion,

$$D = \frac{1.788}{DP} - A \tag{104d}$$

For 9 or more teeth in pinion,

$$D = \frac{1.888}{DP} - A \tag{104e}$$

Dedendum, pinion:

5-tooth pinion,

$$d = \frac{1.557}{DP} - a \tag{104f}$$

6-tooth pinion,

$$d = \frac{1.657}{DP} - a \tag{104g}$$

7-tooth pinion,

$$d = \frac{1.757}{DP} - a \tag{104h}$$

8-tooth pinion,

$$d = \frac{1.788}{DP} - a \tag{104i}$$

9 or more teeth in pinion,

$$d = \frac{1.888}{DP} - a \tag{104j}$$

Whole depth (gear and pinion):

With 5 teeth in pinion,

$$WD = \frac{1.557}{DP} \tag{105a}$$

With 6 teeth in pinion,

$$WD = \frac{1.657}{DP} \tag{105b}$$

With 7 teeth in pinion,

$$WD = \frac{1.757}{DP} \tag{105c}$$

With 8 teeth in pinion,

$$WD = \frac{1.788}{DP} \tag{105d}$$

With 9 or more teeth in pinion,

$$WD = \frac{1.}{DP} \tag{105e}$$

Circular thickness of teeth, gear:

With 5 teeth in pinion,

$$CTh = \frac{1.011}{DP} + 0.8A - \frac{K\ (\text{Table } 28)}{DP} \tag{106a}$$

With 6 teeth in pinion,

$$CTh = \frac{0.971}{DP} + 0.8A - \frac{K}{DP} \tag{106b}$$

With 7 teeth in pinion,

$$CTh = \frac{0.931}{DP} + 0.8A - \frac{K}{DP} \tag{106c}$$

With 8 teeth in pinion,

$$CTh = \frac{1.011}{DP} + 0.7A - \frac{K}{DP} \tag{106d}$$

With 9 teeth in pinion,

$$CTh = \frac{0.976}{DP} + 0.7A - \frac{K}{DP} \tag{106e}$$

With 10 or more teeth in pinion,
(14½ deg. VP)

$$CTh = \frac{1.061}{DP} + 0.6A - \frac{K}{DP} \tag{106f}$$

(17½ deg. VP)

$$CTh = \frac{0.976}{DP} + 0.7A - \frac{K}{DP} \tag{106g}$$

Circular thickness of teeth, pinion:

For all combinations,

$$cth = \frac{3.142}{DP} - CTh \tag{106h}$$

TABLE 27.—ADDENDUM VALUES FOR ONE DIAMETRAL PITCH FOR DIFFERENT RATIOS

(Gleason Works System)

$$\text{Gear ratio } \frac{N}{n} = \frac{\text{number of teeth in gear}}{\text{number of teeth in pinion}}$$

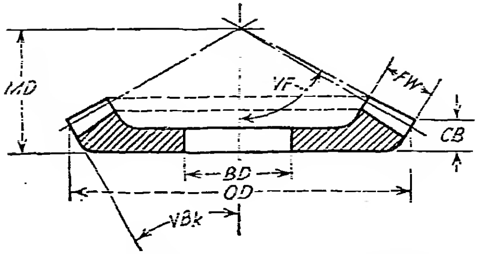
Ratios			Ratios			Ratios			Ratios		
From	To	A	From	To	A	From	To	A	From	To	A
1.00	1.00	0.850	1.15	1.17	0.750	1.41	1.44	0.650	1.99	2.10	0.550
1.00	1.02	.840	1.17	1.19	.740	1.44	1.48	.640	2.10	2.23	.540
1.02	1.03	.830	1.19	1.21	.730	1.48	1.52	.630	2.23	2.38	.530
1.03	1.05	.820	1.21	1.23	.720	1.52	1.57	.620	2.38	2.58	.520
1.05	1.06	.810	1.23	1.26	.710	1.57	1.63	.610	2.58	2.82	.510
1.06	1.08	.800	1.26	1.28	.700	1.63	1.68	.600	2.82	3.17	.500
1.08	1.09	.790	1.28	1.31	.690	1.68	1.75	.590	3.17	3.67	.490
1.09	1.11	.780	1.31	1.34	.680	1.75	1.82	.580	3.67	4.56	.480
1.11	1.13	.770	1.34	1.37	.670	1.82	1.90	.570	4.56	7.00	.470
1.13	1.15	.760	1.37	1.41	.660	1.90	1.99	.560	7.00	∞	.460

TABLE 28.—VALUES OF K FOR CIRCULAR THICKNESS OF TOOTH FORMULAS
(Gleason Works System)

Number of teeth in pinion	Ratios														
	1.00 to 1.25	1.25 to 1.50	1.50 to 1.75	1.75 to 2.00	2.00 to 2.25	2.25 to 2.50	2.50 to 2.75	2.75 to 3.00	3.00 to 3.25	3.25 to 3.50	3.50 to 3.75	3.75 to 4.00	4.00 to 4.50	4.50 to 5.00	5.00 and higher
5	0.020	0.040	0.075	0.110	0.135	0.155	0.170	0.185	0.200	0.215	0.230	0.240	0.255	0.270	0.285
6	.010	.035	.060	.085	.105	.130	.150	.165	.180	.195	.210	.220	.235	.250	.265
7	.000	.025	.050	.075	.095	.115	.135	.155	.170	.185	.195	.205	.220	.235	.250
8	.000	.010	.030	.045	.065	.080	.095	.110	.125	.135	.145	.155	.170	.180	.195
9	.000	.010	.025	.040	.055	.070	.085	.095	.105	.115	.125	.135	.150	.165	.185
10	.020	.055	.085	.105	.125	.125	.110	.120	.130	.140	.150	.155	.160	.170	.180
11	.030	.075	.105	.070	.085	.095	.105	.115	.125	.135	.140	.145	.150	.155	.160
12 to 13	.005	.015	.025	.035	.045	.055	.065	.075	.085	.095	.105	.115	.125	.135	.135
14 to 16	.000	.005	.015	.025	.035	.050	.060	.075	.085	.095	.100	.105	.105	.105	.105
17 to 19	.000	.000	.005	.015	.025	.035	.050	.065	.075	.085	.090	.090	.090	.090	.090
20 up	.000	.000	.000	.005	.015	.025	.040	.050	.055	.060	.060	.060	.060	.060	.060

TABLE 29.—FORM FACTOR Y (LEWIS FORMULAS) FOR SPIRAL-BEVEL GEAR TEETH
(Gleason Works System)

Number of teeth in pinion (n)	Ratios														
	1.00 to 1.25	1.25 to 1.50	1.50 to 1.75	1.75 to 2.00	2.00 to 2.25	2.25 to 2.50	2.50 to 2.75	2.75 to 3.00	3.00 to 3.25	3.25 to 3.50	3.50 to 3.75	3.75 to 4.00	4.00 to 4.50	4.50 to 5.00	5.00 to ..
5	0.297	0.322	0.343	0.361	0.376	0.388	0.398	0.406	0.411	0.416	0.420	0.424	0.431	0.438	0.450
6	.310	.332	.353	.372	.386	.398	.406	.414	.419	.424	.428	.431	.436	.443	.452
7	.318	.331	.347	.300	.373	.384	.392	.398	.405	.410	.415	.419	.426	.432	.439
8	.298	.320	.336	.348	.357	.366	.373	.379	.384	.388	.392	.394	.397	.400	.405
9	.292	.313	.327	.338	.340	.352	.357	.363	.367	.370	.373	.376	.380	.384	.388
10	.315	.338	.353	.363	.371	.345	.326	.342	.351	.357	.363	.367	.371	.374	.377
11	.316	.335	.343	.325	.327	.333	.338	.344	.350	.356	.361	.367	.375	.384	.300
12	.298	.318	.333	.343	.351	.357	.363	.368	.372	.377	.379	.381	.384	.386	.388
13	.302	.320	.334	.343	.351	.358	.365	.371	.376	.381	.384	.386	.388	.391	.393
14	.306	.322	.334	.345	.354	.362	.369	.374	.378	.382	.386	.389	.391	.393	.305
15	.314	.330	.342	.352	.360	.368	.374	.380	.385	.389	.392	.394	.397	.399	.402
16	.322	.335	.347	.358	.367	.374	.381	.386	.390	.394	.397	.400	.402	.404	.406
17-18	.329	.343	.354	.364	.373	.382	.389	.394	.398	.400	.403	.406	.407	.409	.410
19-21	.330	.351	.362	.373	.382	.389	.396	.401	.405	.407	.410	.411	.412	.414	.415
22-25	.351	.363	.373	.382	.391	.398	.403	.407	.410	.412	.413	.414	.415	.417	.418
26-30	.364	.374	.384	.393	.399	.404	.407	.410	.412	.414	.415	.416	.417	.418	.419



The following are suggested as practical limits for the machining of spiral-bevel gears:

Symbol	Dimension	Tolerance
OD	Outside diameter	+0.000 in. −0.005 in.
CB	Crown backing	+ .000 in. − .002 in.
BD	Bore diameter	+ .001 in. − .000 in.
FW	Face width	+ .000 in. − .010 in.
VBk	Back angle	+ 15 min.− 15 min.
VF	Face angle	+ 8 min.− 0 min.

Teeth not to be high at small end.

Formulas for Skew-bevel Gears—Straight-tooth Pinion

$$PDe = \frac{N}{DP} \tag{107}$$

$$\tan VO = \frac{2OS}{PDe} \tag{108}$$

$$PD = \frac{2OS}{\sin VO} \tag{109}$$

$$NP = \frac{3.1416PD}{N} \tag{110}$$

$$\tan vc = \frac{pd}{PDe} \tag{111}$$

$$OD = PD + \frac{di \times PDe}{PD} \tag{112}$$

$$VF = \frac{(VC + VI)PD}{PDe} \tag{113}$$

INTERNAL GEARING—SECTION X

Formulas for Internal Gearing

$$CD = \frac{N - n}{2DP} = \frac{(N - n)CP}{6.2832} = \frac{PD - pd}{2} \tag{114}$$

$$ID = PD - 2A = \frac{N}{DP} - 2A \qquad (115)$$

$$BtD = 2WD + ID = 2.5A + PD \qquad (116)$$

EPICYCLIC GEAR TRAINS—SECTION XI

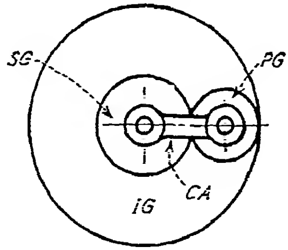


TABLE 30.—SIMPLE SUN-AND-PLANET GEAR VELOCITIES

Symbols for Member Units

Sun gear, *SG*. Planet gear, *PG*. Internal gear, *IG*. Carrying arm, *CA*

(Symbols also denote number of teeth in, or diameters of, gears)

Function Symbols

Stationary member, *SM*. Driving member, *RM*. Driven member, *DM*.

Carrying Arm, *CA*

VSG = velocity, sun gear. VIG = velocity, internal gear.

VCA = velocity, carrying arm. VPG = velocity, planet gear.

SM	*RM*	*DM*	Rotary speed per revolution of driving member			
			VSG	*VCA*	*VIG*	*VPG**
SG	*CA*	*IG*	0	1	$1 + \frac{SG}{IG}$	$\frac{SG}{PG}$
IG	*CA*	*SG*	$1 + \frac{IG}{SG}$	1	0	$\frac{IG}{PG}$
SG	*IG*	*CA*	0	$\frac{IG}{IG + SG}$	1	$\frac{IG}{IG + SG} \times \frac{SG}{PG}$
CA	*IG*	*SG*	$\frac{IG}{SG}$	0	1	$\frac{IG}{PG}$
IG	*SG*	*CA*	1	$\frac{SG}{IG + SG}$	0	$\frac{SG}{IG - SG} \times \frac{IG}{PG}$
CA	*SG*	*IG*	1	0	$\frac{SG}{IG}$	$\frac{SG}{PG}$
PG	*CA*	*SG* & *IG*	1	1	1	0

* Revolutions of planet gear about its own center.

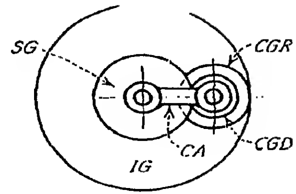


TABLE 31.—COMPOUND SUN-AND-PLANET GEAR VELOCITIES

Symbols for Member Units

Sun gear, *SG*. Internal gear, *IG*. Carrying arm, *CA*.

Compound planet gear { Driver, *CGR*. Driven, *CGD*.

(Symbols also denote number of teeth in, or diameters of, gears)

Function Symbols

Stationary member, *SM*. Driving member, *RM*. Driven member, *DM*. Carrying arm, *CA*.

VSG = velocity, sun gear. *VIG* = velocity, internal gear.

VCA = velocity, carrying arm. *VCPG* = velocity, compound planet gear.

SM	*RM*	*DM*	Rotary speed per revolution of driving member			
			VSG	*VCA*	*VIG*	*VCPG**
SG	*CA*	*IG*	0	1	$\frac{CGR \times SG + CGD \times IG}{CGD \times IG}$	$\frac{SG}{CGD}$
IG	*CA*	*SG*	$\frac{CGR \times SG + CGD \times IG}{CGR \times SG}$	1	0	$\frac{IG}{CGR}$
SG	*IG*	*CA*	0	$\frac{CGD \times IG}{CGR \times SG + CGD \times IG}$	1	$\frac{CGD \times IG}{CGR \times SG + CGD \times IG} \times \frac{SG}{CGD}$
CA	*IG*	*SG*	$\frac{IG \times CGD}{SG \times CGR}$	0	1	$\frac{IG}{CGR}$
IG	*SG*	*CA*	1	$\frac{CGR \times SG}{CGR \times SG + CGD \times IG}$	0	$\frac{CGR \times SG}{CGR \times SG + CGD \times IG} \times \frac{IG}{CGR}$
CA	*SG*	*IG*	1	0	$\frac{SG \times CGR}{IG \times CGD}$	$\frac{SG}{CGD}$
CGR & *CGD*	*CA*	*SG* & *IG*	1	1	1	0

* Revolutions of compound planet gear about its own center.

METHODS OF GEAR PRODUCTION—SECTION XIII

Hob-tooth angle and lead:

$$\tan VHT = \frac{\tan VP}{\cos VHS} \qquad (117)$$

$$LH = \frac{n \times CP}{\cos VHS} \qquad (118)$$

MATERIALS AND HEAT TREATMENT—SECTION XIV

TABLE 32.—FORGED AND ROLLED CARBON STEEL FOR GEARS

Use	Chemical composition, per cent			
	Carbon	Manganese	Phosphorus	Sulphur
Case-hardened........	0.15–0.25	0.40–0.60	0.045 max.	0.05 max.
Untreated............	.25– .50	.50– .80	.045 max.	.05 max.
	.40– .50	.40– .60	.045 max.	.05 max.
Hardened............	.40– .50	.40– .60	.045 max.	.05 max.

TABLE 33.—A.G.M.A. STANDARDS FOR STEEL CASTINGS

Use	Chemical composition, per cent				
	Carbon	Manganese	Phosphorus		Sulphur
			Acid	Basic	
Case-hardened....	0.15–0.25	0.40–0.60	0.06 max.	0.05 max.	0.6 max.
Untreated or hardened..........	.30– .40	.40– .60	.06 max.	.05 max.	.6 max.

HEAT TREATMENTS OF ALLOYED GEAR STEELS

NICKEL STEELS

S.A.E. Steel 2315.

CHEMICAL COMPOSITION, PER CENT

Carbon...........	0.10–0.20	Nickel......................	3.25–3.75
Manganese.......	0.30–0.60	Silicon: basic open hearth.....	0.15–0.30
Phosphorus......	0.04 max.	Silicon: electric and acid open hearth.............	0.15 min.
Sulphur.........	0.05 max.		

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1600–1650°F.

(3) Quench from box in oil.
(4) Reheat to 1500–1550°F.
(5) Quench in oil.
(6) Reheat to 1350–1400°F.
(7) Quench.
(8) Draw at 250–500°F.

S.A.E. Steel 2320.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.15–0.25	Nickel	3.25–3.75
Manganese	0.30–0.60	Silicon: basic open hearth	0.15–0.30
Phosphorus	0.04 max.	Silicon: electric and acid open hearth	0.15 min.
Sulphur	0.05 max.		

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1600–1650°F.
(3) Quench from box in oil.
(4) Reheat to 1500–1550°F.
(5) Quench in oil.
(6) Reheat to 1350–1400°F.
(7) Quench.
(8) Draw at 250–500°F.

S.A.E. Steel 2350 (For gears of large section).

CHEMICAL COMPOSITION, PER CENT

Carbon	0.45–0.55	Nickel	3.25–3.75
Manganese	0.50–0.80	Silicon: basic open hearth	0.15–0.30
Phosphorus	0.04 max.	Silicon: electric and acid open hearth	0.15 min.
Sulphur	0.05 max.		

HEAT TREATMENT

(1) Normalize at 1600–1650°F.
(2) Reheat to 1375–1425°F.
(3) Cool in furnace.
(4) Machine.
(5) Reheat to 1400–1450°F.
(6) Quench in oil.
(7) Draw to required hardness.

Nickel-chromium Steels

S.A.E. Steel 3115.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.10–0.20	Nickel	1.00–1.50
Manganese	0.30–0.60	Chromium	0.45–0.75
Phosphorus	0.04 max.	Silicon: basic open hearth	0.15–0.30
Sulphur	0.05 max.	Silicon: electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1625–1675°F.
(3) Cool in box.
(4) Reheat to 1400–1450°F.
(5) Quench.
(6) Draw at 250–500°F.

S.A.E. Steel 3215.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.10–0.20	Nickel	1.50–2.00
Manganese	0.30–0.60	Chromium	0.90–1.25
Phosphorus	0.04 max.	Silicon: basic open hearth	0.15–0.30
Sulphur	0.045 max.	Silicon: electric and acid open hearth	0.15 min

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1625–1675°F.
(3) Cool in box.
(4) Reheat to 1375–1425°F.
(5) Quench in oil.
(6) Draw at 250–500°F.

S.A.E. Steel 3250.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.45–0.55	Nickel	1.50–2.00
Manganese	0.30–0.60	Chromium	0.90–1.25
Phosphorus	0.04 max.	Silicon: basic open hearth	0.15–0.30
Sulphur	0.045 max.	Silicon: electric and acid open hearth	0.15 min.

S.A.E. Steel 3335.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.30–0.40	Nickel	3.25–3.75
Manganese	0.30–0.60	Chromium	1.25–1.75
Phosphorus	0.04 max.	Silicon: basic open hearth	0.15–0.30
Sulphur	0.045 max.	Silicon: electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1600–1700°F.
(2) Reheat to 1200–1250°F.
(3) Cool slowly in furnace.
(4) Machine.
(5) Reheat to 1425–1475°F.
(6) Quench in oil.
(7) Draw to required hardness.

S.A.E. Steel 3415.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.10–0.20	Nickel	2.75–3.25
Manganese	0.30–0.60	Chromium	0.60–0.95
Phosphorus	0.04 max.	Silicon: basic open hearth	0.15–0.30
Sulphur	0.045 max.	Silicon: electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1600–1650°F.
(3) Cool in box.
(4) Reheat to 1400–1450°F.
(5) Quench in oil.
(6) Draw at 250–500°F.

S.A.E. Steel 3450.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.45–0.55	Nickel	2.75–3.25
Manganese	0.30–0.60	Chromium	0.60–0.95
Phosphorus	0.04 max.	Silicon: basic open hearth	0.15–0.30
Sulphur	0.045 max.	Silicon: electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1550–1650°F.
(2) Reheat to 1250–1300°F.
(3) Cool slowly in furnace.
(4) Machine.
(5) Reheat to 1400–1450°F.
(6) Quench in oil.
(7) Draw to required hardness.

Molybdenum Steels

S.A.E. Steel 4615.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.10–0.20	Nickel	1.50–2.00
Manganese	0.30–0.60	Molybdenum	0.20–0.30
Phosphorus	0.04 max.	Silicon: basic open hearth	0.15–0.30
Sulphur	0.05 max.	Silicon: electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1625–1675°F.

(3) Cool in box.
(4) Reheat to 1475–1525°F.
(5) Quench.
(6) Draw at 250–500°F.

Chromium Steels

S.A.E. Steel 5120.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.15–0.25	Chromium	0.60–0.90
Manganese	0.30–0.60	Silicon, basic open hearth	0.15–0.30
Phosphorus	0.04 max.	Silicon, electric and acid open hearth	0.15 min.
Sulphur	0.05 max.		

HEAT TREATMENT

(1) Normalize at 1600–1700°F.
(2) Carburize at 1650–1700°F.
(3) Cool in box.
(4) Reheat to 1525–1575°F.
(5) Quench.
(6) Draw at 250–500°F.

Chromium-vanadium Steels

S.A.E. Steel 6120.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.15–0.25	Chromium	0.80–1.10
Manganese	0.30–0.60	Vanadium	0.15 min.–0.18 min. desired
Phosphorus	0.04 max.	Silicon, basic open hearth	0.15–0.30
Sulphur	0.045 max.	Silicon, electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Carburize at 1650–1700°F.
(3) Cool in box.
(4) Reheat to 1525–1575°F.
(5) Quench.
(6) Draw at 250–500°F.

S.A.E. Steel 6150.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.45–0.55	Chromium	0.80–1.10
Manganese	0.50–0.80	Vanadium	0.15 min.–0.18 min. desired
Phosphorus	0.045 max.	Silicon, basic open hearth	0.15–0.30
Sulphur	0.05 max.	Silicon, electric and acid open hearth	0.15 min.

HEAT TREATMENT

(1) Normalize at 1650–1750°F
(2) Reheat to 1250–1350°F.
(3) Cool slowly.
(4) Machine.
(5) Reheat to 1525–1625°F.
(6) Quench in oil.
(7) Draw to required hardness.

Silicomanganese Steels

S.A.E. Steel 9260.

CHEMICAL COMPOSITION, PER CENT

Carbon	0.55–0.65	Silicon	1.80–2.20
Manganese	0.60–0.90	basic open hearth	0.15–0.30
Phosphorus	0.045 max.	Silicon electric and acid open hearth	0.15 min.
Sulphur	0.05 max.		

HEAT TREATMENT

(1) Normalize at 1650–1750°F.
(2) Reheat to 1400–1450°F.
(3) Cool slowly
(4) Machine.
(5) Reheat to 1600–1650°F.
(6) Quench in oil.
(7) Draw to required hardness or tests.

BRONZE AND BRASS CASTINGS FOR GEARS

	Per Cent
Copper	86–89
Tin	9–11
Zinc	1– 3
Lead (max.)	0.20
Iron (max.)	0.06

Table 34.—A.G.M.A. Standard Bronzes for Worm-gear Rims

Variety	Chemical composition, per cent				
	Copper	Tin	Phosphorus	Lead	Zinc and impurities
Phosphor-bronze	88–90	10–12	1–3	.....	0.50 max.*
Leaded gun metal	86–89	9–11	0.25 max.	1–2.5	.50 max.

* Includes any lead.

TABLE 35.—A.G.M.A. STANDARDS FOR BUSHINGS AND FLANGES

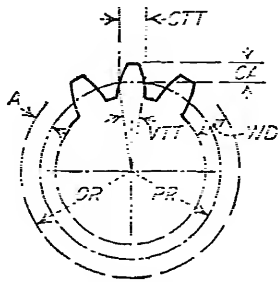
Alloy and use	Chemical composition, per cent								
	Copper	Tin	Lead	Phosphorus	Zinc	Iron	Antimony	Aluminum	Impurities
Bronze bushings	78.5-81.5	9.0-11.0	9.0-11.0	0.05-0.25	0.75 max.	....	....	......	0.25
Brass flanges	83.0-86.0	4.5-5.5	4.5-5.5	4.5-5.5	....	0.35 max.	0.25 max.	None	

Physical Characteristics

Alloy and use	Ultimate strength, lb. per sq. in.	Yield point, lb. per sq. in.	Elongation in 2 in., per cent
Bronze bushings	25,000	12,000	10
Brass flanges	27,000	12,000	10

MEASUREMENT OF GEAR TEETH—SECTION XV

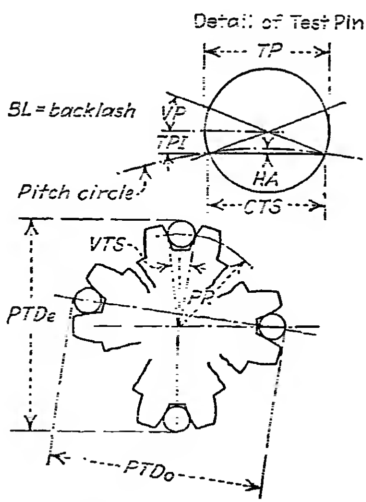
Circular-pitch thickness of gear teeth:



$$CTT = 2PR \times \sin\left(\frac{45 \text{ deg.} \times \text{arc } VTT}{1.5708PR}\right) \tag{119}$$

$$CA = OR - PR \times \cos\left(\frac{45 \text{ deg.} \times \text{arc } VTT}{1.5708PR}\right) \tag{120}$$

Pin measurement of spur gears:

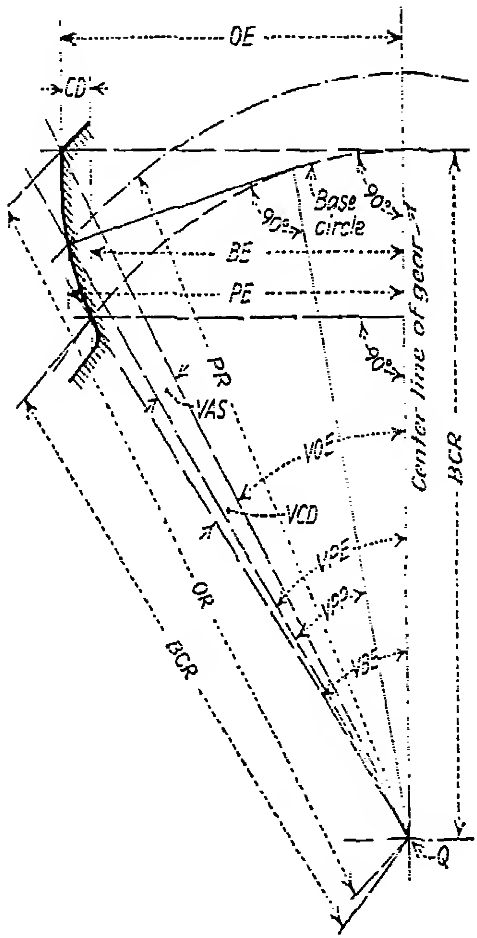


$$TP = \frac{CTS}{\cos VP} \tag{121}$$

$$PD = PTDe - TP - 2(TPI - HA) \tag{122a}$$

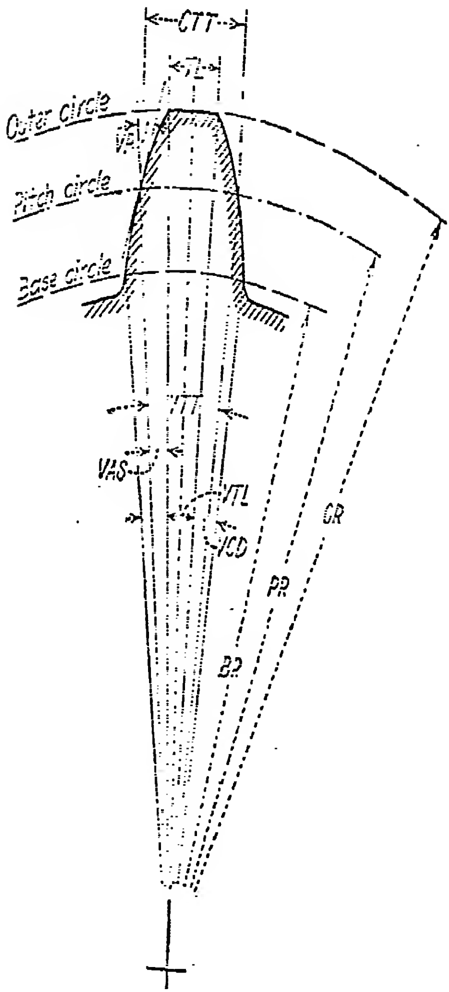
$$PD = \frac{PTDo - TP - 2(TPI - HA)}{\cos \dfrac{90 \text{ deg.}}{N}} \tag{122b}$$

Profile testing:



$$CD = OE - BE \tag{123}$$

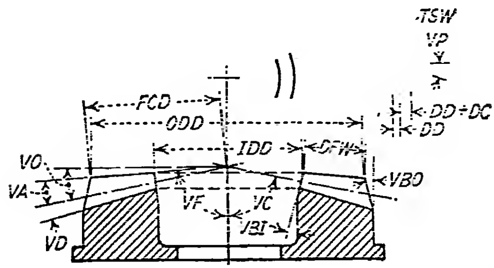
$$VCD = VBE - VOE \tag{123a}$$

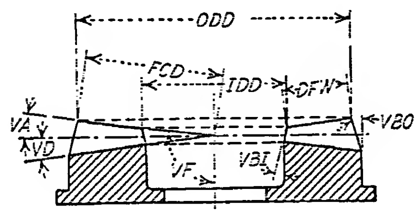


$$TL \text{ (chord)} = OD \times \sin \frac{VTL}{2} \tag{124}$$

ROLLED GEARING—SECTION XVI

Formulas for Bevel-gear Die Rolls





$$DD + DC = \frac{ODD(A + C)}{PD} \tag{125}$$

$$DD = \frac{ODD \times A}{PD} \tag{126}$$

$$DFW = ZE \times FW + 0.125 \tag{127}$$

$$TSW = 2FCD \times \sin \frac{180 \text{ deg.}}{N} \tag{128}$$

VBO and VBI are arbitrary within reasonable limits.

$$N + \text{fraction} = \frac{6.2832PCD}{CP} \quad \text{(bevel die rolls)} \tag{129}$$

$$N = \frac{6.2832PCD}{CP} \quad \text{(crown die rolls)} \tag{129a}$$

N = largest whole number.

$$EPD = \frac{ZE \times N}{DP} \quad \text{(bevel die rolls)} \tag{130}$$

$$= 6.2832PCD \times ZE \quad \text{(crown die rolls)} \tag{130a}$$

$$\sin VC = \frac{ZE \times N}{PCD \times DP} \quad \text{(bevel die rolls)} \tag{131}$$

$$VC = 90 \text{ deg.} \quad \text{(crown die rolls)}$$

$$VO = 90 \text{ deg.} - VC \tag{132}$$

$$VF = VC + VA \quad \text{(bevel die rolls)} \tag{133}$$

$$= 90 \text{ deg.} + VA \quad \text{(crown die rolls)} \tag{133a}$$

$$RD = PD - 2(A + C) \sin VBk \tag{134}$$

$$RCD = \frac{0.5RD}{\sin (VC - VD)} \tag{135}$$

$$FCD = ZE \times RCD + 0.0625 \quad \text{(bevel die rolls)} \tag{136}$$

$$= \frac{0.5ODD}{\cos VA} \quad \text{(crown die rolls)} \tag{136a}$$

$$ODD = 2ZE \times \cos (90 \text{ deg.} - VA) \tag{137}$$

$$IDD = ODD - 2DFW \times \cos (90 \text{ deg.} - VF) \quad \text{(bevel die roll)} \tag{138}$$

$$= ODD - 2DFW \times \cos VA \quad \text{(crown die rolls)} \tag{138a}$$

$$DAC = PD \times \sin VS \quad \text{(helical bevels)} \tag{139}$$

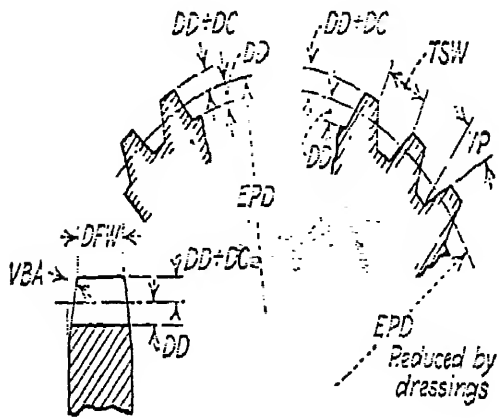
$$DDAC = ZE \times DAC \quad \text{(helical and herringbone bevels)} \tag{139a}$$

$$NTS = TSW \times \cos VS \quad \text{(helical and herringbone bevels)} \tag{140}$$

$$PC = ZE\sqrt{(PD)^2 - (PD + PDI)PFW} \tag{141}$$

$$PTS = \frac{NTS \times PC}{ODD} \tag{142}$$

Formulas for Spur-type Die Rolls



$$N = 24DP + 1 \tag{143}$$

$$EPD = \frac{ZE \times N}{DP} \tag{130}$$

$$DFW = ZE \times FW + 0.125 \tag{127}$$

$$DD = 0.3183CP \times ZE \tag{126a}$$

$$DD + DC = 0.3683CP \times ZE \tag{125a}$$

$$GA = 1.1 \times CP \quad \text{(helical and herringbone gears)} \tag{144}$$

$$\tan VP = \frac{GA}{FW} \quad \text{(helical gears)} \tag{145}$$

$$= \frac{2GA}{FW} \quad \text{(herringbone gears} \tag{145a}$$

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